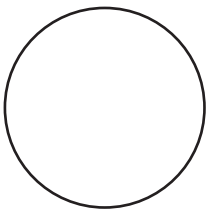
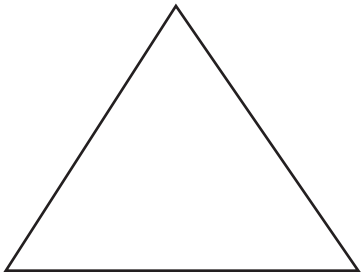
Linear Programming

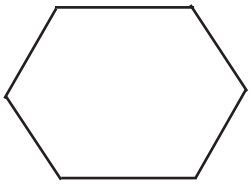
This chapter comes under Paper -I. From this chapter 5 mark questions -1 (1×5=5m), 4 marks questions - 1 (1×4=4m), 2 Mark Questions - 1 (1 ×2=2M) and 1 Mark Question - 1 (1 ×1=1M) and 6 objective bits (6×1/2=3M) altogether we can score 15 Marks easily. Here theory part muset be read by students fully to attempt 1 mark question and objective bits. The information given below will help the students who are going to appear public exams.

Convex set: If P, Q $\in$ X $\Rightarrow$ $\overline{PQ}$ then the plane X is called convex set (or) If the line segment joining any two points of a plane entirely lies in the same plane, then the plane is called convex set.

e.g.

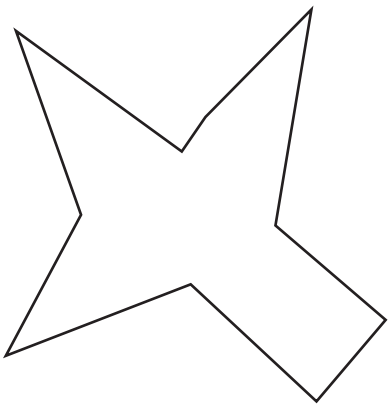


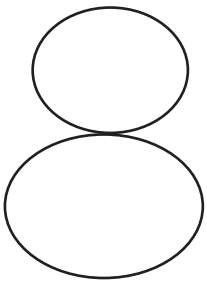


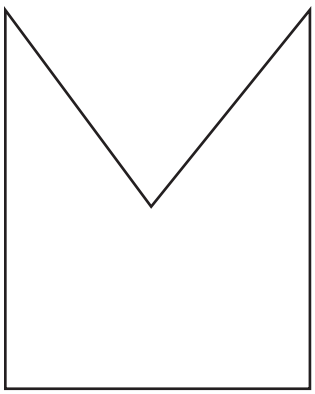


etc

are convex sets.







etc

are not the convex sets.

1. Indicate the polygonal region represented by the system of inequations $x \geq 1$, $y \geq 1$, $x \leq 3$, $y \leq 3$. 2M

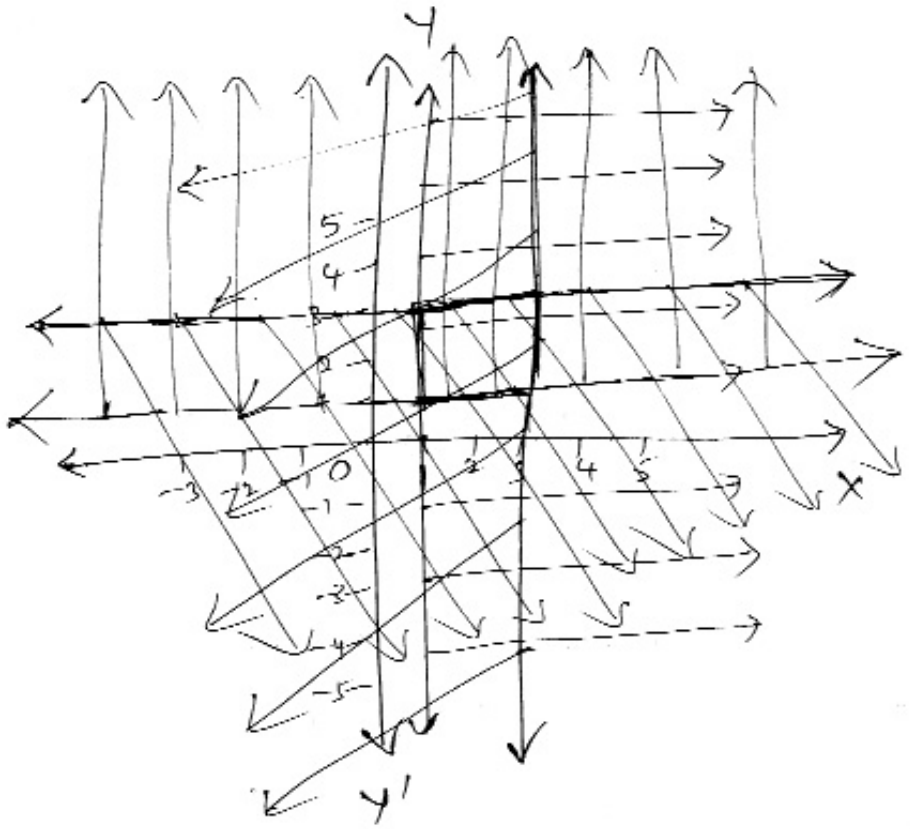
Sol:

i) $x \geq 1$ boundary line $x = 1$ which is parallel to y-axis and shaded region is towards right side of the line

ii) $y \geq 1$ boundary line is $y=1$ which is parallel to x-axis and shaded region is upwards for the line

iii) $x \leq 3$ boundary line is $x=3$ which is parallel to x-axis and shaded region is towards left side of the line.

iv) $y \leq 3$ boundary line is $y=3$ which is parallel to x-axis and shaded region is downwards for the line



Here the four shaded region is the solution set for the given system of inequations.

2. Indicate the polygonal region represented by the system of ineqauation $x \geq 0$, $y \geq 0$, $x + y \leq 1$ 2M

Sol:

i) $x \geq 0$ boundary line is $x=0$ which represents y-axis and shaded region should be right side of y-axis

ii) $y \geq 0$ boundary line is $y=0$ which represents x-axis and shaded region should be upwards for x-axis

iii) $x + y \leq 1$ boundary line is $x+y = 1$

$x + y = 1$

put $x = 0 \Rightarrow y = 1$ (0, 1)

put $y = 0 \Rightarrow x = 1$ (1, 0)

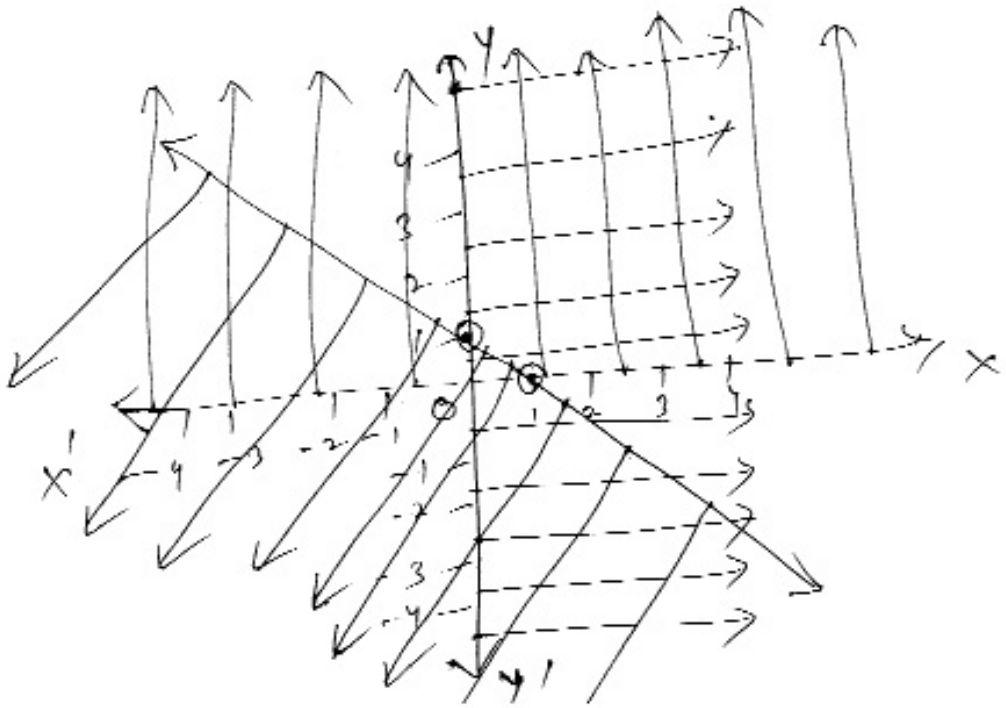
(0, 1), (1, 0)

