# TRIGONOMETRY

- Trigonometry is derived from three greek roots i.e Tri, Gonia and metron which means Three angle measurement.

- The relationship among D, G and C is $\frac{D}{90} = \frac{G}{100} = \frac{C}{\pi/2}$ (or) $\frac{D}{180} = \frac{G}{200} = \frac{C}{\pi}$

- The relationship among $l$, r and $\theta$ is $l = r\theta$

- $\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$

- $\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$

- $\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$

- $\cot\theta = \frac{\text{side adjacent to } \theta}{\text{side opposite to } \theta}$

- $\sec\theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \theta}$

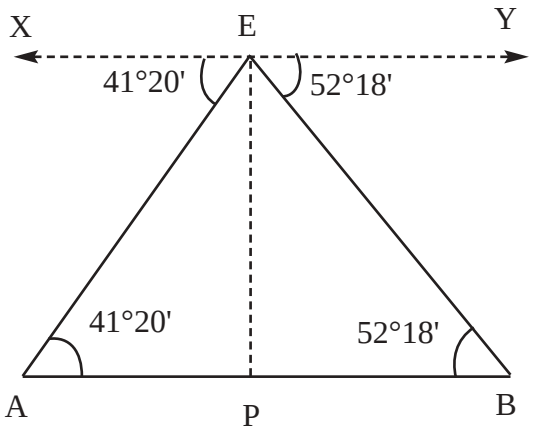
- $\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{side opposite to } \theta}$

- $\sin^2\theta + \cos^2\theta = 1$ $\quad 0 \le \theta \le 360°$
- $\sec^2\theta - \tan^2\theta = 1$ $\quad 0 \le \theta < 90°$
- $\text{cosec}^2\theta - \cot^2\theta = 1$ $\quad 0 \le \theta < 90°$

## 5 Marks Questions

1. An aeroplane at an altitude of 2500 mts observes the angles of depression of opposite points on the two banks of a river to be 41°20' and 52°10'. Find the width of river in metres.

**Sol:** Let the width of the river be AB.
E is the position of an aeroplane.



$\angle AEX = 41°20' \Rightarrow \angle EAP = 41°20'$ [$\because$ Alternate angles]

$\angle BEY = 52°18' \Rightarrow \angle EBP = 52°18'$ [$\because$ Alternate angles]

$$\text{In } \Delta EAP, \tan 41°20' = \frac{EP}{AP}$$

$$\Rightarrow 0.8795 = \frac{2500}{AP}$$

$$\Rightarrow AP = \frac{2500}{0.8795}$$

$$\Rightarrow AP = \frac{25000000}{8795}$$

$\Rightarrow$ AP = 2842.5 m ------ (1)

$$\text{In } \Delta EBP, \tan 52°10' = \frac{EP}{BP}$$

$$\Rightarrow 1.2837 = \frac{2500}{BP}$$

$$\Rightarrow BP = \frac{2500}{1.2877}$$

$$\Rightarrow BP = \frac{25000000}{12877}$$

$\Rightarrow$ BP = 1941.5 m ------ (2)

Width of the river AB = AP + BP

= 2842.5 + 1941.5

= 4784 m

2. A ladder 20 m long is placed against a wall of height 10 mts. Find the distance between the foot of the ladder and the wall. Find also the inclination of the ladder with the horizontal.

**Sol:** Let AB be the ladder.

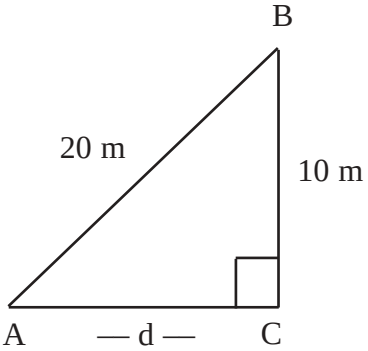
AB = 20 m

BC is the wall.

BC = 10 m

AC is the distance between foot of the ladder and base of the wall.

Let AC = d



$\text{In } \Delta ABC, \sin\theta = \frac{BC}{AB}$

$\Rightarrow \sin\theta = \frac{10}{20}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin 30°$

$\Rightarrow \boxed{\theta = 30°}$

$\text{In } \Delta ABC, \tan\theta = \frac{BC}{AC}$

$\Rightarrow \tan 30° = \frac{10}{d}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{d}$

$\Rightarrow d = 10\sqrt{3}\,m$

$\Rightarrow d = 10 \times 1.732$

$\Rightarrow d = 17.32$ m

$\therefore$ The distance between foot of the ladder and base of the wall = 17.32 m

Inclination of the ladder with the horizontal = 30°

3. The upper past of a tree broken by wind in two parts, makes an angle of 30º with the ground. The top of the tree touches the ground at a distance of 20 m from the foot of the tree. Find the height of the tree before it was broken.

