Real Numbers

Main Points

- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $a+b+c=0 \Rightarrow a^3+b^3+c^3= 3abc$
- $\sqrt[n]{m\sqrt{a}} = \sqrt[mn]{a}$
- $|x| = a$ solution is $x=a$ or $x = -a$
- $|x| = 0$ solution is $x= 0$
- $|x| < a$ solution is $-a< x < a$
- $|x| \leq a$ solution is $-a \leq x \leq a$
- $|x| > a$ solution is $x > a$ or $x \leq -a$
- $|x| \geq a$ solution is $x \geq a$ or $x \leq -a$
- $\lim\limits_{x\to a} \frac{x^n - a^n}{x - a} = n.a^{n-1}$
- $\lim\limits_{x\to a} \frac{x^m - a^m}{x^n - a^n} = m/n\ .a^{m-n}$
- The limiting positon of secant of a circle is tangent.

4 Marks Questions

1. If $a= x+\sqrt{x^2+1}$ then show that $x = \frac{1}{2}\left[a - a^{-1}\right]$

Sol : $a = x+\sqrt{x^2+1}$

$\Rightarrow a- x = \sqrt{x^2+1}$

squaring on both sides,

$(a-x)^2 = \left[\sqrt{x^2+1}\right]^2$

$a^2 - 2ax+x^2 = x^2+1$

$\Rightarrow -2ax = 1-a^2$

$\Rightarrow 2ax = a^2 - 1$

$\Rightarrow x= \frac{a^2-1}{2a}$

$$\Rightarrow x = \frac{1}{2}\left[\frac{a^2-1}{a}\right]$$

$$\Rightarrow x = \frac{1}{2}\left[\frac{a^2}{a}-\frac{1}{a}\right]$$

$$\Rightarrow x = \frac{1}{2}\left[a-a^{-1}\right]$$

2. If $a^{1/3} + b^{1/3} + c^{1/3} = 0$ then show that $(a+b+c)^3 = 27abc$

Sol : $a^{1/3} + b^{1/3} + c^{1/3} = 0$

$a^{1/3} + b^{1/3} = -c^{1/3}$ ..... (I)

C.O.B.S.

$$\left[a^{\frac{1}{3}}+b^{\frac{1}{3}}\right]^3 = \left[-C^{\frac{1}{3}}\right]^3$$

$$\left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{1}{3}}\right)^3 + 3.a^{1/3}.b^{1/3}(a^{1/3}+b^{1/3}) = -\left(c^{\frac{1}{3}}\right)^3$$

$a+ b+3a^{1/3}b^{1/3}(-c^{1/3}) = -c$

$a+ b-3a^{1/3}b^{1/3}c^{1/3} = -c$

$a+ b + c = 3a^{1/3}b^{1/3}c^{1/3}$

C.O.B.S

$(a+b+c)^3 = 27abc$

3. If $y = 3\sqrt{3}+\frac{1}{3\sqrt{3}}$ then show that $3y^3 - 9y = 10$

Sol : y = $3\sqrt{3}+\frac{1}{3\sqrt{3}}$ $\quad y = 3^{\frac{1}{3}}+3^{-\frac{1}{3}}$ ..... (I)

C.O.B.S

$$\Rightarrow y^3 = \left[3^{\frac{1}{3}}+3^{\frac{-1}{3}}\right]^3$$

$$\Rightarrow y^3 = \left(3^{\frac{1}{3}}\right)^3 + \left(3^{\frac{-1}{3}}\right)^3 + 3.3^{\frac{1}{3}}.3^{\frac{-1}{3}}(3^{\frac{1}{3}}+3^{\frac{-1}{3}})$$

$$\Rightarrow y^3 = 3+3^{-1}+3^{\frac{1}{3}-\frac{1}{3}}(y) \qquad [\because \text{from(1)}]$$

$$\Rightarrow y^3 = 3+\frac{1}{3}+3.(1)y$$

$$\Rightarrow y^3 = 3+\frac{1}{3}+3y$$

$\Rightarrow \quad y^3 = \frac{9+1+9y}{3}$

$\Rightarrow \quad 3y^3 - 9y = 10$

4. If *lmn* =1 then show that $\frac{1}{1+l+m^{-1}} + \frac{1}{1+m+n^{-1}} + \frac{1}{1+n+l^{-1}}$

Sol : $lmn = 1 \Rightarrow d = \frac{1}{mn} \Rightarrow l^{-1} = \frac{1}{mn}$

consider L.H.S. $\frac{1}{1+l+m^{-1}} + \frac{1}{1+m+n^{-1}} + \frac{1}{1+n+l^{-1}}$

