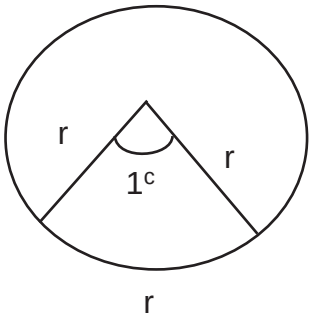
TRIGONOMETRY

This chapter comes under Paper-II. From this chapter 5 Marks question -1 (1×5 = 5M), 4 Marks questions - 2 (2×4 = 8M), 2 Marks questions - 2 (2×2 = 4M), 1 Mark question - 1 (1×1 = 1M) and 6 objective bits will be given. Altogether we can score 21 marks from it. This chapter is not only very important for S.S.C exam but also to higher classes like Intermediate and Engineering etc. The important material relevant to this topic is given below for the sake of the students who are going to appear S.S.C public exams.

- Trigonometry is derived from three "Greek" roots ie Tri + Goma + Metron. Here Tri-means three, Gomia means angle and Metron means measurement, this Trigonometry is the study of "Three angle measurement".
- Trigonometry is an analytical study of a three angled Geometric figure ie "triangle".
- "Hipparchus" established the relationship between the sides and angles of a triangle.
- **Radian:** The angle subtended by an arc of length equals to the radius of the circle at it's centre is called 'radian'.
- $2\pi^c = 360°$, $\pi^c = 180°$
- $90° = 100^g = \pi^c/2$ or $180° = 200^g = \pi^c$
- The relationship among D, G, and C is

$$\frac{D}{90} = \frac{G}{100} = \frac{C}{\pi/2} \text{ or } \frac{D}{180} = \frac{G}{200} = \frac{C}{\pi}$$



- If l is the length of arc of a sector, r is the radius, θ is the angle at the centre of sector then the relationship among them is $\boxed{l = r\theta}$ or $\boxed{r = \frac{l}{\theta}}$ or $\boxed{\theta = \frac{l}{r}}$,

  Here θ should be radian only.
- $1° = 0.01746$ radian ; $1^c = 58°16'$
- $3\pi^c/4 = 135°$ (degrees)
- $120° = 400^g/3$ (grades)
- $270° = 3\pi^c/2$ (radians)

Problem: 1

(1 Mark)

A minute hand of a table clock is 3 cm long. How far it's tip move in 20 minutes?

Sol:

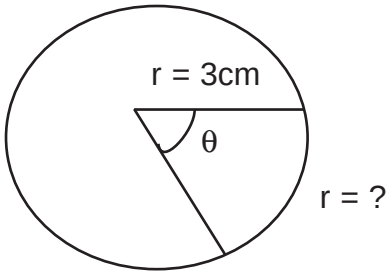
The length of minutes hand

r = 3 cm

Angle made by minutes hand in 60 minutes ---- 360°

20 minutes ------- ?(θ)

$$\theta = \frac{20 \times \overset{6}{\cancel{360}}}{\underset{1}{\cancel{60}}} = 120^\circ = 120 \times \frac{\pi^c}{180} = \frac{2\pi^c}{3}$$



We know that $\boxed{l = r\theta}$

The distance travelled by tip of minutes hand

$l = r\theta$

$$= \cancel{3} \times \frac{2\pi}{\cancel{3}}$$

$$= 2 \times \frac{22}{7}$$

$$= \frac{44}{7} \text{cm}$$

Problem: 2

(1 Mark)

A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second.

Sol:

A wheel makes

in 1 minute $\rightarrow$ 360 revolutions

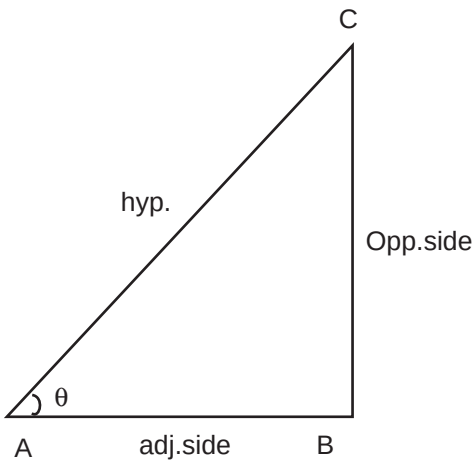
i.e. 60 seconds $\rightarrow$ $360 \times 2\pi^c$

1 second $\rightarrow$ $\frac{\overset{6}{\cancel{360}}}{\underset{1}{\cancel{60}}} \times 2\pi^c$

1 second $\rightarrow 12\pi^c$

$\therefore$ The wheel makes $12\pi^c$ in 1 second.