(0, 0): $x + y \leq 1$

$0 + 0 \leq 1$

$0 \leq 1$ (True)

$\therefore$ Shaded region should be towards the origin including boundary line.



$\therefore$ Here triple shaded region is the solution set

Linear Programming Problem (L.P.P.):

- Which contains an objective function subject to certain constraints which are expressed in the form of Linear inequation is called Linear Programming Problem.

Objective Function :

- A function which is to be maximised or minimised in L.P.P is called an objective function or Profit function.

Note: The solution of Linear inequation is either closed convex polygon or open convex polygon.

The Fundamental Theorem :

- The objective function of L.P.P will be maximised or minimised at any are of the vertices of Convex polygon.

Feasible region :

- The region formed by the constrains of L.P.P is called Feasible region.

Feasible Solution (L.P.P.):

- Every point in Feasible region of L.P.P. is called Feasible Solution.

Optimum Solution :

- The Solution which makes the objective function either minimum or maximum is called

"Optimum Solution".

Iso-Profit Lines:

- The system of Parallel lines obtained form the objective function in L.P.P. are called Isoprofit lines.

Properties of isoprofit lines:

1. Iso-Profit lines are parallel to each other.
2. If iso-profit line co-incides with the boundary of convex polygon then the LPP has infinite soluiton.
3. No point in the feasible region makes the objective function either maximum or minimum then the no.of solutin of Lpp are 0
4. If the isoprofit line moves away from the origin then the profit will be increased.
5. If the iso-profit line moves towards the origin then the profit will be minimised.

3. A sweet shop makes gift packets of sweet combines two special types of sweets A and B which weigh 7kg. Atleast 3kg of A and no more than 5 kgs of B should be used. The shop makes a profit of Rs. 15 on A and Rs 20 on B per kg. Determine the product mix so as to obtain maximum profit. [Construct the Linear Programming Problem, graph is not required]. 4M

Sol:

These are two types of sweets i.e. A and B.
let the weight 'A' type sweets be x kgs and 'B' type sweets be y kgs.
$\therefore x \geq 0, y \geq 0$ .....(1)
The Packet weight should not increase 7 kgs.
$x + y \leq 7$ .....(2)
In each packet atleast 3 kgs of A type and not more than 5 kgs of B type should be used
i.e. $x \geq 3$ .....(3)
and $y \leq 5$ .....(4)
Profit on 'A' type sweets per kg is Rs 15 and 'B' type sweets per kg is Rs. 20
$\therefore$ Objective function or profit function
$f = 15x + 20y$

$\therefore$ The linear programming problem is
Maximise $f = 15x + 20y$ subject to the Constrants $x+y \leq 7, x \geq 3, y \leq 5, x \geq 0, y \geq 0$.

4. Maximinse $f = 2x + y$ subject to the constraints $2x+y \geq 8, y \leq 4, x \leq 3, x \geq 0$ and $y \geq 0$

Sol:

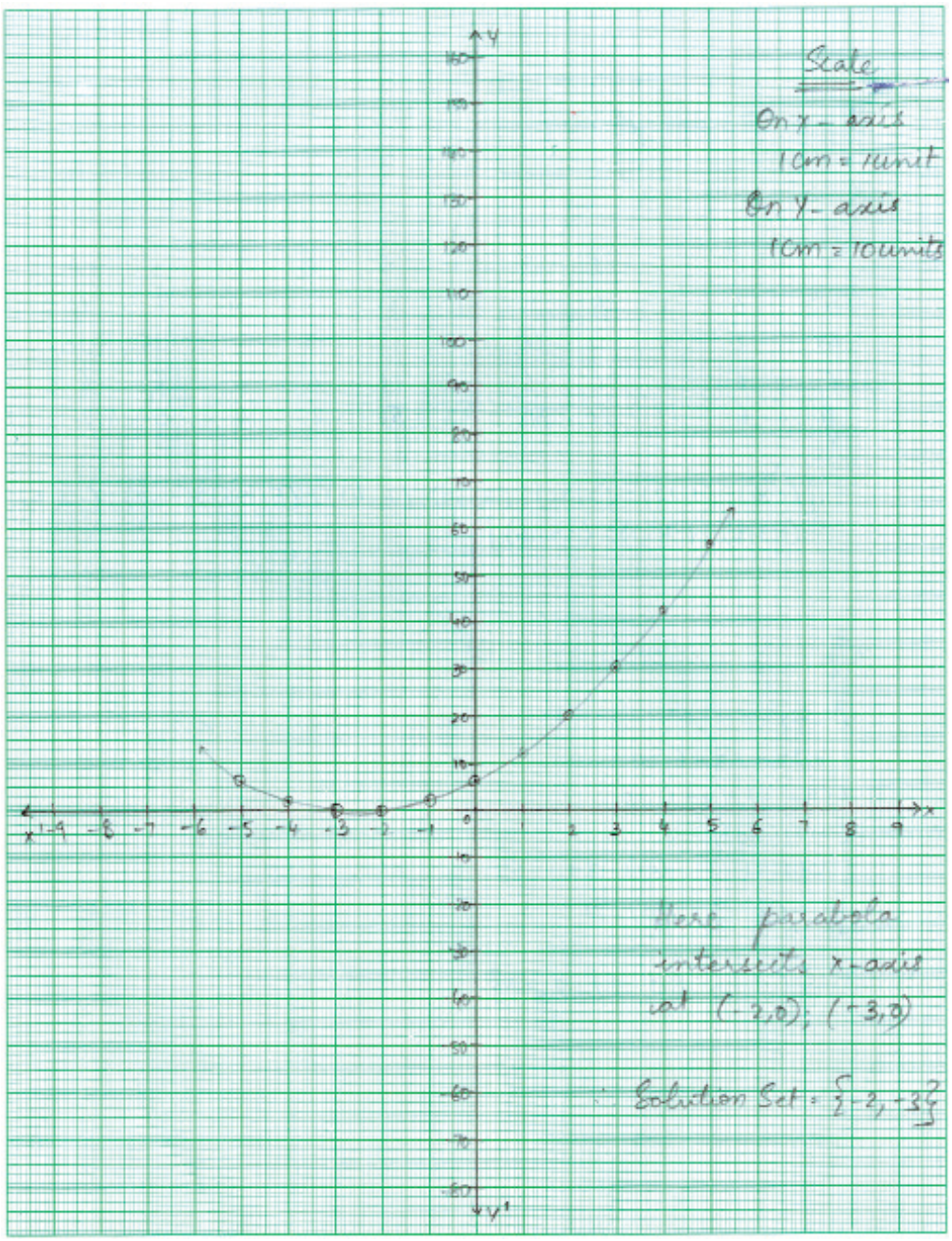
i) $x \geq 0, y \geq 0$ represents 1^{st} quadrant
ii) $y \leq 4$, boundary line is $y = 4$
iii) $x \leq 3$ boundary line is $x = 3$
iv) $2x+y \geq 8$, boundary line is $2x + y = 8$
put $x= 0 \Rightarrow 2(0) + y = 8 \Rightarrow 0 + y = 8 \Rightarrow y = 8$ (0, 8)
put $y = 0 \Rightarrow 2x + 0 = 8 \Rightarrow 2x = 8 \Rightarrow x = 8/2 \Rightarrow x = 4$ (4, 0)
{(0, 8); (4, 0)}

(0, 0): $2(0) + 0 \geq 8 \Rightarrow 0 + 0 \geq 8 \Rightarrow 0 \geq 8$ (F)

shaded region is on the other side of origin.

