STATEMENTS & SETS

Main points:

- A Sentence which is either true or false but not both is called statement.
- If two simple statements are combined with the connectives then the statements so formed are called compound statements.
- conditional : $p \Rightarrow q$
- conditional : $p \Rightarrow q$
- Inverse : $(\sim p) \Rightarrow (\sim q)$
- Contrapositive : $(\sim q) \Rightarrow (\sim p)$
- A compond statement which contains only 'T' in the last column of it is truth table is called tantology.
- A compond statement which contains only 'F' in the column of it is truth table is called contradiction.
- '$\forall$' is the symbol for iniversal quantifier.
- '$\exists$' is the symbol for Existential quantifier.
- A and B are disjoint sets i) and ii) only
  i) $A \cap B = \phi$ ii) $n(A \cap B) = 0$ iii) $n(A \cup B) = n(A) + n(B)$
- $A \subset B$ and $B \subset C$ then $A \subset C$
- If $A \subset B$ and $B \subset C$ then $A = B$
- $x \in (A \cup B) \Leftrightarrow x \in A \vee x \in B$
- $x \notin (A \cup B) \Leftrightarrow x \notin A \wedge x \notin B$
- $x \in (A \cap B) \Leftrightarrow x \in A \wedge x \in B$
- $x \notin (A \cap B) \Leftrightarrow x \notin A \vee x \notin B$
- $x \in (A - B) \Leftrightarrow x \in A \wedge x \notin B$
- $x \notin (A - B) \Leftrightarrow x \notin A \vee x \in B$

- $x \in A \Rightarrow x \in B$ then $A \subset B$
- $x \in B \Rightarrow x \in A$ then $B \subset C$
- Basic Theorem
- $A \subset B \ \& \ B \subset C \Leftrightarrow A = B$

Laws of algebra of statement in Sets:

Name of the Law	Sets	Statement
Idempotent Laws	$A \cup A = A$; $A \cap A = A$	$p \vee p = p$; $p \wedge p = p$
Associative Laws	$A \cup (B \cup C) = (A \cup B) \cup C$	$p \vee (q \vee r) = (p \vee q) \vee r$
	$A \cap (B \cap C) = (A \cap B) \cap C$	$p \wedge (q \wedge r) = (p \wedge q) \wedge r$
Commutative laws	$A \cup B = B \cup A$,	$p \vee q = q \wedge p$
	$A \cap B = B \cap A$,	$p \wedge q = q \wedge p$
Distributive law	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$p \wedge (q \vee r) = (p \wedge q) \vee (q \wedge r)$
Identity law	$A \cup \phi = A$,	$p \vee f = p$, $p \vee t = t$
	$A \cap \mu = A$,	$p \wedge f = f$, $p \wedge t = p$,
complement law	$A \cup A' = \mu$, $A \cap A' = \phi$	$p \vee (\sim p) = t$, $p \wedge (\sim p) = f$
	$(A')' = A$, $\mu' = \phi$, $\phi' = \mu$	$\sim(\sim p) = p$, $\sim t = f$, $\sim f = t$,
Demograms law	$(A \cup B)' = A' \cap B'$	$\sim(p \vee q) = (\sim p) \wedge (\sim q)$
	$(A \cap B)' = A' \cup B'$	$\sim(p \wedge q) = (\sim p) \vee (\sim q)$
	$A - (B \cup C) = (A - B) \cap (A - C)$	
	$A - (B \cap C) = (A - B) \cup (A - C)$	

4 Marks Questions

1. prove that $A - (B \cup C) = (A - B) \cap (A - C)$ for any three non-empty sets A, B and C.

