Progressions

Main Points

- **In A.P.**

  i) common difference $d = t_2 - t_1$

  ii) n^{th} term $t_n = a + (n-1)\,d$

  iii) sum to n term $S_n = \frac{n}{2}\,[2a+(n-1)d]$ (or) $S_n = \frac{n}{2}\,[a+l\,]$

- $\Sigma\, 1 = n$
- $\Sigma_n = \frac{n(n+1)}{2}$
- $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$
- $\Sigma n^3 = \frac{n^2(n+1)^2}{4}$ (or) $\left[\frac{n(n+1)}{2}\right]^2$ or $\left[\sum n\right]^2$

- **In G.P.**

  (i) Common ratio $r = \frac{t_2}{t_1}$

  (ii) n^{th} term $tn = a.r^{n-1}$

  (iii) sum of n term $S_n = \frac{a\left[1-r^n\right]}{1-r}$ if $r < 1$ (or) $S_n = \frac{a\left[r^n-1\right]}{r-1}$ if $r > 1$

- For only two numbers a and b

  i) A.M $= \frac{a+b}{2}$

  ii) G.M $= \sqrt{ab}$

  iii) H.M $= \frac{2ab}{a+b}$

- A,M., G.M. and H.M. of any two numbers A,G,H, respectively
  i) will be in G.P.
  ii) $A \geq G \geq H$

4 Marks Questions

1. If 7 times to the 7th term of an A.P. is equals to 11 times to the 11th term, then show that 18th term of it is zero.

Sol : In an A.P. $7\,t_7 = 11.t_{11}$

$\Rightarrow 7[a+6d] = 11\,[a+10d]$

$\Rightarrow 7a+42d = 11a+110d$

$\Rightarrow 7a-11a + 42d - 110d = 0$

$\Rightarrow -4a - 68d = 0$

$\Rightarrow -4a\,[a+17d] = 0$

$\Rightarrow a + 17d = 0$

$\Rightarrow a + (18-1)\,d = 0$

$\Rightarrow t_{18} = 0$

2. If the sum of the first 'n' natural numbers is S_1, that of their squares is S_2 and cubes is S_3, then show that $9S_2^2 = S_3\,(1+8S_1)$

Sol : $\Sigma_n = S_1 \quad \frac{n(n+1)}{2}$

$\Sigma_n^2 = S_2 \quad \frac{n(n+1)(2n+1)}{6}$

$\Sigma_n^3 = S_3 \quad \frac{n^2(n+1)^2}{4}$

Consider RHS

$$S_3\,[1+8S_1] = \frac{n^2(n+1)^2}{4}\left[1+\cancel{8}\,\frac{n(n+1)}{\cancel{2}}\right]$$

$$= \frac{n^2(n+1)^2}{4}\left[1+4n^2+4n\right]$$

$$= \frac{n^2(n+1)^2}{4}\left[(2n)^2+2.(2n).1+1^2\right]$$

$$= \frac{n^2(n+1)^2}{4}\left[(2n)+1\right]^2$$

Multiplying and dividing with a

$$= \frac{9}{9}\quad\frac{n^2(n+1)^2(2n+1)^2}{4}$$

$$= 9.\ \frac{n^2(n+1)^2(2n+1)^2}{36}$$

$$= 9.\left[\frac{n(n+1)(2n+1)}{6}\right] \qquad = 9.\ S_2^2 \text{ L.H.S}$$

3. Find the sum to 'n' terms in the series 7, 77, 777,7777,.....

Sol : Let S= 7+ 77+ 777+ ..... n terms

= 7[1+11+111+..... n terms]

Multiplying and dividing with 9

$= \frac{7}{9}\times 9$ [1+11+111+ ..... n terms]

$= \frac{7}{9}$ [9+99+999+ ..... n terms]

$= \frac{7}{9}$ [(10–1) + (100–1)+ (1000+1)+ ..... n terms]

$= \frac{7}{9}$ [(10+100+1000+ ..... n terms) - (1+1+1+..... n terms)]

$= \frac{7}{9}$ [(10+100+1000+ ..... n terms) –n] ..... (i)