Vertices of polygon are (3, 2); (3, 4); (2, 4)
f = 2x + y;
at (3, 2); f = 2(3) + 2 = 6 + 2 = 8
at (3, 4); f = 2(3) + 4 = 6 + 4 = 10
at (2, 4); f = 2(2) + 4 = 4 + 4 = 8

∴ Maximum value of f is 10 at (3, 4)



5. Minimise f = x+y, subject to the constraints x+y ≥ 6; 2x+y ≥ 8, x ≥ 0, and y ≥ 0.

sol:

f = x+y; x + y ≥ 6; 2x+y ≥ 8; x ≥ 0 and y ≥ 0
i) x ≥ 0; y ≥ 0 represents 1st Quadrant
ii) x + y ≥ 6; boundary line is x+y = 6
put x = 0 ⇒ 0 + y = 6 ⇒ y = 6 (0, 6)
put y = 0 ⇒ x + 0 = 6 ⇒ x = 6 (6, 0)
= {(0, 6); (6, 0)}

(0, 0): $0 + 0 \geq 6 \Rightarrow 0 \geq 6$ (F)

$\therefore$ Shaded region is one the other side of the origin

iii) $2x + y \geq 8$, boundary line is $2x + y = 8$

put $x = 0 \Rightarrow 2(0) + y = 8 \Rightarrow 0 + y = 8 \Rightarrow y = 8$ (0, 8)

put $y = 0 \Rightarrow 2x + 0 = 8 \Rightarrow 2x = 8 \Rightarrow x = 8/2 \Rightarrow x = 4$ (4, 0)

{(0, 8); (4, 0)}

(0, 0): $2(0) + 0 \geq 8 \Rightarrow 0 + 0 \geq 8 \Rightarrow 0 \geq 8$ (F)

$\therefore$ Shaded region is on the other side of the origin.

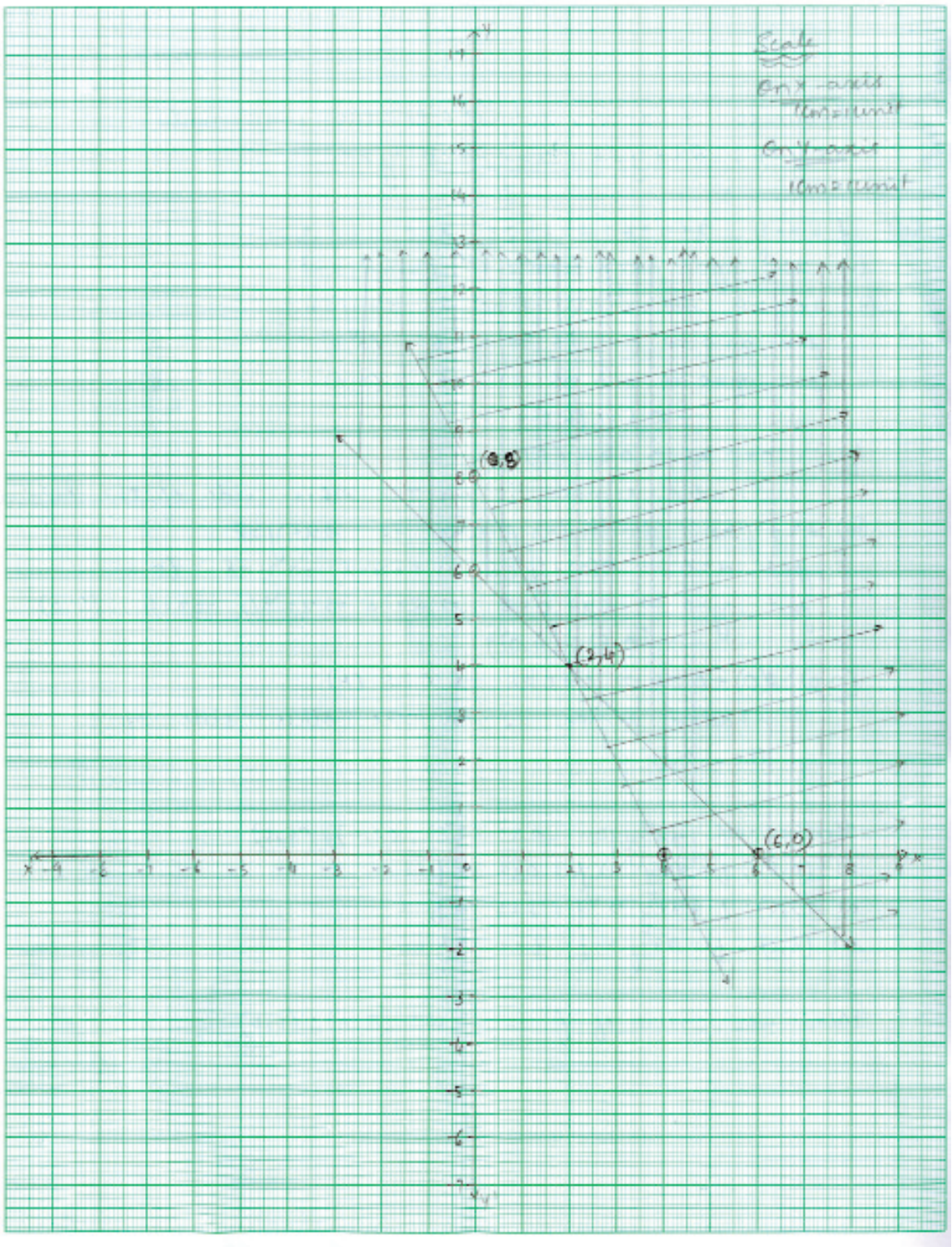
Vertices of polygon are (0, 8), (2, 4), (6,0)