$= \frac{1}{1+\frac{l}{mn}+\frac{1}{m}} + \frac{1}{1+m+\frac{1}{n}} + \frac{1}{1+n+mn}$

$\Rightarrow \frac{1}{\frac{mn+n}{mn}} + \frac{1}{\frac{n+mn+1}{n}} + \frac{1}{1+n+mn}$

$\Rightarrow \frac{mn}{mn+1+n} + \frac{n}{n+mn+1} + \frac{1}{1+n+mn} = \frac{mn+n+1}{mn+l+n} = 1/1 = 1$

5. Evaluate $\lim_{x \to a} \frac{\sqrt{x+a} - \sqrt{2a}}{x-a}$

Sol : $\lim_{x \to a} \frac{\sqrt{x+a} - \sqrt{2a}}{x-a}$

$= \frac{\sqrt{a+a} - \sqrt{2a}}{a-a}$

$= \frac{\sqrt{2a} - \sqrt{2a}}{a-a} = \frac{0}{0}$ which is ideterminate form

consider $\lim_{x \to a} \frac{\sqrt{x+a} - \sqrt{2a}}{x-a} \times \frac{\sqrt{x+a} + \sqrt{2a}}{\sqrt{x+a} + \sqrt{2a}}$

$= \lim_{x \to a} \frac{\left(\sqrt{x+a}\right)^2 - \left(\sqrt{2a}\right)^2}{(x-a)\left(\sqrt{x+a} + \sqrt{2a}\right)}$

$= \lim_{x \to a} \frac{x+a-2a}{(x-a)\left(\sqrt{x+a} + \sqrt{2a}\right)}$

$$= \lim_{x \to a} \frac{\cancel{x-a}}{\cancel{(x-a)}\left(\sqrt{x+a}+\sqrt{2a}\right)}$$

$$= \lim_{x \to a} \frac{1}{\sqrt{x+a}+\sqrt{2a}}$$

$$= \frac{1}{\sqrt{a+a}+\sqrt{2a}}$$

$$= \frac{1}{\sqrt{2a}+\sqrt{2a}}$$

$$= \frac{1}{2\sqrt{2a}}$$

6. Solve $\left|1-\frac{m}{3}\right| > 8$

Sol : $|x|$ > a solution is x > a or x < –a

$\left|1-\frac{m}{3}\right| > 8$ solution is 1– m/3 > 8 (or) 1–m/3 < –8

$\Rightarrow$ – m/3 > 8–1 (or) – m/3 < – 8–1
$\Rightarrow$ – m/3 >7 (or) – m/3 < – 9
$\Rightarrow$ – m>21 (or) –m < – 27
$\Rightarrow$ m < – 21 (or) m > 27

2 Marks Questions

1. If $a^x = b$, $b^y = c$, $c^z = a$ then show that abc = 1

Sol : $a^x = b$, $b^y = c$, $c^z = a$

consider $a^x = b$

$\Rightarrow (c^z)^x = b$

$\Rightarrow (c^{zx}) = b$

$\Rightarrow (b^y)^{zx} = b$

$\Rightarrow (b)^{xyz} = b^1$

Bases are equal, powers shared to equated

$\Rightarrow$ xyz = 1

2. solve $|5x-1| \leq 0$

Sol : $|5x-1| \leq 0$ since modules of a real number is never negative

$|5x-1| = 0$

$|x| = 0$ solution is x= 0

$|5x-1| = 0$ solution is 5x–1 = 0

$\Rightarrow 5x = 1 \qquad \Rightarrow x = 1/5$

3. Evalute $\underset{x\ a}{Lt}\ \dfrac{x^{\frac{1}{4}} - a^{\frac{1}{4}}}{x^4 - a^4}$

Sol : $\underset{x\ a}{Lt}\ \dfrac{x^{\frac{1}{4}} - a^{\frac{1}{4}}}{x^4 - a^4} = \dfrac{x^{\frac{1}{4}} - a^{\frac{1}{4}}}{x^4 - a^4} = 0/0$ which is Indeterminate form

consider $\underset{x\ a}{Lt}\ \dfrac{x^{\frac{1}{4}} - a^{\frac{1}{4}}}{x^4 - a^4} = \dfrac{\frac{1}{4}}{\frac{4}{1}}.a^{\left(\frac{1}{4}\right)-(4)}$

$= \dfrac{1}{4\times 4}\cdot a^{\frac{1-16}{4}} = \dfrac{1}{16}a^{\frac{-15}{4}} \qquad \Rightarrow \qquad \dfrac{1}{16a^{\frac{15}{4}}}$

4. Find the limit of the series 1– 1/2 + 1/4 –1/8 + ...

Sol : 1-1/2 + 1/4 - 1/8 + ...