Trigonometric Ratio's:



- $\sin\theta = \frac{\text{side opp. to } \theta}{\text{hypotenuse}} = \frac{BC}{AC}$
- $\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$
- $\tan\theta = \frac{\text{side opp. to } \theta}{\text{side adj. to } \theta} = \frac{BC}{AB}$
- $\cot\theta = \frac{\text{side adj. to } \theta}{\text{side opp. to } \theta} = \frac{AB}{BC}$
- $\sec\theta = \frac{\text{hyp}}{\text{side adj. to } \theta} = \frac{AC}{AB}$

- $\operatorname{cosec}\theta = \frac{\text{hyp}}{\text{side opp. to } \theta} = \frac{AC}{BC}$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}; \cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\sin\theta.\cos\theta = 1$; $\cos\theta.\sec\theta = 1$; $\tan\theta.\cot\theta = 1$

Trigonometric Values

ratio \ θ	0°	30°	45°	60°	90°
sin	0	1/2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	∞
cot	∞	√3	1	1/√3	0
sec	1	2/√3	√2	2	∞
cosec	∞	2	√2	2/√3	1

Trigonometric Identities:

I. $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$; $\cos^2\theta = 1 - \sin^2\theta$

II. $\sec^2\theta - \tan^2\theta = 1$; $\sec^2\theta = 1 + \tan^2\theta$; $\sec^2 - 1 = \tan^2\theta$

III. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$; $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$; $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

Problem: 3

(2 Marks)

If $\tan(A-B) = \frac{1}{\sqrt{3}}$, $\sin A = \frac{1}{\sqrt{2}}$ find B in circular measure.

Sol:

$\tan(A-B) = 1/\sqrt{3}$

$\tan(A-B) = \tan 30°$

$A - B = 30°$ ____ (1)

$\sin A = 1/\sqrt{2}$

$\sin A = \sin 45°$

$\boxed{A = 45°}$ ____ (2)

Substitute (2) in (1)

$45° - B = 30°$

$-B = 30° - 45°$

$-B = -15°$

$\boxed{B = 15°}$

Now $A = 45^\circ = \cancel{45}^{1} \times \frac{\pi^c}{\cancel{180^c}_{4}} = \frac{\pi^c}{4}$

$B = 15^\circ = \cancel{15}^{1} \times \frac{\pi^c}{\cancel{180}_{12}} = \frac{\pi^c}{12}$

Problem: 4

Find the value of $32\cot^2\frac{\pi^c}{4} - 8\sec^2\frac{\pi^c}{3} + 8\cot^3\frac{\pi^c}{6}$

Sol:

$$32\cot^2\frac{\pi^c}{4} - 8\sec^2\frac{\pi^c}{3} + 8\cot^3\frac{\pi^c}{6}$$

$$= 32\cot^2\frac{\overset{45^\circ}{\cancel{180^\circ}}}{\underset{1}{\cancel{4}}} - 8\sec^2\frac{\overset{60^\circ}{\cancel{180^\circ}}}{\underset{1}{\cancel{3}}} + 8\cot^3\frac{\overset{30^\circ}{\cancel{180^\circ}}}{\underset{1}{\cancel{6}}}$$

$= 32\cot^2 45^\circ - 8\sec^2 60^\circ + 8\cot^3 30^\circ$

$= 32(1)^2 - 8(2)^2 + 8.(\sqrt{3})^3$

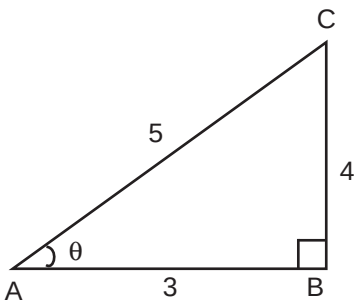
$= 32 - 8(4) + 8 \times 3\sqrt{3}$

$= \cancel{32} - \cancel{32} + 24\sqrt{3}$

$= 24\sqrt{3}$

Problem: 5

If 4 cotA = 3 then find $\frac{\sin A + \cos A}{\sin A - \cos A}$



Sol:

