ANALYTICAL GEOMETRY

This chapter comes under Paper - II. From this chapter 4 mark questions-2 (2×4=8m), 2 marks questions - 2 (2×2=4m), 1 Mark Questions - 2 (2 ×1=2M) and 6 objective bits (6×1/2=3M), altogether 17 Marks, very easily can be scored. Even though there are many models in this chapter, there are few questions which will very often repeated. The following material related to this chapter will help alot for the students who are going to appear S.S.C. public exams.

- "Rene Descarter" is known as "Father of Analytical Geometry".
- If inclination of a line is θ, then slope m = tan θ
- Slope of a line passing through (x_1, y_1) and (x_2, y_2) i.e. $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Slope of x-axis is '0'
- Slope of x-axis is indefined
- The distance between two points A (x_1, y_1) and B (x_2, y_2) is $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The distance between origin to a point (x, y) is $\sqrt{x^2 + y^2}$ units.
- The mid point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Q. Find the point on x-axis that is equidistant from (2, 3) and (4, – 5) **2M**

sol: Let A (2, 3) B(4, 5) and P (x, 0) is a point on x-axis such that PA = PB.

P (x, 0) A (2, 3), P (x, 0) B (4, –2)

PA = PB

$\sqrt{(2-x)^2 + (3-0)^2} = \sqrt{(4-x)^2 + (-2-0)^2}$

$(2 - x)^2 + 9 = (4 - x)^2 + 4$

$4 + x^2 - 4x + 9 = 16 + x^2 - 8x + 4$

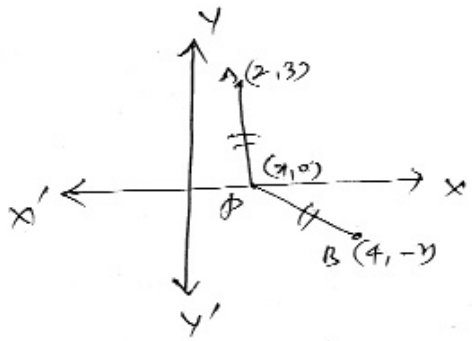
$8x - 4x + 9 = 16 + 4 - 4 - 9$

$4x = 7$

$x = 7/4$

P (x, 0) = (7/4, 0)

∴ Required point on x-axis is (7/4, 0)



2. If the triangle formed with A (8, – 10) B (7, –3) and C (0, p) is right angled with right angle at B, find "p". **4M**

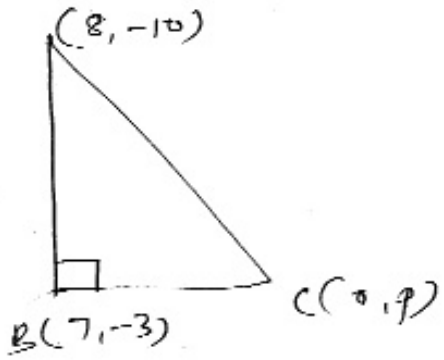
sol: A (8, – 10) B (7, – 3) C (0, p)

A (x_1, y_1) B (x_2, y_2)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

A (8, – 10) B (7, – 3)

$$AB = \sqrt{(7-8)^2 + (-3-(10))^2}$$



$$= \sqrt{(-1)^2 + (7)^2}$$

$$= \sqrt{1+49} = \sqrt{50} \text{ units}$$

B (7, – 3) C (0, p)

$$BC = \sqrt{(0-7)^2 + (p-(-3))^2}$$

$$= \sqrt{49 + (p+3)^2} \text{ units}$$

A (8, – 10) C (0, p)

$$AC = \sqrt{(0-8)^2 + (p-(-10))^2}$$

$$\sqrt{64 + (p+10)^2} \text{ units}$$

Given that ΔABC is right angled at B.

