Areas of Bounded Regions

THEOREM Let f(x) be a continuous function defined on [a, b]. Then, the area bounded by the curve y=f(x), the x-axis and the ordinates x = a and x = b is given by

$$\int_a^b f(x)\,dx \text{ or, } \int_a^b y\,dx$$

The area bounded by the curve x = f(y), the y axis and the abscissae y = c and y = d is given by

$$\int_c^d f(y)\,dy \text{ or, } \int_c^d x\,dy$$

(1) Let f(x) = maximum $\{x^2, (1-x)^2, 2x(1-x)\}$. Determine the area of the region bounded by the curve y=f(x), x-axis, x = 0 and x = 1.

(2) Let f(x) be defined by f(x) = max (4sinx, 4-2 sinx), $0 \le x \le 2\pi$. Draw a sketch of y=f(x) and compute the area bounded by the curves y=f(x), the y-axis, the x-axis and the ordinate at $x = 2\pi$.

(3) Find the area lying on the same side of the axis of x, as the positive part of the axis of y and which is contained by $y^2 = 4ax$, $x^2 + y^2 = 2ax$ and x=y + 2a..