4 cot A = 3

$\Rightarrow \cot A = 3/4 = \frac{\text{side adj. to } \theta}{\text{side opp. to } \theta}$

By Pythagorean Theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 3^2 + 4^2$

$AC^2 = 9 + 16$

$AC^2 = 25 \Rightarrow AC = 5$

$\sin A = \frac{4}{5}, \cos A = \frac{3}{5}$

$$\frac{\sin A + \cos A}{\sin A - \cos A} = \frac{\frac{4}{5} + \frac{3}{5}}{\frac{4}{5} - \frac{3}{5}}$$

$$= \frac{7 / \cancel{5}}{1 / \cancel{5}} = 7$$

Problem: 6

(2 Marks)

Show that $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$

Proof:

L.H.S

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$\frac{\sqrt{(1-\cos\theta)^2}}{\sqrt{1-\cos^2\theta}}$$

$$= \frac{1-\cos\theta}{\sqrt{\sin^2\theta}}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$= \operatorname{cosec}\theta - \cot\theta$

= R.H.S

Problem: 7

(4 Marks)

Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$

Proof:

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - \left[\sec^2\theta - \tan^2\theta\right]}{\tan\theta - \sec\theta + 1} \qquad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$$

$$= \frac{(\tan\theta + \sec\theta) - \left[(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)\left[1 - (\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)\left(1 - \sec\theta + \tan\theta\right)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta}{1}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \frac{\sin\theta + 1}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta}$$

Problem: 8

(4 Marks)

If $\sec\theta + \tan\theta = P$ then show that $\sin\theta = \frac{P^2 - 1}{P^2 + 1}$

Proof:

Given that $P = \sec\theta + \tan\theta$

R.H.S=

$$\frac{P^2 - 1}{P^2 + 1} = \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$= \frac{\sec^2\theta - 1 + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \tan^2\theta + 1 + 2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta\left(\tan\theta + \sec\theta\right)}{2\sec\theta\left(\sec\theta + \tan\theta\right)}$$

$$= \frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{1/\cancel{\cos\theta}}$$

$= \sin\theta$

$= L.H.S$

Problem: 9

(4 Marks)

Eliminate θ from $x = a\sin\theta - b\cos\theta$, $y = a\cos\theta + b\sin\theta$

Sol:

$x = a\sin\theta - b\cos\theta$, $y = a\cos\theta + b\sin\theta$

Consider $x = a\sin\theta - b\cos\theta$

Squaring on both sides

$x^2 = (a\sin\theta - b\cos\theta)^2$

$x^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$ ____ (1)

Consider $y = a\cos\theta + b\sin\theta$

Squaring on both sides

$y^2 = (a\cos\theta + b\sin\theta)^2$

$y^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta$ ____ (2)

(1) + (2)

$$x^2 + y^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cancel{\sin\theta}\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cancel{\cos\theta}.\sin\theta$$

$\Rightarrow x^2 + y^2 = a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow x^2 + y^2 = a^2(1) + b^2(1)$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

Problem: 10

(1 Mark)

If $x = \sec\theta + \tan\theta$, $y = \sec\theta - \tan\theta$ then find the relationship between x and y.

Sol:

$x = \sec\theta + \tan\theta$

$y = \sec\theta - \tan\theta$

Consider $xy = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta = 1$

- $\sin(90-\theta) = \cos\theta$, $\cos(90-\theta) = \sin\theta$,
  $\tan(90-\theta) = \cot\theta$, $\cot(90-\theta) = \tan\theta$
  $\sec(90-\theta) = \operatorname{cosec}\theta$, $\operatorname{cosec}(90-\theta) = \sec\theta$

- $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$
  $\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$
  $\sec(-\theta) = \sec\theta$, $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Problem: 11

(4 Marks)

Show that sec(270–θ) = –cosecθ

Sol:

