Functions

Main ponts :

- A relation in which no two ordered pairs have the same first co-ordinate is called a function
  (or)
  f : A→B is said to be a function if ∀ x ∈ A there exist unique y∈ B Such that f(x)= y.
- n(A) = p, n(B) = q then
  i) n (AxB) = pq
  ii) number of relations from A to B are 2^{pq}
  iii) number of proper district relations from A to B are $2^{pq}–1$
  iv) number of functions from A to B are q^p
- In f: A → B, A is called Domain, B is called condomain. f(A) is called Range.
- A function is said to be bijection if it is both one-one and onto.
- Identity function is its own inverse
- Any line drawn Paralel to Y-axis i.e Perpendicler to x–axis should not cut the given graph of a relation more than once then the graph of relation is a function.

4 Marks Questions

1. Let f,g,h are functions such that f(x) = x+2, g(x) = 3x–1, h(x) = 2x then show that ho(gof) = (hog)of?

Sol : f (x) = x+2, g(x) = 3x–1, h(x)= 2x

L.H.S. ho(gof) :

consider gof = gof (x)
= g [f(x)] = g [x +2]
= 3(x+2) –1
= (3x+6) –1
= 3x+5

Now ho(gof) = ho(gof) (x)
= h [gof (x)]
= h (3x+5)
= 2(3x+5)
= 6x+10 ..... (i)

R.H.S : (hog)of

Consider hog = hog (x)
= h [g(x)]
= h [3x–1]
= 2(3x–1)
= 6x–2

Now (hog)of = (hog)of(x)
= (hog)[f(x)]
= (hog)(x+2)
= 6 (x+2) –2
= 6x+12–2

= 6x+ 10 ..... (ii)

From (i) & (ii)

ho(gof) = (hog)of

2. f : R→R , g : R→R , h : R →R and f(x) = x–1 g(x) = x^2 –2, h(x) = x^3 – 3 where x∈R then find i) (fog)oh ii) fo(goh) iii) what do you notice

Sol : f : R→R, g : R→R , h : R →R

f(x) = x–1, g(x) = x^2 –2, h(x) = x^3 –3

i) (fog)oh

consider fog = fog(x) = f [g(x)]

= f (x^2 –2)

= (x^2 – 2)–1

= x^2 –3

Now (fog)oh = (fog)oh (x) = (fog) [h(x)]

= (fog)(x^2 –3)

= (x^3–3)^2 –3

= x^6 – 6x^3 + 9 –3

= x^6 – 6x^3+6 ..... (i)

ii) fo(goh)

consider goh = goh (x) = g[h(x)]

= g(x^3–3)

= (x^3–3)^2 – 2

= (x^6 – 6x^3+ 9)–2

= x^6 –6x^3+ 7

Now fo(goh) = fo(goh)(x)

= f [goh(x)]

= f [x^6 –6x^3+7]

= x^6–6x^3+7–1

= x^6–6x^3 +6 ..... (ii)

From (i) & (ii) (fog)oh = fo(goh)

3. Let f,g, h are functions that f(x) = x +2, g(x) = 3x–1 and h(x) = 2x then show that ho(gof) = (hog)of

Sol : f(x) = x+2 , g(x) = 3x–1, h(x) = 2x

LHS : ho(gof)

Let us take gof = gof(x) = g [f(x)]

= g [x+2]

= 3 (x+2)–1

= 3x+ 6 –1

= 3x+5

ho(gof) = ho(gof) (x) = h [gof (x)]

= h [3x+ 5]

= 2 (3x+5)

= 6x+ 10 --------- (i)

RHS : (hog)of

Let us take hog = hog (x) = h [g(x)]

= h [3x–1]

= 2(3x–1)

= 6x–2

(hog)of = (hog)of(x) = (hog) [f(x)]

= (hog) [x+5]

= 6(x+2) –2

= 6x+10 --------- (ii)

From (i) and (ii) ho(gof) = (hog)of

4. f : R–{2}→R be defined by f(x) = $\frac{x^2-4}{x-2}$ verify f is bijection (or) not.