Here 10, 100, 1000..... are in G.P

G = 10, r = 100/10 = 10

$S_n = \frac{a(r^n-1)}{r-1} = \frac{10(10^n-1)}{10-1}$

$= \frac{10}{9}(10^n-1)$ ..... (ii)

Sub (ii) in (i)

$= \frac{7}{9}\left[\frac{10}{9}.(10^n-1)-n\right]$

$= \frac{7}{9}\left[\frac{10^{n+1}-10-9n}{9}\right]$

$= \frac{7}{81}\left[10^{n+1}-9n-10\right]$

4. If A.M, G.M, H.M of two numbers are A,G, H, respectively. then show that A≥G≥H

Sol : Let the two numbers be a and b

A.M. i.e. A = $\frac{a+b}{2}$

G.M. i.e. G = $\sqrt{ab}$

H.M. i.e. H = $\frac{2ab}{a+b}$

Consider A–G = $\frac{a+b}{2} - \sqrt{ab}$

$= \frac{a+b-2\sqrt{ab}}{2} = \frac{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{a}.\sqrt{b}}{2}$

$= \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0$

Consider A– G ≥ 0

$\Rightarrow A \geq G$ ..... (i)

$\Rightarrow \frac{A}{G} \geq 1 \quad \geq 1$ ..... (ii)

We know that $G^2 = A.H.$

$\Rightarrow \frac{G}{H} = \frac{A}{G}$

$\Rightarrow \frac{G}{H} \geq 1 \qquad \left[\because \frac{A}{G} \geq 1\right]$

$\Rightarrow G \geq H$ ..... (iii)

From (i) & (iii) $A \geq G \geq H$

5. If (b+c), (c+a) and (a+b) are in H.P. then show that $\frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}$ will also be in H.P.

Sol : (b+c), (c+a) and (a+b) are in H.P.

$\Rightarrow \frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ will be in A.P.

$\Rightarrow \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$\Rightarrow \frac{(b+c)-(c+a)}{(c+a)(b+c)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$

$\Rightarrow \frac{b+c-c-a}{b+c} = \frac{c+a-a+b}{a+b}$

$\Rightarrow \frac{b-a}{b+c} = \frac{c-b}{b+a}$

$\Rightarrow (b-a)(b+a) = (c-b)(c+b)$

$\Rightarrow b^2 - a^2 = c^2 - b^2$

$\Rightarrow a^2, b^2$ and c^2 will be in A.P.

$\Rightarrow \frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}$ will be in H.P

6. If g_1, g_2, g_3 are three G.M's between m and n then show that $g_1 g_3 = g_2^2 = mn$

Sol : g_1, g_2, g_3 are 3 G.M's between m and n

$\Rightarrow m, g_1, g_2, g_3, n$ will be in G.P.

$$\Rightarrow \frac{g_1}{m} = \frac{g_2}{g_1} = \frac{g_3}{g_2} = \frac{n}{g_3}$$

$\Rightarrow \frac{g_1}{m} = \frac{n}{g_3}$ | $\frac{g_2}{g_1} = \frac{g_3}{g_2}$

$\Rightarrow g_1 g_3 = mn$..... (i) | $g_2^2 = g_1 g_3$ ..... (ii)

From (i) & (ii)

$g_1 g_3 = g_2^2 = mn$

2 Marks Questions

1. Inset 4 A.M's between 3 and 33?

Sol : Let a_1, a_2, a_3, a_4 be 4 A.M.'s between 3 & 33

$\therefore$ 3, a_1, a_2, a_3, a_4, 33 will be in A.P.

$a=3, \quad t_6 = 33$

$a+5d = 33$

$3+5d = 33$

$5d = 33-3$

$5d = 30$

$d = 6$

$\therefore$ Required A.M.'s are

$a_1 = 3 + d = 3 + 6 = 9$

$a_2 = a_1 + d = 9 + 6 = 15$

$a_3 = a_2 + d = 15 + 6 = 21$

$a_4 = a_3 + d = 21 + 6 = 27$

2. Which term of A.P. 10, 8, 6, ... is –28 ?

Sol : 10, 8, 6, ... are in A.P

$a = m\ 10, d = 8 - 10 = -2$

Let $t_n = -28$

$a+(n-1)\,d = -28$
$10+(n-1)\,(-2) = -28$
$10+[-2n+2]\,d = -28$
$10 -2n+2 = -28$
$12-2n = -28$
$-2n = -28-12$
$-2n = -40$
$n = 20$
there fore $t_{20} = -28$

3. Insert Five G,M's between $\frac{1}{3}$ and 243?

Sol : Let g_1, g_2, g_3, g_4, g_5 be Five G.M's between $\frac{1}{3}$ and 243.