$AC^2 = AB^2 + BC^2$

$$\left[\sqrt{64 + (p+10)^2}\right]^2 = \left(\sqrt{50}\right)^2 + \left[\sqrt{49 + (p+3)^2}\right]^2$$

$$64 + p^2 + 20p + 100 = 50 + 49 + p^2 + 6p + 9$$

$$20p - 6p = 108 - 164$$

$$14p = -56$$

$$p = -4$$

3. If p (6, – 1) Q (1, 3) and R (x, 8), find the value of 'x'. So that PQ = QR **2M**

sol: p (6, –1) Q (1, 3) R (x, 8)

A (x_1, y_1) and B (x_2, y_2) $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

p (6, –1) Q (1, 3); Q (1, 3) R(x, 8)

PQ = QR

$$\Rightarrow \sqrt{(1-6)^2 + (3-(-1))^2} = \sqrt{(x-1)^2 + (8-3)^2}$$

$\Rightarrow \sqrt{(-5)^2+(4)^2} = \sqrt{(x-1)^2+(5)^2}$

$\Rightarrow 25+16=(x-1)^2+25$

$\Rightarrow (x-1)^2 = 16$

$\Rightarrow (x-1) = \sqrt{16}$

$\Rightarrow (x-1) = \pm 4$

i) if x – 1 = 4
x = 4 + 1
x = 5

ii) if x – 1 = – 4
x = – 4 + 1
x = – 3
∴ x = 5 or – 3

Internally dividing point.

- Internally dividing point in the given ratio m:n $P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

- Externally dividing point in the given ratio m:n $P(x,y) = \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

4. Find the co-ordinates of the points of trisection of a segmant joining A (–3, 2) and B(9, 5) **4M**

sol: The points which divides the segment in the ratio of 1: 2 and 2: 1 are called "Trisecting Points".

i) A (–3, 2) B (9, 5) ; m : n = 1: 2
x_1, y_1 x_2, y_2

Internally dividing point $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$\left(\frac{1(9)+2(-3)}{1+2}, \frac{1(5)+2(2)}{1+2}\right)$

$\left(\frac{9-6}{3}, \frac{5+4}{3}\right)$

$\left(\frac{3}{3}, \frac{9}{3}\right)$

= (1, 3)

i) A (–3, 2) B (9, 5); m : n = 2: 1

x_1, y_1 x_2, y_2

Internally dividing point $= \left(\frac{2(9)+1(-3)}{2+1}, \frac{2(5)+1(2)}{2+1}\right)$

$\left(\frac{18-3}{3}, \frac{10+2}{3}\right)$

$\left(\frac{15}{3}, \frac{12}{3}\right)$

= (5, 4)

∴ The required trisecting points are (1, 3) and (5, 3)

5. In what ratio the segment joining the points (–3, 2) and (6, 1) divided by y-axis. **4M**

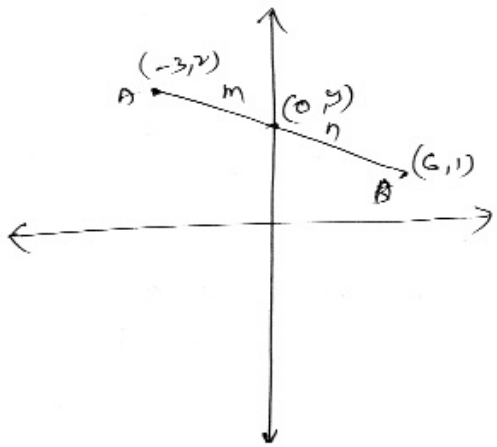
sol: A (–3, 2) B (6, 1)

Let the dividing point on y-axis be (0, y) in the ratio of m:n

A (–3, 2) B (6, 1)

x_1, y_1 x_2, y_2

∴ Internally dividing point $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$



$(0,y) = \left(\frac{m(6)+n(-3)}{m+n}, \frac{m(1)+n(2)}{m+n}\right)$

$\Rightarrow 0 = \frac{6m-3n}{m+n}$

$\Rightarrow$ 0 = 6m – 3n

$\Rightarrow \not{}6m = \not{}3n$

$\Rightarrow \frac{m}{n} = \frac{\not{3}}{\not{6}_2}$

$\Rightarrow \frac{m}{n} = \frac{1}{2}$

$\Rightarrow$ m:n = 1:2

∴ Required dividing ratio by y-axis is 1:2

6. One end of the diameter of a circle is (2, 3) and the centre is (–2, 5). Find the co-ordinates of the other end. **4M**

sol: One end of the diameter (x_1, y_1) = (2, 3)