Here $a = 1, r = \dfrac{t_2}{t_1} = \dfrac{-\frac{1}{2}}{1} = \dfrac{1}{2} < 1 \ \therefore s_\infty = \dfrac{a}{1-r}$

$\Rightarrow S_\infty = \dfrac{1}{1-(-r_2)} = \dfrac{1}{1+\frac{1}{2}}$

$\Rightarrow \dfrac{1}{\frac{2+1}{2}} = \dfrac{1}{\frac{3}{2}} = \dfrac{1}{6}$

5. Solve $|2x-5| = 7$

Sol : $|2x-5| = 7$

$|x|$ = a solution is x= a or x= – a

$|2x-5| = 7$ solution is 2x − 5=7 (or) 2x-5 = − 7

$\Rightarrow$2x = 7+5 (or) 2x = –7+5
$\Rightarrow$2x = 12 (or) 2x = –2
$\Rightarrow$x = 6 (or) x = –1

6. Evalute $\underset{x\ 0}{Lt}\ \dfrac{(1+x)^4 - 1}{(1+x)^3 - 1}$

Sol : $\underset{x\ 0}{Lt}\ \dfrac{(1+x)^4 - 1}{(1+x)^3 - 1} = \dfrac{(1+0)^4 - 1}{(1+0)^3 - 1} = 0/0$ which is indeterminate form

$\underset{x\ 0}{Lt}\ \dfrac{(1+x)^4 - 1}{(1+x)^3 - 1}$ Hence put 1+x = t, as x $\to$0

$\Rightarrow$t $\to$1

$$\lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1} = \lim_{t \to 1} \frac{t^4 - 1^4}{t^3 - 1^3}$$

$= \frac{4}{3}.(1)^{4-3}$ $\quad \because \lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n}.a^{m-n}$

$= \frac{4}{3}.(1) = \frac{4}{3}$

1 Marks Questions

1. **If $a^2 = 0.04$ then find a^3**

Sol : $a^2 = 0.04$

$\Rightarrow a^2 = 0.04 = \frac{4}{100}$

$\Rightarrow a^2 = \left(\frac{2}{10}\right)^2$

$\Rightarrow a = \frac{2}{10}$

C.O.B.S.

$\Rightarrow a^3 = \left(\frac{2}{10}\right)^3$

$\Rightarrow a^3 = \frac{2}{10^3}^3$

$\Rightarrow a^3 = \frac{8}{1000}$

$\Rightarrow a^3 = 0.008$

2. **If $\left(x^{\frac{2}{3}}\right)^p = x^2$ then find p**

Sol : $\left(x^{\frac{2}{3}}\right)^p = x^2$

$\Rightarrow \left(x^{\frac{2P}{3}}\right) = x^2$

Bases are equal, powers to be equated

$\Rightarrow \frac{2p}{3} = 2$

$\Rightarrow 2p = 2.3 \quad \Rightarrow p = 3$

3. If $x^{\frac{1}{2}} = 0.2$ then find $x^{\frac{3}{2}}$

Sol : $x^{\frac{1}{2}} = 0.2 \Rightarrow x^{\frac{1}{2}} = \frac{2}{10}$

cubing on both sides

$\Rightarrow \left(X^{\frac{1}{2}}\right)^3 = \left(\frac{2}{10}\right)^3$

$\Rightarrow x^{3/2} = \frac{8}{1000} \Rightarrow x^{3/2} = 0.008$

4. If $\lim_{x \to 2} \frac{X^2-4}{X-2}$

Sol : $\lim_{x \to 2} \frac{X^2-4}{X-2} = \frac{2^2-4}{2-2} = 0/0$ which is indeterminate form.