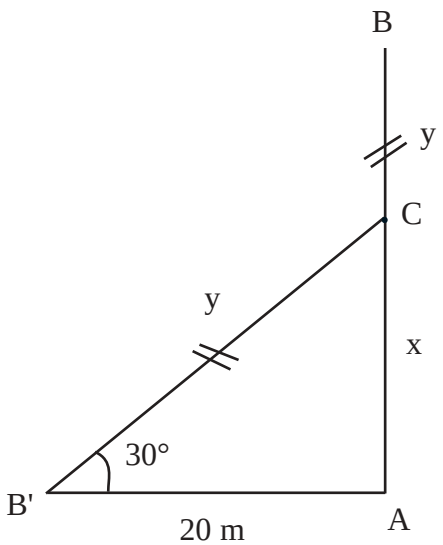
**Sol:** Let AB be the tree before it was broken.

C is the broken point.

B'C is the broken part.

$\angle AB'C = 30°$

AB' = 20 m



In $\Delta AB'C \quad \tan 30° = \frac{AC}{AB'}$

$\frac{1}{\sqrt{3}} = \frac{x}{20}$

$x = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\sqrt{3}}{3} m$

$\therefore x = \frac{20\sqrt{3}}{3} m$

In $\Delta AB'C \quad \cos 30° = \frac{AB'}{B'C}$

$\frac{\sqrt{3}}{2} = \frac{20}{B'C}$

$\frac{\sqrt{3}}{2} = \frac{20}{y}$

$\sqrt{3}.y = 40$

$y = \frac{40}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$y = \frac{40\sqrt{3}}{3} m$

$\therefore$ The height of the tree before it was broken.

= x+y

$= \frac{20\sqrt{3}}{3} + \frac{40\sqrt{3}}{3} = \frac{60\sqrt{3}}{3} = 20\sqrt{3} m$

## 4 Marks Questions

1. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$

**Proof:** L.H.S

$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \frac{\tan\theta + \sec\theta - \left[\sec^2\theta - \tan^2\theta\right]}{\tan\theta - \sec\theta + 1}$

$= \frac{\tan\theta + \sec\theta - \left[(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta)\left[1 - (\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$

$$= \frac{(\tan\theta+\sec\theta)(1-\cancel{\sec\theta}+\tan\theta)}{\tan\theta-\cancel{\sec\theta}+1}$$

$= \tan\theta + \sec\theta$

$$= \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

= R.H.S

2. If $\sec\theta + \tan\theta = P$, then show that $\sin\theta = \frac{P^2-1}{P^2+1}$

**Proof:** $\sec\theta + \tan\theta = P$

R.H.S

$$\frac{P^2-1}{P^2+1}$$

$$= \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$$

$$= \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$$

$$= \frac{\sec^2\theta-1+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\tan^2\theta+1+2\sec\theta\tan\theta} \qquad \left[\begin{array}{l}\because \sec^2\theta-\tan^2\theta=1\\ \sec^2\theta-1=\tan^2\theta\\ \sec^2\theta=\tan^2\theta+1\end{array}\right]$$

$$= \frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta}$$

$$= \frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$$

$$= \frac{\cancel{2}\tan\theta\left(\cancel{\tan\theta+\sec\theta}\right)}{\cancel{2}\sec^2\theta\left(\cancel{\sec\theta+\tan\theta}\right)}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$= \sin\theta$

= L.H.S

3. If $x\cos\theta + y\sin\theta = a$, $x\sin\theta - y\cos\theta = b$, then eliminate 'θ'.

**Sol:** $x\cos\theta + y\sin\theta = a$

Squaring on both sides

$(x\cos\theta + y\sin\theta)^2 = a^2$

$x^2\cos^2\theta + y^2\sin^2\theta + 2xy\cos\theta\sin\theta = a^2$ ------- (1)

$x\sin\theta - y\cos\theta = b$

Squaring on both sides

$(x\sin\theta - y\cos\theta)^2 = b^2$

$x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta = b^2$ ------- (2)

Adding (1) and (2)

$$x^2\cos^2\theta + y^2\sin^2\theta + \cancel{2xy\cos\theta\sin\theta} + x^2\sin^2\theta + y^2\cos^2\theta - \cancel{2xy\sin\theta\cos\theta} = a^2 + b^2$$