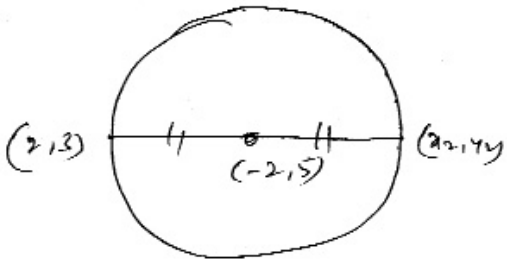
Centre of Circle = (–2, 5)

Let the other end of diameter be (x_2, y_2)

We know that centre of the circle is the midpoint of diameter

Mid Point $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$(-2, 5) = \left(\frac{2 + x_2}{2}, \frac{3 + y_2}{2}\right)$



$-2 = \frac{2 + x_2}{2}$	$5 = \frac{3 + y_2}{2}$
$-4 = 2 + x_2$	$10 = 3 + y_2$
$-4 - 2 = x_2$	$10 - 3 = y_2$
$-6 = x_2$	$7 = y_2$
$x_2 = -6$	$y_2 = 7$

$\therefore$ other end of diameter = (–6, 7)

- Area of triangle formed by (x_1, y_1), (x_2, y_2), (x_3, y_3)
  i.e. $\Delta = 1/2\ |x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)|$ sq.units
- $\Delta = 0$ Given points are collinear.

7. Prove that (4, –3), (1, 3) and (5, –5) are collinear i.e. they lie on same straight line. 2M

sol: (4, –3), (1, 3) and (5, –5)

x_1, y_1 x_2, y_2 x_3, y_3

To prove the given points are collinear, it is enough to prove that $\Delta = 0$

$\Delta = 1/2\ |x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)|$ sq.units

$= 1/2\ |4 (3 - (-5)) + 1 (-5 - (-3)) + 5 (-3 - 3)|$ sq.units

$= 1/2\ |4 (8) + 1 (-2) + 5 (-6)|$ sq.units

$= 1/2\ |32 - 2 - 30|$ sq.units

$= 1/2\ |0| = 0$

Here $\Delta = 0$ given points are collinear.

8. If the three points A (p, 2), B (–3, 4) and C (7, –1) are collinear, find the value of 'p'. 2M

sol: A (p, 2), B (–3, 4) and C (7, –1)

x_1, y_1 x_2, y_2 x_3, y_3

A, B, C are collinear $\Leftrightarrow \Delta = 0$

$\Delta = 1/2\ |x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)|$sq.units

$0 = 1/2\ |p (4 - (-1)) + (-3) (-1 - 2) + 7 (2 - 4)|$

$2 \times 0 = |p (5) + (-3) (-3) + 7 (-2)|$

$0 = |5p + 9 - 4|$

$0 = |5p - 5|$ [$\because$ $|x| = 0$ solution is $x = 0$]

$\Rightarrow 5p - 5 = 0$

$\Rightarrow \frac{5}{1}p = \frac{5}{1}$

$\Rightarrow p = 1$

9. If the area of the triangle formed with the vertices (t, 2t), (–2, 6) and (3, 1) is 5 square units, then fine "t". 4M

sol: (t, 2t) (– 2, 6) (3, 1)

x_1, y_1 x_2, y_2 x_3, y_3

Given that $\Delta = 5$ square units

$\Delta = 1/2\ |x_1 (y_2 - y_3) + x_2(y_3 - y_1) + x_3 (y_1 - y_2)|$ sq.units

$5 = 1/2\ |t (6 - 1) + (-2)(1 - 2t) + 3 (2t - 6)|$ sq.units

$10 = |t (5) + (- 2) (1 - 2t) + 3 (2t - 6)|$

$10 = |5t - 2 + 4t + 6t - 18|$

$10 = |15t - 20|$

$\Rightarrow 15t - 20 = 10$ or $15t - 20 = -10$ [$|x| = a$ solution is $x = a$ or $x = -a$]

$\Rightarrow 15t = 10 + 20$ or $15t = 20 - 10$

$\Rightarrow \frac{15}{1}t = \frac{30}{2}$ or $\frac{15}{3}t = \frac{10}{2}$

$\Rightarrow t = 2$ or $2t = 3$

$\Rightarrow t = 2$ or $t = 2/3$

1. Gradient form of slope of a line is $y=mx$, $m \rightarrow$ slope

2. Slope intercept form of a line is
   $y = mx + c$; $m \rightarrow$ slope, $c \rightarrow$ y-intercept.

3. Slope point form of a line is
   $y - y_1 = m(x - x_1)$; $m \rightarrow$ slope, $(x_1, y_1) \rightarrow$ point

4. Two - points form of line is

   $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ (x_1, y_1) & (x_2, y_2) are two points.