$\therefore \frac{1}{3}, g_1, g_2, g_3, g_4, g_5, 243$ will be in G.P

$a = 1/3,\ t_7 = 243$

$a.r^6 = 243$

$\frac{1}{3} r^6 = 243$

$r^6 = 243\times3$

$r^6 = 3^5\times3^1$

$r^6 = 3^6 \Rightarrow r = 3$

$g_1 = ar = \frac{1}{3}\times3 = 1$

$g_2 = g_1.r = 1\times3 = 3$

$g_3 = g_2.r = 3\times3 = 9$

$g_4 = g_3.r = 9\times3 = 27$

$g_5 = g_4.r = 27\times3 = 81$

$\therefore$ Required G.M's are 1, 3, 9, 27,81

4. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p,q,r respectively then show that $q^2 = rs$

Sol : In G.P. $t_5 = p,\ t_8 = q,\ t_{11} = S$

$\Rightarrow ar^4 = p,\ ar^7 = q,\ ar^{10} = S$

consider LHS $q^2 = (ar^7)^2$

$= a^2(r^7)^2$

$= a^2 r^{14}$ ..... (i)

R.H.S **r.s** $= ar^4.\ ar^{10}$

$= a^2.\ r^{4+10}$

$= a^2 . r^{14}$ ..... (ii)

From (i) and (ii) $\mathbf{q^2 = rs}$

5. Express $1.\overline{15}$ in $\frac{p}{q}$ form

Sol : $1.\overline{15} = 1.151515.....$

$= 1 + 0.15 + 0.0015 + 0.000015 + \cdots\cdots\cdots \infty$ ... (i)

$a = 0.15, r = \frac{0.0015}{0.15} = \frac{1}{100} < 1$

$S_\infty = \frac{a}{1-r}$

$= \frac{0.15}{1-\frac{1}{100}}$

$= \frac{\frac{15}{100}}{\frac{99}{100}}$

$= \frac{15}{99}$ ..... (ii)

substitute ii) in i)

$1.\overline{15} = 1 + \frac{15}{99}$

$= \frac{99+15}{99} = \frac{114}{99} = \frac{38}{33}$

1 Mark Questions

1. Find the 17th term in $t_n = \frac{n(n+3)}{n+2}$

Sol : $t_n = \frac{n(n+3)}{n+2}$

$\Rightarrow t_{17} = \frac{17+(17+3)}{17+2} = \frac{17\times 20}{19} = \frac{340}{19}$

2. Determine k so that k+2, 4k–6, 3k–2 are the three terms of A.P?

Sol : K+2, 4k–6, 3k–2, are in A.P.

$\Rightarrow (4k-6)-(k+2)=(3k-2)-(4k-6)$

$\Rightarrow 4k-6-k-2=3k-2-4k+6$

$\Rightarrow 3k-8=-k+4$

$\Rightarrow 4k-8=4$

$\Rightarrow 4k=12 \quad \Rightarrow k=3$

3. If $\frac{-7}{2}$, x, $\frac{-2}{7}$ are in G.P. find x?

Sol : $\frac{-7}{2}$, x, $\frac{-2}{7}$ are in G.P.