L.H.S

sec(270 – θ)

= sec[90 + 180 – θ)

Let 180 – θ= x

= sec[90 + x]

= sec[90–(–x)]

= cosec(–x)

= –cosec x

= –cosec(180–θ)

= –cosec[90 + 90 – θ]

Let 90 – θ=y

= –cosec [90 + y]

= –cosec [90–(–y)]

= –sec(–y)

= –secy

= –sec(90–θ)

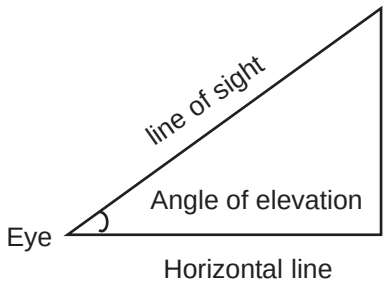
= –cosecθ

= R.H.S

- sin(180 + θ) = –sinθ
- cos(270 + θ) = –sinθ
- tan(90 + θ) = –cotθ
- sec(270 – θ) = –cosecθ
- cot(360 + θ) = cotθ
- cosec (360 – θ) = –cosecθ

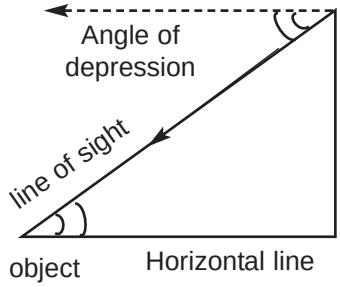
Angle of Elevation:

If the object is at a heigher level than the eye then the angle between the horizontal line drawn through the observers eye and the line joining the eye to an object is called "Angle of Elevation".



Angle of Depression:

If the object is at a lower level than the eye, then the angle between the horizontal line drawn through the observers eye and the line joining the eye to any object is called "Angle of Depression".



Problem: 12

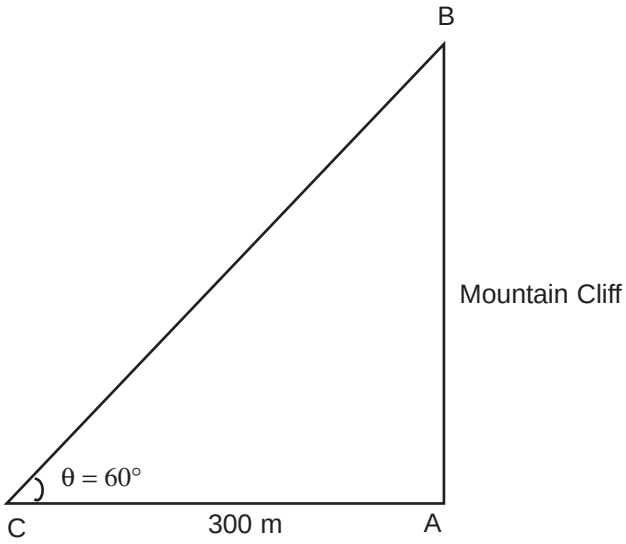
(2 Marks)

Find the height of mountain cliff, if the angle of elevation of it's top, from a point 300m from it's foot is found to be 60°.

Sol:

Let AB is the mountain cliff. AC is the distance between cliff and observing point.

$\angle BCA = 60°$



In ΔABC, $\tan 60° = \frac{AB}{AC}$

$$\sqrt{3} = \frac{AB}{300}$$

$\Rightarrow AB = 300\sqrt{3}$ m

$\Rightarrow AB = 300 \times 1.732$

$\Rightarrow AB = 519.6$m

$\therefore$ Height of mountain cliff = 519.6m

Problem: 13

(4 Marks)

The upper part of a tree, broken by wind into two parts, makes an angle of 30° with the ground. The top of the tree touches the ground at a distance of 20m from the foot of the tree. Find the height of the tree before it was broken.

Sol:

Let AB is the tree before it was broken.