$\lim_{x \to 2} \frac{X^2-4}{X-2} = \lim_{x \to 2} \frac{X^2-2^2}{X-2}$

$= \lim_{x \to 2} \frac{(X+2)\cancel{(X-2)}}{\cancel{X-2}} = \lim_{x \to 2} X+2$

$\Rightarrow 2+2 = 4$

5. simplify $\left(x^{\frac{1}{4}}+y^{\frac{1}{4}}\right)\left(x^{\frac{1}{4}}-y^{\frac{1}{4}}.y^{\frac{1}{a}}+y^{\frac{1}{2}}\right)$

Sol : $(a+b)(a^2 - ab + b^2) = a^3 + b^3$

$\left(x^{\frac{1}{4}}+y^{\frac{1}{4}}\right)\left(x^{\frac{1}{4}}-y^{\frac{1}{4}}.y^{\frac{1}{a}}+y^{\frac{1}{2}}\right)$

$= \left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)\left[\left(x^{\frac{1}{4}}\right)^2 - x^{\frac{1}{4}}.y^{\frac{1}{4}} + \left(y^{\frac{1}{4}}\right)^2\right]$

$= \left(x^{\frac{1}{4}}\right)^3 + \left(y^{\frac{1}{4}}\right)^3 = x^{\frac{3}{4}} + y^{\frac{3}{4}}$

Fill in the Blanks

1. $(64)^{-2/3}$ = ...

2. The value of $a^{\frac{2}{3}}\left(\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{4}}\right)^4\right)$ = ...

3. $(0.001)^{1/3}$ = ...

4. $|x|$ = -7 solution .....

5. modulas of a real number is never .....

6. Is the limit of a function approaches to Zero then the function is called .....

7. $\lim_{x \to \infty} \frac{3x-1}{2x+7}$ = ...

8. $\lim_{x \to \infty} \frac{(1+x)^{\frac{1}{2}}-1}{x}$ = ...

9. $\lim_{x \to p} \frac{x^a - p^a}{x-p}$ = ...

10. $\lim_{x \to 3} \frac{x^3-27}{x-3}$ = ...

Answers

1) 1/16 2) a^2 3) 0.1 4) doesnot exists 5) negative 6) infinitesinal
7) 3/2 8) 1/n 9) $a.p^{a-1}$ 10) 27

Multiple choice Questions

1. $16^{1.25}$ = ...
   a) 32 b) 64 c) 16 d) 5/4

2. $\sqrt[3]{x} + \sqrt[3]{y} + \sqrt[3]{z} = 0$ then x+y+z = ...

   a) 3myz b) 9xyz c) 27xyz d) $(3xyz)^{1/3}$

3. $\sqrt{2^x} = 16$ then x= .....

a) 8 b) 4 c) 2 d) 10

4. $\sqrt{\sqrt{3^x}} = 81$ then x = ...

a) 16 b) 8 c) 4 d) 2

5. If $5^{x+3} = 5^{x+3}$ then x = ...

a) 0 b) 1 c) 3 d) –3

6. If $|x+1| < 6$ then solution set is .....

a) { x/–7 <x<5} b) { x/–7< x ≤ 5}

c) {x/–7≤ x≤ 5} d) { x/–7 < x ≤ 5}

7. $|x| \leq$ a solution is .....

a) –a ≤ x ≤ a b) –a ≤ x≤ a

c) –a ≥ x ≥ a d) x > a or x < a

8. $\lim_{x \to 0} \frac{(1+x)^n - 1}{x}$ = ...

a) n b) 1/n c) – n d) n^2

9. $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$ = ...

a) m/n .a^{m-n} b) m/n. a^{m+n} c) n/m. a^{m-n} d) n/m. a^{m+n}

10. $\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$ = ...

a) 3 b) 9 c) 27 d) 1

11. $\lim_{x \to 0} \frac{(1+x)^4 - 1}{(1-(x)^3 - 1}$ = ...

a) 4/3 b) 3/4 c) –4/3 d) –3/4

12. $\lim_{x \to 0} \frac{x^2 + 5n}{x}$ = ...

a) 0 b) 1 c) 5 d) 2

13. $|x| = 5$ solution is .....

a) 5 b) -5 c) 5 (or) -5 d) doesn't exist

14. If $2^n - 2^{n-1} = 4$ then x^n = ...

a) 1 b) 27 c) 3 d) 9

15. $a^{m/n}$ = ...

a) $\frac{a^m}{a^n}$ b) $a^m.a^n$ c) $(a^m)^{1/n}$ d) $(a^n)^m$

Answers

1) a 2) d 3) a 4) a 5) d 6) a 7) a 8) a
9) a 10) c 11) c 12) a 13) d 14) b 15) c