Sol: f : R –{2} → R

$f(x) = \frac{x^2-4}{x-2} = \frac{x^2-2^2}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2$

i) $\forall x_1, x_2, \in$ **R–{2}**

$f(x_1) = f(x_2)$

$\Rightarrow x_1+2 = x_2+2$

$\therefore$ f is one – one ..... (i)

ii) Let y= f(x) = x+2

$\Rightarrow$ y= x+2

$\Rightarrow$ y– 2 = x

$\Rightarrow$ x = y–2

Here for y=4 $\nexists$ x=2 in R–{2}

$\therefore$ f is not an onto

$\therefore$ f is not a bijecton

5. Let 'f ' be given by f(x) = x+2 and f has the domain { x/ 2 ≤ x ≤ 5}. find f^{-1} and it's domain and range.

Sol:

Domain of f = { x/ 2 ≤ x ≤ 5}

Range of f = {x+2/ 2 ≤ x ≤ 5}

= { y / 45 x+2 ≤ 7}

= { y / 4 ≤ y≤ 7 }

f(x) = x+2

Let y = f(x) = x+2

$\Rightarrow$ y = f(x)

$\Rightarrow$ $f^{-1}(y) = x$ --------- (i)

y= x+2

$\Rightarrow$ y–2 = x --------- (ii)

From (i) and (ii)

$f^{-1}(y) = y-2$

$\Rightarrow \quad f^{-1}(x) = x-2$

$\therefore$ The doman of $f^{-1} = \{ y/ \ 4 \le y \le 7\}$

$\therefore$ The range of $f^{-1} = \{x/ \quad 2 \le x \le 5\}$

2 Marks Questions

1. f : R–{3}→ R defined by f(x) = $\frac{x+3}{x-3}$ then show that $f\left(\frac{3x+3}{x-1}\right) = x$

Sol : $f:R-\{3\} \to R$

$$f(x) = \frac{x+3}{x-3}$$

$$\text{L.H.S} = f\left(\frac{3x+3}{x-1}\right) = \frac{\frac{3x+3}{x-1}+3}{\frac{3x+3}{x-1}-3}$$

$$= \frac{3x+3+3(x-1)}{\cancel{x-1}} \Big/ \frac{3x+3-3(x-1)}{\cancel{x-1}}$$

$$= \frac{3x+\cancel{3}+3x-\cancel{3}}{\cancel{3x}+3-\cancel{3x}+3} \qquad = \frac{6x}{6} = x$$

2. f : R–{2}→ R be defined by f (x) = $\frac{2x+3}{x-2}$ then show that $f = \left(\frac{2x+3}{x-2}\right) = x$

Sol : f: R – {2}→ R be defined by f (x) = $\frac{2x+1}{x-2}$

$$f\left(\frac{2x+1}{x-2}\right) = \frac{2\left(\frac{2x+1}{x-2}\right)+1}{\frac{2x+1}{x-2}-2}$$

$$= \frac{\frac{4x+2}{x-2}+1}{\frac{2x+1}{x-2}-2}$$

$$= \frac{\frac{4x+\cancel{2}+x-2}{\cancel{x-2}}}{\frac{2x+1-2(x-2)}{\cancel{x-2}}} = \frac{5x}{2x+1-2x+4} = \frac{5x}{5} = x \quad \text{R.H.S}$$

3. f(x) = x+2, g (x) = x^2–x–2 then find $\frac{g(1)+g(2)+g(3)}{f(-4)+f(-2)+f(2)}$

