Permutations and Combinations

FACTORIAL

The continued product of first n natural numbers is called the " n factorial" and is denoted by $\underline{n}$ or n! i.e. $n! = 1 \times 2 \times 3 \times 4 \times ... \times (n-1) \times n$

EXPONENT OF PRIME *p* in n!

Let p be a prime number and n be a positive integer. Then, the last integer amongst 1, 2, 3… ., (n-1), n which is divisible by p is $\left[\frac{n}{p}\right] p$, where $\left[\frac{n}{p}\right]$ denotes the greatest integer less than or equal to $\frac{n}{p}$.

(1) Find the number of zeros at the end of 100!.

(2) Find the largest positive integer n for which 35! Is divisible by 3^n.

SYMBOL ${}^nPr$ OR, P(n, r) :

If n is a natural number and r is a positive integer satisfying $0 \le r \le n$, then the natural number $\frac{n!}{(n-r)!}$ is denoted by the symbol ${}^nP_r$ or, P(n, r)

PROPERTIES OF ${}^nC_r$ or, C(n, r)

We shall now discuss some important properties of ${}^nC_r$.

PROPERTY I ${}^nC_r = {}^nC_{n-r}$ for $0 \le r \le n$

PROPERTY II Let n and r be non-negative such that r ≤ n. Then, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

PROPERTY III Let n and r be non-negative integers such that $1 \le r \le n$. Then,

$${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$$

PROPERTY IV If $1 \le r \le n$, then $n \cdot {}^{n-1}C_{r-1} = (n-r+1) {}^nC_{r-1}$.

PROPERTY V If ${}^nC_x = {}^nC_y \Rightarrow$ x = y or, x+y=n.

PROPERTY VI If If n is even, then the greatest value of ${}^nC_r (0 \le r \le n)$ is ${}^nC_{n/2}$

PROPERTY VII If n is odd then the greatest value of ${}^nCr \ (0 \le r \le n)$ is

$$\frac{{}^nC_n + 1}{2} \text{ or, } \frac{{}^nC_n - 1}{2}$$

FUNDAMENTAL PRINCIPLES OF MULTIPLICATION

If there are two jobs such that one of them can be completed in m ways, and when it has been completed in any one of these m ways, second job can be completed in n ways; then the two jobs in succession can be completed in m × n wasy.

FUNDAMETNAL PRINCIPLE OF ADDTION

If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in (m+n) ways.

(1) Five persons entered the lift cabin on the ground floor of an 8 floor house. Suppose each of them can leave the cabin independently at any floor beginning with the first. Find the total number of ways in which each of the five persons can leave the cabin (i) at any one of the 7 floors (ii) at different floors.

COMBINATIONS Each of the different selections made by taking some or all of a number of distinct objects or items, irrespective of their arrangements or order in which they are placed, is called a combination.

PERMUTATIONS Each of the different arrangements which can be made by taking some or all of a number of distinct objects is called a permutation.

THEOREM 1: Let r and n be positive integers such that $1 \le r \le n$. Then prove that the number of all permutations of n distinct items or objects taken r at a time is.

THEOREM 2: The number of all permutations (arrangements) of n distinct objects taken all at a time is n!.

THEOREM 3: The number of ways of selecting items or objects from a group of n distinct items or objects is

PERMUTATIONS UNDER CERTAIN CONDITIONS

THEOREM: The number of all permutations (arrangements) of n different objects taken r at a time,

(i) When a particular object is to be always included in each arrangement is ${}^{n-1}C_{r-1} \times r!$

(ii) When a particular object is never taken in each arrangement is ${}^{n-1}C_{r-1} \times r!$

(iii) When two specified objects always occur together is ${}^{n-2}C_{r-1} \times r! \times 2!$

(1) Find the number of ways of arranging m white and n black balls in a row (m > n) so that no two black balls are placed

PERMUTATIONS OF OBJECTS NOT ALL DISTINCT

THEOREM: The number of mutually distinguishable permutations of n things, take all at a time, of which p are alike of one kind, q alike of second such that p+q=n, is

$$\frac{n!}{p!q!}$$

(1) Find the number of ways of selecting two integers a and b from the set $\{1, 2, \ldots, 5n\}, n \in N$ so that $a^4 - b^4$ is divisible by 5.