Proof : In order to prove $A - (B \cup C) = (A - B) \cap (A - C)$, according to Basic Theorem we have to prove that

i) $A-(B \cup C) \subset (A-B) \cap (A-C)$

ii) $(A-B) \cap (A-C) \subset A-(B \cup C)$

i) Let $x \in A-(B \cup C)$

$\Rightarrow x \in A \wedge x \notin (B \cup C)$

$\Rightarrow x \in A \wedge (x \notin B \wedge x \notin C)$

$\Rightarrow (x \in A \wedge x \notin B) \wedge (x \in \wedge x \notin C)$

$\Rightarrow x \in (A-B) \wedge x \in (A-C)$

$\Rightarrow x \in (A-B) \cap (A-C)$

$\therefore A-(B \cap C) \subset (A-B) \cap (A-C)$ ------(1)

ii) Let $x \in (A-B) \cap (A-C)$

$\Rightarrow (x \in A \wedge x \notin B) \wedge (x \in \wedge x \notin C)$

$\Rightarrow x \in A \wedge (x \notin B \wedge x \notin C)$

$\Rightarrow x \in A \wedge x \notin (B \cup C)$

$\Rightarrow x \in (A-B) \vee x \in (A-C)$

$\Rightarrow x \in A-(B \cup C)$

$\therefore (A-B) \cap (A-C) \subset A-(B \cup C)$ ------(2)

From (1) & (2)

$A-(B \cup C) = (A-B) \cap (A-C)$

2. $(p \cap q)' = p' \cup q'$

Proof: In order to prove $(p \cap q)' = p' \cup q'$

according to basic theorem, we have to prove that

i) $(p \cap q)' \subset p' \cup q'$

i) $p' \cup q' \subset (p \cap q)'$

i) Let $x \in (p \cap q)'$

$\Rightarrow x \notin (p \cap q)$

$\Rightarrow x \notin p \vee x \notin q$

$\Rightarrow x \in p' \vee x \in q'$

$\Rightarrow x \in p' \cup q'$

$\therefore (p \cap q)' \subset p' \cup q'$.....(1)

ii) Let $x \in p' \cup q'$

$\Rightarrow x \in p' \vee x \in q'$

$\Rightarrow x \notin p \vee x \notin q$

$\Rightarrow x \notin (p \cap q)'$

$\therefore p' \cup q' \subset (p \cap q)'$.....(2)

From (1) & (2) $(p \cap q)' = p' \cup q'$

3. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for any three sets A, B, C

Proof: In order to prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

according to 'Basic Theorem' we have to prove that

i) $A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$

i) $(A \cap B) \cup (A \cap C) \subset A \cap (B \cup C)$

i) Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A \wedge x \in (B \cup C)$

$\Rightarrow x \in A \wedge (x \in B \vee x \in C)$

$\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$\Rightarrow x \in (A \cap B) \vee x \in (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$ ..... (i)

ii) Let $(A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \vee (A \cap C)$

$\Rightarrow (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$\Rightarrow x \in A \wedge x \in (B \cup C)$

$\Rightarrow x \in A \cap (B \cup C)$

$\therefore (A \cap B) \cup (A \cap C) \subset A \cap (B \cup C)$ .....(ii)

From (i) & (ii)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Prove $A - (B \cap C) = (A - B) \cup (A - C)$ for any three non-empty sets A, B, and C.

Proof: In order to prove $A - (B \cap C) = (A - B) \cup (A - C)$

according to basic theorem, we have to prove that

i) $A - (B \cap C) \subset (A - B) \cup (A - C)$ ii) $(A - B) \cup (A - C) \subset A - (B \cap C)$

i) Let $x \in A - (B \cap C)$

$\Rightarrow x \in A \wedge x \notin (B \cap C)$

$\Rightarrow x \in A \wedge (x \notin B \vee x \notin C)$

$\Rightarrow (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C)$

$\Rightarrow x \in (A - B) \vee x \in (A - C)$

$\Rightarrow x \in (A - B) \cup (A - C)$

$\therefore A - (B \cap C) \subset (A - B) \cup (A - C)$ --------- (I)

ii) $x \in (A - B) \cup (A - C)$

$\Rightarrow x \in (A - B) \vee x \in (A - C)$

$\Rightarrow (x \in A \wedge x \notin B) \vee (x \notin A \wedge x \notin C)$

$\Rightarrow x \in A \wedge (x \notin B \vee x \notin C)$

$\Rightarrow x \in A \wedge x \notin (B \cap C)$

$\Rightarrow x \in A - (B \cap C)$

$\therefore (A - B) \cup (A - C) \subset A - (B \cap C)$ --------- (II)