at (0, 8); $f = x+y \Rightarrow f = 0 + 8 \Rightarrow f = 8$

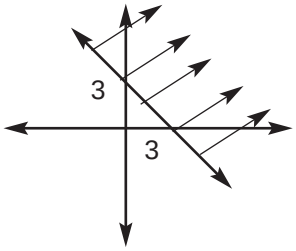
at (2, 4); $f = x+y \Rightarrow f = 2 + 4 \Rightarrow f = 6$

at (6, 0); $f = x+y \Rightarrow f = 6 + 0 \Rightarrow f = 6$

Minimum Value of f is 6 at (2, 4); (6, 0)



Objective bits

1. If x > 0, y < 0 then (x, y) lies in _____ quadrant.
2. 

   In the adjacent figure the graph represents _____
3. The solution set of $x \geq y$ and $x \leq y$ is _____
4. The line x=k is _____ to y-axis and 'k' units away from _____ axis.
5. The y=0 represents _____ axis
6. The solution set of Linear programming problem lies in _____ quadrant.
7. If y=mx+c passes through origin, C = _____
8. If a < 0 and $a \in R$ then (–a, a) lies in _____ quadrant.
9. The value of f = 2x + 3y at the intersecting point of x = 2 and y = 3 _____
10. If f = 3x + 4y is an objective function, the equatin of isoprofit line which passes through (3, 4) is _____ or _____

Answers:

1. IV
2. $x + y \geq 3$
3. x = y
4. Parallel, y - axis, y-axis
5. x
6. I
7. 0
8. IV
9. 13
10. 3x+4y = 25, 3x+4y–25 = 0

Assignment:

1. Define "iso-profit line" **1M**
2. Indicate the polygonal region represented by the system of inequation $x \geq 0$, $y \leq 4$ and $x \geq y$ **2M**
3. Maximise f = 5x + 7y subject to the constraints $2x + 3y \leq 12$, $3x + y \leq 12$, $x \geq 0$ and $y \leq 0$ **5M**
4. Minimise f=3x+2y, subject to the constraints $x + y \geq 1$, $x \geq y$, $0 \leq x \leq 1$, $y \geq 0$. **5M**
5. A shop keeper sells not more than 30 shirts of each colour. At least twice as many white ones are sold as green ones. If the profit on each of the white be Rs. 20 and that of green be Rs. 25, how many of each kind be sold to give him a maximum profit? [Construct only the Linear Programming Problem] **4M**