Sol: f(x) = x + 2, g(x) = x^2– x – 2

g (x) = x^2 – x – 2

g (1) = 1^2 – 1 – 2 = – 2

g (2) = 2^2 – 2 – 2 = 0

g (3) = 3^2 – 3 – 2 = – 4

f (– 4) = – 4 + 2 = – 2

f (– 2) = – 2 + 2 = 0

f (2) = 2 + 2 = 4

then $\frac{g(1)+g(2)+g(3)}{f(-4)+f(-2)+f(2)} = \frac{-2+0+4}{-2+0+4} = \frac{2}{2} = 1$

5. if f(x) = $\frac{x+1}{x-1}$ **x ≠1 then show that f(x) + f(1/x) = 0**

Sol : f(x) = $\frac{x+1}{x-1}$ (x ≠1)

$\Rightarrow f(1/x) = \frac{\frac{1}{x}+1}{\frac{1}{x}-1} = \frac{\frac{1+x}{x}}{\frac{1-x}{x}}$

$= \frac{1+x}{1-x}$

L.H.S f(x) + f (1/x) = $\frac{x+1}{x-1} + \frac{1+x}{1-x}$

$= \frac{(x+1)(1-x)+(1+x)(x-1)}{(x-1)(1-x)} = \frac{x-x^2+1-x+x-1+x^2-x}{(x-1)(1-x)}$

$= \frac{0}{(x-1)(1-x)} = 0$

6. f : (x) =x^2+2x+3, then find $\frac{f(x+h)-f(x)}{h}$ **, h ≠ 0**

Sol: f(x)= x^2 +2x+3

f(x+h) = $(x+h)^2$ + 2 (x+h)+3

= x^2+ 2xh+ h^2+2x+2h+3

$= \frac{f(x+h)-f(x)}{h}$

$$= \frac{[x^2 + 2xh + h^2 + 2x + 2h + 3] - [x^2 + 2x + 3]}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 2x + 2h + 3 - x^2 - 2x - 3}{h}$$

$$= \frac{h(2x + h + 2)}{h}$$

$= 2x+h+2$

7. If f(x) = x+2, g(x) = x^2–3, Find fog and gof. what do you notice?

Sol: f(x) = fog (x)

= f [g(x)]
= f [x^2–3]
= (x^2+3)+2
= x^2–1
gof = gof (x)
= g [f(x)]
= g [x+2]
= $(x+2)^2$ –3
= x^2 +4x +4 –3
= x^2+4x+1
we noticed that gof ≠ fog

1 Marks Questions

1. Define onto functions

A. f: A → B is said to be on to function if ∀ y∈ B, ∃ x∈ A such that f(x) = y

(or)

f: A → B is said to be onto function if the Range = Co-domain i.e. f(A) = B.

Ex: 1) f: R → R, f(x) = 5x + 4 is onto function
2) f: R → R f(x) = x^3 is onto function.
3) f: R → R f(x) = x^2 is not an onto function.

2. Define zeros of functions

A. f: R → R, and for which x∈ A, f(x)= 0 then such x is called zeros of function.

3. Define identity functions

A. I: A→A is said to be an Identify function, " x∈ A, I(x) = x.
f: R→R, f is identify function then $f^{-1}(7) = 7$

4. Define equal functions

A. f and g are equal functions if i) They are defined on the same domain and ii) f(x) = g(x)

Ex: 1. f: R – {1} → R, f(x) = x+1

g: R – {1}$\to$ R, $g(x) = \frac{x^2 - 1}{x - 1}$

Here f = g.

5. Define Real functions

A. f: A $\to$ B, and A $\subseteq$ R, B $\subseteq$ R, then f is called Real function.

6. f(x) = x+2, g(x) = x^3 - 3 then find fog (x)

A. fog(x) = f [g(x)] = f(x^2 –3) = (x^2 – 3)+2 = x^2–1

7. Define constant function

A. f: A$\to$B is said to be a constant function if $\forall$ x$\in$ A, f(x) = k when k$\in$B.

Note: Range of a constant function is a singleton set.