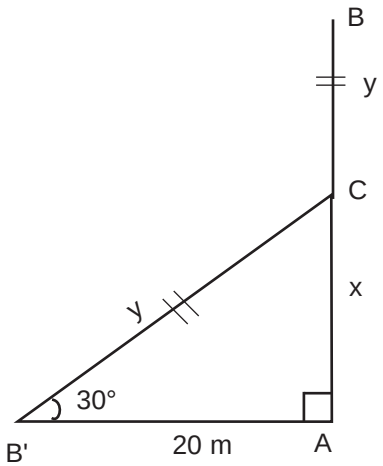
C is broken point.

CB' is broken part.

$\angle CB'A = 30°$

AB' = 20m

Let AC = x, BC = y = B'C



In $\Delta CB'A$, $\tan 30° = \frac{AC}{AB'}$

$$\frac{1}{\sqrt{3}} = \frac{x}{20}$$

$$\Rightarrow x = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{20\sqrt{3}}{3} \text{m} \quad \text{____ (1)}$$

In $\Delta CB'A$, $\cos 30° = \frac{AB'}{CB'}$

$$\frac{\sqrt{3}}{2} = \frac{20}{y}$$

$$y = \frac{20 \times 2}{\sqrt{3}}$$

$$y = \frac{40\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$y = \frac{40\sqrt{3}}{3} \quad \text{____} (2)$$

The height of tree before it was broken = x + y

$$= \frac{20\sqrt{3}}{3} + \frac{40\sqrt{3}}{3}$$

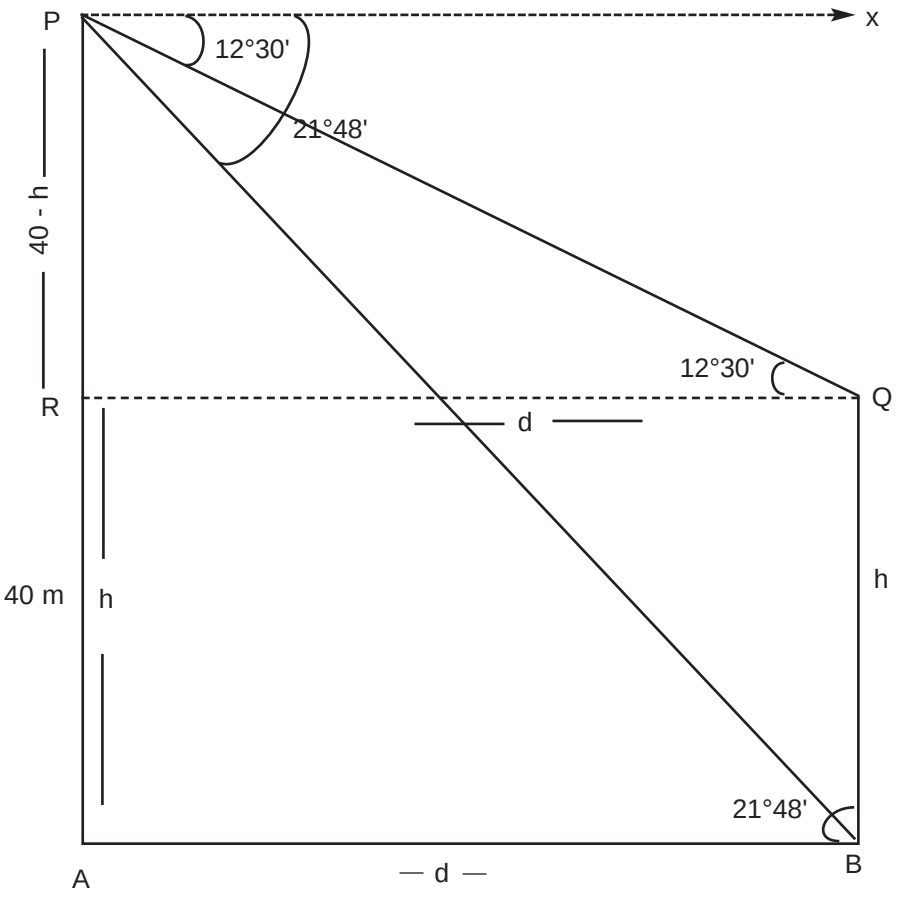
$$= \frac{\overset{20}{\cancel{60}}\sqrt{3}}{\cancel{3}}$$

$= 20\sqrt{3}$
$= 20 \times 1.732$
$= 34.64m$

Problem: 14

There are two temples, one on each bank of a river, just opposite to each other. One of the temple A is 40m high. As observed from the top of this temple 'A', the angles of depression of the top and foot of the other temple 'B' are 12°30' and 21°48' respectively. Find the width of the river and height of the temple "B".