$\Rightarrow x^2(\cos^2\theta + \sin^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = a^2 + b^2$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

4. Solve $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4$

**Sol:** $\Rightarrow \dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4$

$$\Rightarrow \frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$

$$\Rightarrow \frac{\cos\theta + \cancel{\cos\theta\sin\theta} + \cos\theta - \cancel{\cos\theta\sin\theta}}{1-\sin^2\theta} = 4$$

$$\Rightarrow \frac{\overset{1}{\cancel{2}}\cos\theta}{\cos^2\theta} = \overset{2}{\cancel{4}}$$

$\Rightarrow \cos\theta = 2\cos^2\theta$

$\Rightarrow 2\cos^2\theta - \cos\theta = 0$

$\Rightarrow \cos\theta[2\cos\theta - 1] = 0$

$\Rightarrow \cos\theta = 0$ (or) $2\cos\theta - 1 = 0$

$\Rightarrow \cos\theta = \cos 90°$ (or) $2\cos\theta = 1$

$\Rightarrow \theta = 90°$ (or) $\cos\theta = 1/2$

$\Rightarrow \theta = \pi^c/2$ (or) $\theta = 60° = \pi^c/3$

$$\therefore \theta = \frac{\pi^c}{2} \text{ (or) } \frac{\pi^c}{3}$$

## 2 Marks Questions

1. Prove that

$$4\left[\sin^4 30° + \cos^4 60°\right] - 3\left[\cos^2 45° - \sin^2 90°\right] = 2$$

**A. Proof:** L.H.S

$$4\left[\sin^4 30° + \cos^4 60°\right] - 3\left[\cos^2 45° - \sin^2 90°\right]$$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= \cancel{4}^{1}\left[\frac{\cancel{2}^{1}}{16_{\cancel{4}_{2}}}\right] - 3\left[\frac{1-2}{2}\right]$$

$$= \frac{1}{2} - 3\left(-\frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{3}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

$$= \text{R.H.S}$$

2. Show that

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

**A. Proof:** L.H.S

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1^2 + \cos^2\theta}}$$

$$= \frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{\sin^2\theta}}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta - \cot\theta$$

$$= \text{R.H.S}$$

3. $\sin(A-B) = \frac{1}{2}$, $\cos(A+B) = \frac{1}{2}$ find A and B

**Sol:** $\sin(A-B)=\frac{1}{2}$ $\cos(A+B)=\frac{1}{2}$

$\sin(A-B)=\sin 30°$ $\cos(A+B)=\cos 60°$

A–B = 30° ------ (1) A+B = 60° ------- (2)

From (1) and (2)

$A-B=30°$

$A+B=60°$

$2A=90°$

$A=45°$

Substitute A value in (1)

45° – B = 30°

–B = 30 – 45°

$-B=-15°$

$B=15°$

∴ A = 45°, B = 15°

4. Show that cos (270°+θ) = sinθ

**A.** **Proof:** cos ($270^0$+θ)

= cos [90 + 180 + θ)

Let us take 180 + θ = x,

= cos (90 + x)

= cos [90–(–x)]

= sin (–x)

= –sinx

= –sin [180+θ]

= –sin[90+90+θ]

Let us take 90 + θ = y

= –sin(90+y)

= –sin[90–(–y)]

= –cos (–y)

= –cos y

= –cos [90–(θ)]

= –sin(–θ)

= –(–sinθ)

= sinθ

= R.H.S

## 1 Mark Questions

1. Define 'radian'

A. An angle made by an arc of a circle equals to its radius subtended at it's centre is called 'radian'.

2. Show that $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta.\operatorname{cosec}^2\theta$

**A. Proof:** L.H.S

$\sec^2\theta + \operatorname{cosec}^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta.\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta.\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta} . \frac{1}{\sin^2\theta}$$

$= \sec^2\theta.\operatorname{cosec}^2\theta$

R.H.S

3. If $x = \cos\theta + \sin\theta$, $y = \cos\theta.\sin\theta$, then eliminate $\theta$.

**Sol.** $x = \cos\theta + \sin\theta$

Squaring on both sides

$(x)^2 = (\cos\theta + \sin\theta)^2$

$x^2 = \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta$

$x^2 = 1 + 2y$

$\therefore \boxed{x^2 = 1 + 2y}$ or $\boxed{x^2 - 2y = 1}$

4. Find the value of $\sin 22½° . \cos 22½°$

**Sol.** $\sin 22½° . \cos 22½°$

$$= \frac{2}{2} . \sin 22½° . \cos 22½°$$

$$= \frac{1}{2} . \sin 2.(22½°)$$

$$= \frac{1}{2} . \sin 45°$$

$$= \frac{1}{2} . \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

5. $x = \sec\theta + \tan\theta$, $y = \sec\theta - \tan\theta$ then find xy.