$\Rightarrow \frac{t_2}{t_1}=\frac{t_3}{t_2}$

$\Rightarrow \frac{x}{-\frac{7}{2}}=\frac{-\frac{2}{7}}{x}$

$\Rightarrow x^2=\frac{-\cancel{7}^{1}}{\cancel{2}^{1}}=\frac{-\cancel{2}^{1}}{\cancel{7}^{1}}$

$\Rightarrow x^2=1$

$\Rightarrow x=\sqrt{1}$

$\Rightarrow x=\pm 1$

4. The common ratio of G.P is $\frac{-4}{5}$ and the sum to infinity is $\frac{80}{9}$ then find the first term?

Sol : $r=\frac{-4}{5}$, $S_\infty=\frac{80}{9}$

$S_\infty=\frac{a}{1-r}$

$\frac{80}{9}=\frac{a}{1-\left(\frac{-4}{5}\right)}$

$\frac{80}{9}=\frac{a}{\frac{5+4}{5}}$

$$\frac{80}{9} = \frac{5a}{9}$$

$5a = 80$

$a = 16$

$\therefore$ first term is 16

5. The first term of a G.P is 2 and the sum to infinity is 6, then find the common ratio?

Sol : $a = r, \quad S_\infty = 6$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow 6 = \frac{2}{1-r}$$

$\Rightarrow 6(1-r) = 2$

$\Rightarrow 6 - 6r = 2$

$\Rightarrow -6r = 2-6$

$\Rightarrow -6r = -4$

$\Rightarrow r = 4/6$

$$\Rightarrow r = \frac{2}{3}$$

$\therefore$ common ratio is $\frac{2}{3}$

Multiple choice Questions

1. If a,b,c are in A.P then c+a= .....
   a) a b) 2b c) 3b d) 4b

2. If there are 'n' A.M's between a and b then common difference is = ...
   a) $\frac{b-a}{n+1}$ b) $\frac{n+1}{b+a}$ c) $\frac{b-a}{n+1}$ d) $\frac{ab}{n+1}$

3. 1.2+ 2.3+ 3.4 + 4.5 + ... sum to firsr 5 terms is
   a) 70 b) 5 c) 20 d) 40

4. The 10^{th} term of A.P. 13, 8, 3, –2, ... is .....
   a) –32 b) 32 c) –57 d) 57

5. The sum to 'n' term of the series 1+4+9+16+ ..... is
   a) $\frac{n(n+1)}{2}$ b) $\frac{n^2(n+1)^2}{4}$ c) $\frac{n(n+1)(2n+1)}{6}$ d) $\frac{n(n+1)^2}{4}$

6. Sum of the cubes of the first five natural numbers is .....
   a) 15 b) 225
   c) 55 d) 40

7. The common ratio of the G.P is 1/2, -1/4, 1/8..... is
   a) –1/2 b) 1/2
   c) 2 d) –2

8. s_∞ = ...
   a) $\frac{a}{1+r}$ b) $\frac{a}{1-r}$ c) $\frac{1-r}{a}$ d) $\frac{1+r}{a}$

9. If tan A, tan B, tan C, are in A.P, then cot A, cot B , cot C are in .....progression.
   a) Arthmetic b) Geometric c) Harmon d) None

10. A.M. and G.M of two numbers is 16 and 8 respectively. then their harmonic mean is .....
    a) 4 b) 12/5 c) 32/3 d) 6 4

11. Harmonic mean of a and b is .....
    a) $\frac{ab}{2}$ b) $\frac{a-2}{2}$ c) $\frac{2ab}{a+b}$ d) $\sqrt{ab}$

12. If A, G, and H are A.M., G.M. and H.M. of only two numbers then the relation between them is .....
    a) $A^2 = GH$ b) $G^2 = AH$ c) $H^2 = AG$ d) $G = AH$

13. If $\frac{1}{a}\ \frac{1}{b}\ \frac{1}{c}$ are in H.P then .....
    a) $b = \frac{2ac}{a+c}$ b) $b = \sqrt{ac}$ c) $b = \frac{a^2 - c^2}{4}$ d) $c = 2b - a$