From (I) and (II)

$A - (B \cap C) = (A - B) \cup (A - C)$

5. prove that $(A \cap B)' = A' \cup B'$

proof : In order to prove $(A \cap B)' = A' \cup B'$,

According to "Basic Theorem" we have to prove that

i) $(A \cap B)' \subset A' \cup B'$ ii) $A' \cup B' \subset (A \cap B)'$

i) Let $x \in (A \cap B)'$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \in A' \vee x \in B'$

$\Rightarrow x \in A' \cup B'$

$\therefore (A \cap B)' \subset A' \cup B'$ -------(1)

ii) Let $x \in A' \cup B'$

$\Rightarrow x \in A' \vee x \in B'$

$\Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \notin (A \cap B)$
$\Rightarrow x \in (A \cap B)'$
$A' \cup B' \subset (A \cap B)'$ --------(2)
From (1) and (2)
$(A \cap B)' = A' \cup B'$

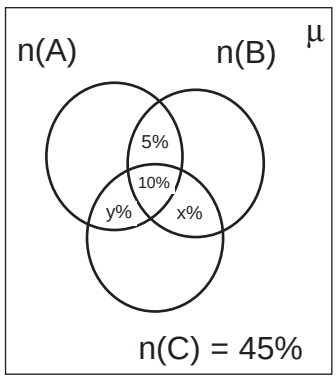
6. Prove that $(A \cup B)' = A' \cap B'$

proof : In order to proove $(A \cup B)' = A' \cap B'$
according to basic theorem we have to prove that
i) $(A \cup B)' \subset A' \cap B'$
ii) $A' \cap B' \subset (A \cup B)'$

i) Let $x \in (A \cup B)'$
$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A \wedge x \notin B$
$\Rightarrow x \in A' \wedge x \in B'$
$\Rightarrow x \in A' \cap B'$
$\Rightarrow (A \cup B)' \subset A' \cap B'$ ---------- (I)

ii) Let $x \in A' \cap B'$
$\Rightarrow x \in A' \wedge x \in B'$
$\Rightarrow x \notin A \wedge x \notin B$
$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \in (A \cup B)'$
$A' \cap B' \subset (A \cup B)'$ ---------- (II)
From (I) and (II)
$(A \cup B)' = A' \cap B'$

7. In a city they are three major news papers A,B,C, of which atleast two are read by 35% of the population . It is known that news paper C is read by 45% of the populaton and the newspaper A and B. are seen by 15 % and that all the Three are read by 10%. what present of read only news paper C ?



Sol : $n(A \cap B \cap C) = 10\%$ $n(A \cap B) = 15\%$
The news papers, at least two are read by 35%
i.e., x% + y% + 5% + 10% = 35%
$\Rightarrow$ x% + y% + 15% = 35%
$\Rightarrow$ x% + y% = 35% – 15%
$\Rightarrow$ x% + y% = 20% --------(1)
The population who read only newspaper C.
i.e., only n(C) = n(C) – [x% + y% + 10%]
= 45% –[x% + y% + 10%]
= 45% –[20% + 10%] [$\therefore$ from (1)]
= 45% – 30%
= 15%

2 Marks Questions

1. Define Conjunction and write its truth table?

A. If two simple statements are combined with the connective 'AND' then the compond statement formed is called **conjunction.**

Conjuction connective is '$\wedge$'

Truth table of conjunction

P	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Define Disjunction and write its truth table?

A. If two simple statemets are combined with the connective 'OR' then the compond statement formed is called '**Disjunction**'

Disjunction connective is '$\vee$'

Truth table of Disjunction

P	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3. Define conditional and write its truth table?