Ex: f: R$\to$R, f(x) = 100 and f is a constant function then f(2) = 100

Multiple Choice Questions

1. f : N$\to$N, f (x) = x+1 then the range of f is ---------- ( )
   a) { 0, 1, 2} b) {2, 3, 4,----}
   c) {1, 2, 3, ---} d) {2, 3, 4}

2. f(x) = x^2 – 3x+2, then f(–2) = ---------- ( )
   a) 12 b) 4 c) –12 d) 0

3. A = {3,4} B= {4, 5, 6} then f(2) = ---------- ( )

4. f : R$\to$R, f (x) = x then f(2) = ---------- ( )

5. f = { (x,3) / x$\in$N} is a ---------- function ( )
   a) one to one b) on to c) constant d) identity

6. f : R$\to$R, and f(x) = 3x+2 then f^{-1} = ---------- ( )
   a) 3x–2 b) 2x+3 c) $\frac{x-2}{3}$ d) 2x+3

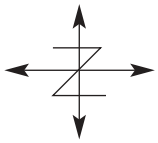
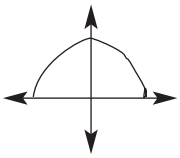
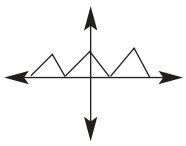
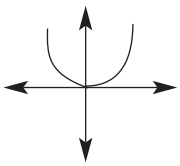
7. f : A $\to$B, g : B$\to$C then ( )
   a) gof : A $\to$ C b) gof : C $\to$A c) gof : A $\to$ B d) gof f: B $\to$ A

8. f(x) = 2x–1, g(x) = x then gof (3) = ---------- ( )
   a) 3 b) 5 c) 7 d) 9

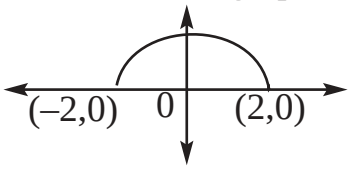
9. If $f(x) = x^2 + 4x - 12$ then zero's of f are ---------- ( )

a) { –6, 2} b) { 6, 2} c) { 3,2} d) { –3,–2}

10. which of the following is not a relation ( )

a)  b)  c)  d) 

11. The zeros of the graph of the following relation is ---------- ( )



a) { 0,0} b) {-2, 2} c) { –2, 2, 0} d) { 0, –1}

12. f(x) = 2x–3 then zero of the function is ( )

a) 2/3 b) 3/2 c) 0 d) -3/2

13. ---------- function it's own inverse

a) inverse function b) identity functiion c) ---------

14. The range of constant function is --------- set ( )

a) Empty set b) fi c) Informate set
d) ---------

15. f : A→B and A & B are subjects of the set of real numbers then R is ----- ( )

a) Real vasiable function b) Real valued function
c) Real function d) None

Fill in the blanks

1. If f = {(4,6), (5,7), (6,8), (7,9) then $f^{-1}(8)$ = ...

2. If y = f(x) = x+5 then $f^{-1}(x)$ = ...

3. $f^{-1}(y) = y-3$ then f(y) = ...

4. If I (x)= x, $\forall$ x$\in$ A then I is ..... function

5. f= { (1,2), (2,3), (3,4), (4,1)} then fof = ...

6. f N→N, $f(x) = \frac{12}{x-3}$ $x \neq 3$ then domain of f = ...

7. The range of f(x)= 3 is .....

8. The zero of f(x) = 3x+5 is .....

9. If f(x) x^2+x+4, then 24 is the image of = ...

10. If f(x) = $\frac{x(x-1)}{2}$ then f(x+2) =.....

Key

1) 6 2) x–5 3) 7 4) identity 5) {(1,3),(2,4),(3,4) 2,4)}

6) {4,5,6,7,9,15} 7) {3} 8) –5/3 9) {–5,4} 10) $\frac{x^2+2x+2}{2}$

Multiple choice:

1) b 2) a 3) d 4) b 5) c 6) c 7) a
8) b 9) a 10) a 11) b 12) b 13) c 14) b
15) c