14. Geometric mean of 5 and 125 is .....
    a) 100 b) 65 c) 25 d) 625

Answers

1. b 2. c 3. c 4. a 5. c 6. b 7. a 8. b 9. c 10. c 11.c 12. b

13. d 14. c 15. a

Fill in the Blanks

1. If a = –3, d= –2 then 20^{th} term of A.P. is .....

2. If n^{th} term of A.P. is .....

3. If there are 5 A.M'S between x , y then d = ...

4. A.M. of a–3, a–1, a+1 and a+3 is .....

5. If P, Q, R, be p^{th}, q^{th}, r^{th} terms of an A.P. then P (q–r) + Q (r–p) + R (p–q) = ...

6. The n^{th} term of a G.P. is $2.(0.5)^{n-1}$ then r = ...

7. The first term of a G.P. is 'a' and common ratio is '*l*' the sum to n terms is .....

8. The H.P. between 10 and 15 is .....

9. The H.M. and A.M. of two numbers is 2 and 8 respectively then G.M. is .....

10. The A.M. of x and y is .....

11. if x, y, z are in A.P then y= .....

12. The 4th term of H.P. then y = ...

13. 1+ 1/3 + 3 1/2+ 3 1/3+ .....∞ = ...

14. If tan A, tan B, tan C are in ..... progression.

15. The relationship among A.M. G.M. and H are i.e. A,G, H is ..... and .....

16. Geometric mean of 5 and 125 is .....

17. The 10th term 5, 2,1, 1/2, 1/4, ... is

18. In 1.3 + 3.5 + 5.7 + ...

19. 1.2 + 2.3 + 3.4+... sum to 5 terms is .....

20. $\Sigma n = 10$ then Σn^3 = ...

21. Σn^3 = b25 then Σn = ...

22. In 15, 10, 5.... S_7 = ...

23. If $1^3+ 2^3 + 3^3+ ... + m^3 = 3025$ then n= .....

24. the n^{th} term of the series 1+(1+3) + (1+3+5) + ...

25. sum to r terms of 4, –8, 16, –32, ... is .....

26. The name of the progression 1/5+ 1/7+ 1/9 ..... is .....

27. If $t_n = \frac{n}{n+1}$ then t_4 = ...

28. In G.P. $t_6 = 24$, $t_{13} = 3/16$ then t_{20}= .....

29. The 10^{th} term of the series $n + \frac{4n}{3} + \frac{5n}{3} + 2n + ...$ is ....

30. $1^2+2^2+ 3^2+..... 10^2$= .....

31. The sum to n terms is 1, 8, 27, 64 .... is .....

32. The 7^{th} term of 1/3, 1/7, 1/11 is .....

33. In the progression 1, –1/2, 1/4.... then t_7 = ...

34. Harmonic mean of 2 and 6 is .....

35. If a,b,c are three consecutive terms of A.P then k^a, k^b, k^c are is .....

36. In a G.P. a =1, r = $\sqrt{2}$ then S_{20}= .....

37. If $\frac{-2}{7}$, x, $\frac{-7}{2}$ are three consecutive terms of G.P. then x = ...

38. $1^3+2^3+ 3^3+.... 10^3$=.....

39. The sum of first 100 natural numbers is .....

40. If $t_n = (-1)\, n.n^2$ then t_{10}= .....

Answers

1. –14 **2.** 2 **3.** $\frac{y-a}{6}$ **4.** a **5.** 0 **6.** 0.5 **7.** na **8.** $\frac{20}{3}$ **9.** 4

10. $\frac{2xy}{x+y}$ **11.** $\frac{2xz}{x+z}$ **12.** 2 **13.** 3/2 **14.** Arithmetic **15.** $G^2 = AH$; $A \geq G \geq H$ **16.** 25

17. $2^{1/8}$ or 1/256 **18.** (2n–1) (2n) or $4n^2–2n$ **19.** 70 **20.** 100 **21.** 25 **22.** 0

23. 10 **24.** n^2 **25.** $\frac{4}{3}[1-(2)]$ **26.** Harmonic progression **27.** 4/5 **28.** $\frac{3}{2^{11}}$ **29.** 4x

30. 385 **31.** $\frac{n^2(n+1)^2}{4}$ **32.** 1/27 **33.** 1/64 **34.** 3 **35.** Geometric

36. $\frac{(\sqrt{2})^{20}-1}{\sqrt{2}-1}$ or $\frac{2^{10}-1}{\sqrt{2}-1}$ or $(\sqrt{2}+1)(2^{10}-1)$ **37.** ± 1 **38.** 2725 **39.** 5050 **40.** 100