5. Two intercepts form of a line

   $\frac{x}{a} + \frac{y}{b} = 1$; $a \rightarrow$ x - intercept, $b \rightarrow$ y - intercept

6. Standard form of a straight line is $ax+by+c = 0$ if and only if $|a| + |b| \neq 0$

- If two straight lines are parallel, their equation differ only by a constant.

- .Two lines are parallel if and only if their slopes are equal.

- If the product of slopes of any two line is "–1" then they are perpendicular to each other but converse need not be true.

10. Find the x-intercept and y-intercept of the line 2x–6y+7 = 0 **1M**

sol: To get x - intercept, put y = 0

2x – 6 (0) + 7 = 0

2x = – 7

x = – 7/2

x - intercept (a) = – 7/2

To get y - intercept, put x = 0

2 (0) – 6y + 7 = 0

– 6y = – 7

y = 7/6

∴ y - intercept b = 7/6

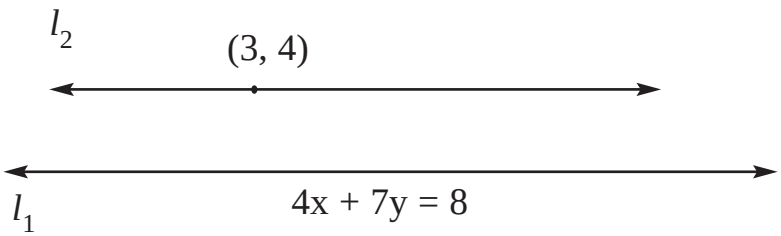
11. Find the equation of a line passing through (3, 4) and parallel to 4x+7y=8. **1M**

sol: Given line

l_1 : 4x + 7y = 8

7y = – 4x + 8

$$y = \left(\frac{-4}{7}\right)x + \frac{8}{7}$$



y = mx + c

slope (m1) = –4/7

Let the required line be l_2

and it's slope be m_2

Since $l_2 \parallel l_1 \Rightarrow m_2 = m_1$

$\Rightarrow m_2 = -4/7$

l_2 – slope m_2 = – 4 / 7 and point (3, 4)

x_1, y_1

Slope point form of line is

$y - y_1 = m(x - x_1)$

y – 4 = – 4/7 (x – 3)

7 (y – 4) = – 4 (x – 3)

7y – 28 = – 4x + 12

4x + 7y – 28 – 12 = 0

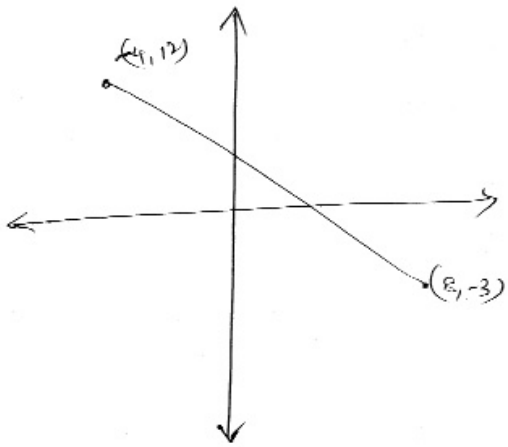
4x + 7y – 40 = 0

∴ Required line is 4x + 7y – 40 = 0

12. Find the area of the triangle enclosed between the co-ordinate axes and the line passing through (8, – 3) and (– 4, 12). 4M

sol: (8, – 3) (– 4, 12)

x_1 y_1 x_2 y_2



The points form of a line is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-3) = \frac{12 - (-3)}{-4 - 8}(x - 8)$$

$$y + 3 = \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{-12}}}(x - 8)$$

$$y + 3 = \frac{-5}{4}(x - 8)$$

4 (y+3) = – 5 (x – 8)
4y + 12 = – 5x + 40
5x + 4y + 12 – 40 = 0
5x + 4y – 28 = 0

To get x-intercept put y = 0.
5x – 28 = 0
5x = 28
x = 28/5
∴ x-intercept (a) = 28/5