A. If two simple statements are comditioned with the connective 'IF-THEN' then the compond statement formed is called **conditional.**

The connective symbol of conditional is "$\rightarrow$" or "$\Rightarrow$"

Truth table of conditional.

P	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

4. Define Bi- conditional and write its truth table?

A. If two simple statements are combined with the connective "IF AND ONLY IF" or "IFF" then the compond statement formed is called **'Bi-Conditional'**

The connective symbol of bio-conditional is "$\leftrightarrow$" "$\Leftrightarrow$"

Truth table of Bi -Conditional

P	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

5. Prove that $(A')' = A$

A. In order to prove $(A')' = A$, according to Basic Theorem. We have to prove that

(i) $(A')' \subset A$ ii) $A \subset (A')'$

i) Let $x \in (A')'$

$\Rightarrow x \notin A'$

$\Rightarrow x \in A$

$\therefore (A')' \subset A$ -------(i)

ii) Let $x \in A$

$\Rightarrow x \notin A'$

$\Rightarrow x \in (A')'$

$\therefore A \subset (A')'$ -------(ii)

From (i) and (ii)

$(A')' = A$

6. Prove that $A \cap B' = A - B$

proof: In order to prove $A \cap B' = A - B$, according to "Basic theorem" we have to prove that

$A \cap B' \subset A - B$ $\quad$ $A - B \subset A \cap B'$

i) Let $x \in A \cap B'$

$\Rightarrow x \in A \wedge x \in B'$

$\Rightarrow x \in A \wedge x \notin B$

$\Rightarrow x \in A - B$

$A \cap B' \subset A - B$ ---------- (i)

ii) Let $x \in A - B$

$\Rightarrow x \in A \wedge x \notin B$

$\Rightarrow x \in A \wedge x \in B'$

$\Rightarrow x \in A \cap B'$

$A - B \subset A \cap B'$ ---------- (ii)

From (i) and (ii) $A \cap B' = A - B$

7. Prove that $\sim (p \Rightarrow q) = p \wedge (\sim q)$

A :

P	q	$(p \Rightarrow q)$	$\sim (p \Rightarrow q)$	$\sim q$	$p \wedge (\sim q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

$\therefore \sim (p \Rightarrow q) = p \wedge (\sim q)$

8. Write the convesse inverse and contra positive of If a polygon is a square then it is a rectangle

Sol : conditional : If a polygon is a square then it is a rectangle.

convesse : If a polygn is a rectangle, then it is a square

inverse : If a polygon is not a square , then it is not a rectangle

contra positive : If a polygon is not a rectangle, then it is not a square.

1 Mark Questions

1. Define equivalence of statements?

A. Two statements are said to be logically equivalent if the last columns of their truth tables are identical.

2. Prove that $\sim(\sim p) \equiv p$

A.

P	~p	~(~p)
T	F	T
F	T	F

3. Mention the symbols of universal quantities and existential quantities.

A. The symbol of universal quantities is $\forall$

The symbol of Existential quantities is $\exists$

4. If $n(A \cup B) = 51, n(A) = 20, n(B) = 44$, find $n(A \cap B)$

Sol : $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$51 = 20 + 44 - n(A \cap B)$