Sol:



AP & BQ are two temples.
AP = 40m
AB is the width of river.
$\angle QPX = 12°30' \Rightarrow \angle RQP = 12°30'$,
$\angle BPX = 21°48' \Rightarrow \angle ABP = 21°48'$
Let AB = d $\Rightarrow$ RQ = d
Let BQ = h $\Rightarrow$ AR = h $\Rightarrow$ PR = 40–h

Consider ΔPQR, $\tan 12°30' = \frac{40-h}{d}$

$$0.2217 = \frac{40-h}{d} \quad \text{---- (1)}$$

In ΔPBA, $\tan 21°48' = 40/d$
$0.4000 = 40/d$

$$d \times \frac{4000}{10000} = 40$$

$$d = \frac{\overset{1}{\cancel{4}}0 \times 10,\cancel{000}}{\cancel{4}\,\cancel{000}_{1}}$$

$$\boxed{d = 100m}$$

Substitute 'd' value in (1)

$$0.2217 = \frac{40-h}{100}$$

$$\frac{2217}{10,\cancel{000}} \times 100 = 40 - h$$

$$\frac{2217}{100} = 40 - h$$

22.17 = 40–h
h = 40 – 22.17
h = 17.83m
$\therefore$ Width of the river (d) = 100m.
The height of temple B = 17.83m

Objective Bits:

1. If $\sin\theta = \cos\theta$ then θ = _____
2. If $\tan\theta - \sec\theta = 1$ then $\tan\theta + \sec\theta$ = _____
3. $\text{cosec}^2 9° - \cot^2 9°$ = _____
4. $\sin 420°$ = _____
5. $\sin^2 9° + \sin^2 81°$ = _____
6. If $\tan\theta = a/b$ then $\sin\theta$ = _____
7. If $\sin 2\theta = \cos 3\theta$ then (i) $\tan 5\theta$ =
   (ii) $\cot 5\theta$ =

8. $\frac{\sin 18^\circ}{\cos 72^\circ} =$ _____
9. $\sin 15^\circ \cos 15^\circ =$
10. $1^\circ =$ _____ radians
11. $x = \sec\theta - \tan\theta$, $y = \sec\theta + \tan\theta$ then the relationship between x and y is ____
12. $\cos 240^\circ =$ _____
13. The relationship among D, G, and C is
14. $\sin^2 58^\circ + \cos^2 58^\circ =$_____
15. An angle made by a side of a 24 sided regular polygon at it's centre is ____

Answers:

1. $\theta = \frac{\pi^c}{4}$ or 45°
2. –1
3. 1
4. $\sqrt{3}/2$
5. 1
6. $\frac{a}{\sqrt{a^2 + b^2}}$
7. i) ∞ ii) 0
8. 1
9. 1/4
10. 0.01746
11. xy = 1
12. –1/2
13. $\frac{D}{90} = \frac{G}{100} = \frac{C}{\pi/2}$ or $\frac{D}{180} = \frac{G}{200} = \frac{C}{\pi}$
14. 1
15. 15°

Assignment:

(2 Marks)

1. The angles of a triangle are in A.P and the greatest is three times the least. Then find the angles of triangle in circular measure.
2. Prove that $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) = 2$

(1 Mark)

3. Find the value of $\cos 0^\circ + \sin 90^\circ + \sqrt{2}.\sin 45^\circ$
4. Show that $\tan^2\theta + \tan^4\theta = \sec^4\theta - \sec^2\theta$

(4 Marks)

5. Solve $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

(2 Marks)

6. Show that $\cos(270 + \theta) = \sin\theta$

(4 Marks)

7. A ladder 20m long is placed against a vertical wall of height 10m. Find the distance between the foot of the ladder and the wall, also find the inclination of the ladder with the horizontal.

(4 Marks)

8. An aeroplane at an altitude of 2500 mts observes the angles of depression of opposite points on the two banks of a river to be 41°20' and 52°10'. Find the width of the river in metres.