To get y-intercept put x = 0
4y – 28 = 0
4y = 28
y = 7
∴ y - intercept (b) = 7

Area of triangle enclosed with the coordinate axes = 1/2 |ab| sq. units.

$$= \frac{1}{2}\left|\frac{28}{5} \times 7\right|$$

$$= \frac{1}{2}\left|\frac{196}{5}\right|$$

$$= \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{98}{\cancel{196}}}{5} = \frac{98}{5}$$

13. Find the equation of the line that cuts off intercepts a and b on the x-axis and y-axis such that a+b = 2 and ab = – 3 **4M**

sol: a+b = 2, ab = – 3

x - intercept = a

y - intercept = b

we know that

$(a-b)^2 = (a+b)^2 - 4ab$

$(a-b)^2 = 2^2 - 4(-3)$

$(a-b)^2 = 4 + 12$

$(a-b)^2 = 16$

$a - b = \sqrt{16}$

$(a-b) = \pm 4$

i) $a - b = 4$ .....(1)

$a + b = 2$ .....(2)

$2a = 6$

$a = 3$

substitute a value in (2)

$3 + b = 2$

$b = 2 - 3$

$b = -1$

∴ Two intercepts form of a line is

$\frac{x}{a} + \frac{y}{b} = 1;$

$\frac{x}{3} + \frac{y}{-1} = 1$

$\frac{-x + 3y}{-3} = 1$

$-x + 3y = -3$

$-x + 3y + 3 = 0$

$x - 3y - 3 = 0$

ii) $a - b = -4$

$a + b = 2$

$2a = -2$

$a = -1$

Substitute a value in (2)

a + b = 2
– 1 + b = 2
b = 2 + 1 = 3
b = 3

Two - intercept form of a line is

$$\frac{x}{-1}+\frac{y}{3}=1$$

$$\frac{3x+(-y)}{-3}=1$$

3x – y = – 3
3x – y + 3 = 0

∴ Required line equations are x – 3y – 3 = 0 and 3x – y + 3 = 0

Important Objective bits

1. Slope of a line $\frac{x}{a}+\frac{y}{b}=1$ is <u>– b/a</u>
2. Inclination of a line is 150° then the slope of a line is <u>-1/√3</u>
3. If the area of a triangle is 96 sq. units then the area of triangle formed by midpoint of sides is <u>24</u> sq.units
4. Centroid of a triangle formed by the vertices (–1, 0), (5, – 2) and (8, – 2) is <u>(4, – 4/3)</u>
5. Centroid divides the median in the ratio of <u>2:1</u> from the vertex and <u>1:2</u> from the midpoint of opposite side.
6. The ratio divided by x-axis to the line segment joining the points (4, 6) and (–7, –1) is 6:1
7. The slope of a line perpendicular to the line 5x – 2y + 4 = 0 is –2/5
8. y = mx + c cuts the x-axis at (–c/m, 0) and y-axis at (0, c)
9. One end of the diameter of a circle is (3, 2) and the centre is (0, 0) then the other end of the circle is (–3, –2)
10. Area of the triangle enclosed between the co-ordinate axes and the line joining the points (4, 0) and (0, 5) is 10 sq.units.

Assignment:

1. If P (2, 5), Q (x, – 7) find the possible values of x such that PQ = 13 **2M**
2. Find the ratio in which P (3, 0) divides the line joint of A (1, a) and B(7, –4). Hence find 'a'. **2M**
3. Find the area of the triangle formed by the points (–2, 3), (–7, 5) and (3, –5) **2M**
4. Find the equation of a line whose slope is 4/5 and which bisects the line join of P (1, 2) and Q (4,–3). **4M**
5. Find the area of triangle formed by the line 2x – 4y –7 = 0 with the co-ordinate axes.