$51 = 64 - n(A \cap B)$ $n(A \cap B) = 64 - 51$, $n(A \cap B) = 13$

5. If $n(A \cup B) = 50, n(A) = 21, n(A \cap B) = 14$, find $n(B)$

Sol : $n(A \cup B) = 50$

$n(A) = 21$

$n(A \cap B) = 14$

$n(A \cup B) = n(A) + n(B) + n(A \cap B)$

$50 = 21 + n(B) - n(A \cap B)$

$50 = 7 + n(B)$

$50 - 7 = n(B)$

$n(B) = 43$

6. If A = {all primes less than 20}, B = {0,1,2,3,4,5,6,7,8,9} then find $A \cap B$

Sol : A { 2,3,5,7,11,13,17,19}

B= { 0,1,2,3,4,5,6,7,8,9}

$A \cap B$ = { 2,3,5,7,11,13,17,19} $\cap$ { 0,1,2,3,4,5,6,7,8,9}

= { 2, 3, 5, 7}

7. Write the set builder form of A = { 1,8, 27, 64, 125}

sol : A = { 1,8, 27, 64, 125}

A= { 1^3, 2^3, 3^3, 4^3, 5^3}

Set builder form of A = { $n^3 / x \in n$, $x \leq 5$}

8. Show that $A \cup B = \phi$ implies $A = \phi$ and $B = \phi$

Sol: we have $A \subset A \cup B$, $B \subset A \cup B$,

Given that $A \cup B = \phi$

$A \subset A \cup B \Rightarrow A \subset \phi$

also we know that $\phi \subset A$

$\therefore$ A = ϕ

Similerly we can show that B = ϕ

9. If A$\subset$ B, then show that B$\subset$ A, then show that B'$\subset$A'

Given that A$\subset$B

To Prove B'$\subset$A'

Let $x \in B' \Rightarrow x \notin B$

$\Rightarrow x \notin A \Rightarrow x \in A'$

$\therefore$ B' $\subset$ A'

Fill in the Blanks

1. If A $\subset$ B then A$\cup$B = ..., A$\cap$B = ...
2. If A $\subset$ B and B $\subset$ C then .....
3. ~(p$\Leftrightarrow$q) = ... or .....
4. A conditional is equalent to it's .....
5. The symbol for existential quantifier is .....
6. A true statement can't imply a ..... statement.
7. The set builder form of A = { 1, 1/2, 1/3, 1/4, 1/5} is .....
8. A $\subset$ B and B $\subset$ C $\Leftrightarrow$ A = B is known as .....
9. A$\cap$B' = ...

Answers

1) B, A	2) A= B	3) (~p) $\Leftrightarrow$ q) or p $\Leftrightarrow$ (~q)
4) contrapositive	5) $\exists$	6).....
7) A= {1/x/ $x \in N, x \leq 3$	8) basic theorem	9) A–B

Multiple choice

1. ~(p$\Leftrightarrow$q) = ...
   a) p $\vee$ (~q) b) p $\wedge$ (~q) c) (~p) $\wedge$q d) (~p) $\vee$ q
2. converse of p $\Rightarrow$q is .....
   a) (~p) $\Rightarrow$ (~q) b) p $\Rightarrow$ ~q c) q $\Rightarrow$ p d) (~q) $\Rightarrow$ (~p)
3. p $\vee$ (q$\wedge$r) = ...
   a) (p$\vee$q) $\wedge$ (p$\vee$r) b) (p$\vee$q)$\vee$(p$\vee$r) c) (p$\vee$q)$\wedge$r d) p $\wedge$ (q$\vee$r)
4. Identity set over the intersection of sets is .....
   a) ϕ b) μ c) both d) None
5. A and B are disjoint sets then n (A$\cap$B) = ...
   a) 0 b) 1 c) 2 d) 3
6. p $\wedge$ p $\equiv$p is ..... law
   a) commutative b) idenpotent c) Demorgans d) Distributive
7. p $\vee$ ~p is a = ...

a) Contradiction b) Tautology
c) neither contradiction nor tautology d) None

8. If n(A) = 6 then the number of proper subsets of A are .....
a) 30 b) 32 c) 33 d) 31

9. If n(A–B) = 12, n (B–A) = 15 , n(A∩B) = 7 then n (A∪B) = ...
a) 30 b) 32 c) 33 d) None

10. In series network p∧q, the current flows only when .....
a) p is closed, q is closed b) p is opened, q is opened
c) p is closed, q is opened d) p is opened, q is closed

Answers

1) B 2) C 3) A 4) B 5) A

6)B 7) B 8) D 9) D 10) A