**Sol:** $x = \sec\theta + \tan\theta$,

$y = \sec\theta - \tan\theta$

$xy = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$xy = \sec^2\theta - \tan^2\theta$

$\boxed{xy = 1}$

## Multiple Choice Questions
## (1/2 Mark Each)

1. A sexgesimal of 72° is equal in circular measure to ———

a) $\frac{\pi}{5}$ b) $\frac{2\pi}{5}$ c) $\pi$ d) $\frac{2\pi}{3}$

2. A minute hand of a table clock is 3 cm long. How far it's top moves in 20 minutes?

a) 3 cm b) 9 cm c) $\frac{22}{7}$ cm d) $\frac{44}{7}$ cm

3. 1° = ——— radians.

a) 0.07146 b) 0.01746 c) 0.01764 d) 0.01756

4. The relationship among D, G and C is ———

a) $\frac{D}{90} = \frac{G}{200} = \frac{C}{\pi/2}$ b) $\frac{D}{180} = \frac{C}{200} = \frac{C}{\pi}$

c) $\frac{D}{180} = \frac{G}{200} = \frac{C}{\pi}$ d) $\frac{D}{90} = \frac{G}{100} = \frac{C}{\pi}$

5. If $8\tan\theta = 15$ then $\cot\theta$ = ———

a) 8/15 b) 15/8 c) 8/17 d) 17/8

6. $x = a\sec\theta$, $y = a\tan\theta$ then $x^2 - y^2$ = ———

a) $a^2$ b) a c) 0 d) 1

7. $\tan^2 60° - \sec^2 60°$ = ———

a) 1 b) –1 c) $\sqrt{2}$ d) 0

8. If $\tan\theta$ is not defined, then $\theta$ = ———

a) 90° b) 45° c) 60° d) 0°

9. The value of $\sin\theta$ in terms of $\sec\theta$ is ———

a) $\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$ b) $\sqrt{\sec^2\theta-1}$ c) $\frac{1}{\sqrt{\sec^2\theta-1}}$ d) $\frac{1}{\sec\theta}$

10. $\cos0° + \sqrt{2}.\sin 45° + \sin A = 3$ then A = ———
a) 90° b) 30° c) 45° d) 60°

11. cosec (270+θ) = ———
a) –secθ b) secθ c) cosecθ d) –cosecθ

12. sin 30° = 1/2 then cos 60° = ———
a) √2 b) √3 c) 1/2 d) √3/2

13. Which of the following is not a trigonometric identity?
a) $\sin^2\theta + \cos^2\theta = 1$ b) $\sec^2\theta + \tan^2\theta = 1$
c) $\text{cosec}^2\theta = 1 + \cot^2\theta$ d) $\sec^2\theta - \tan^2\theta = 1$

14. 1 radian ———
a) 57°18' b) 56°17' c) 57°16' d) 51°67'

15. If $\sin2\theta = \cos 3\theta$ then $\tan 5\theta$ = ———
a) 0 b) ∞ c) 1 d) –1

**KEY**

| | | | | | |
|---|---|---|---|---|---|
| 1) b | 2) d | 3) b | 4) b | 5) a | 6) a |
| 7) b | 8) 1 | 9) a | 10) a | 11) a | 12) c |
| 13) b | 14) c | 15) b | | | |

## Fill in the Blanks
## (1/2 Mark Each)

1. 1° = ——— radians.
2. $1^c$ = ——— (degrees)
3. If $\tan\theta + \cot\theta = 2$, then $\tan^2\theta + \cot^2\theta$ = ———
4. If $\sin2\theta = \cos 3\theta$ then $\tan 5\theta$ = ———
5. cos 720° = ———
6. $\sin^2 1° + \sin^2 89°$ = ———
7. $\cos0°.\cos1°.\cos2°.\cos3°..... \cos 180°$ = ———
8. $\cot\theta + \text{cosec}\theta = x$ then $\cot\theta - \text{cosec}\theta$ = ———
9. If $\tan(15°+B) = \frac{1}{\sqrt{3}}$ then B = ———
10. Angle made by a side of a hexagon at it's centre is ———

**KEY**

1) 0.01746

2) 57°16'

3) 2

4) $\infty$

5) 1

6) 1

7) 0

8) $-\frac{1}{x}$

9) 15°

10) 60°