FUNCTIONS or MAPPINGS

This topic comes under Paper-I. It is very important chapter for scoring purpose. There are few models which will come compulsory in every exam from this topic. Even for the slow learners also it is a helping chapter to score good marks.

From this chapter 2 marks questions-1 (1×2 = 2M), 1 mark questions-1 (1×1=1M), 4 Marks questions-2 (2×4 = 8M) and 6 objective bits (6×1/2 = 3M) altogether 14 marks very easily can be scored. The important material related to this topic is given below which will help the students who are going to appear S.S.C. exam.

Function: f: A $\rightarrow$ B is said to be a function if every element of A is assigned with unique element of B.

- Here A is called Domain, B is called co-domain.
- The set of images in the codomain is called Range and it is denoted by f(A).
- f: A $\rightarrow$ B, if a$\in$A and b$\in$B then b is called image if "a", denoted f(a) = b and 'a' is called pre-image of b, denoted by $f^{-1}(b) = a$

- n(A) = p, n(B)=q
  i) n (A × B) = pq
  ii) No. of relations from A to B are 2^{mn}
  iii) No.of distinct relations from A to B are $2^{pq} - 1$
  iv) No. of functions from A to B are q^p

Type of functions

one-one function:.f: A$\rightarrow$B is said to be one-one function if $\forall$ $x_1, x_2 \in$ A, $f(x_1)=f(x_2) \Rightarrow x_1 = x_2$ where $f(x_1), f(x_2) \in$ B.

(or)

f: A$\rightarrow$B is said to be one-one function if distinct elements of A have distinct images in B.

e.g. 1) f: R $\rightarrow$ R, f(x) 3x+2 is one-one function.
2) f: R $\rightarrow$ R, $f(x) = x^2$ is not one-one function.

Note: One-One function is also called as "injection".

Onto function: f: A $\rightarrow$ B is said to be onto function if $\forall$ y$\in$B, $\exists$ x$\in$A such that f(x) = y

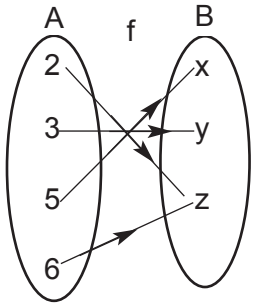
(or)

f: A $\rightarrow$ B is said to be onto function if the Range = Co-domain i.e. f(A) = B.

e.g. 1) f: R $\rightarrow$ R, f(x) = 5x + 4 is onto function
2) f: R $\rightarrow$ R $f(x) = x^3$ is onto function.
3) f: R $\rightarrow$ R $f(x) = x^2$ is not onto function.

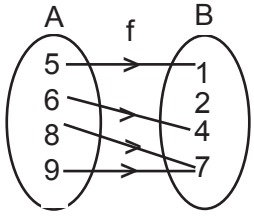
Note: f: A $\rightarrow$ B and Range $\neq$ co-domain i.e. f(A) $\neq$ B or Range $\subset$ codomain i.e. f(A) $\subset$ B, then f is called into function.

e.g. 1



Range = {x, y, z} = co-domain
$\therefore$ f is onto

e.g. 2



Range = {1, 4, 7}
Co-domain = {1, 2, 4, 7}
Here Range $\neq$ Co-domain
$\therefore$ f is into

One-One onto function or Bijection: f: A $\rightarrow$ B is said to be bijection if and only if f is both one-one and on to.
f is both one-one and on to
e.g. f: R $\rightarrow$ R and f(x) = 5x + 7 is bijection.

Inverse of a function: f: A $\rightarrow$ B is a function, inverse function of f is formed by interchanging the first and second co-ordinates of ordered pairs of function 'f'.
Note:
1. Inverse of a function may be a function or may not.
2. Inverse of a function is a function if f is bijection.

Inverse function: f A $\rightarrow$ B is function and f is bijection then f^{-1}: B$\rightarrow$ A is called inverse function of f. **Note:** f is bijection $\Rightarrow$ f^{-1} exists.

Problems

1. If f:R $\to$ R and f(x) = 2x + 1, find f^{-1} (if exists)

Sol:

f:R$\to$ R f(x) = 2x + 1

i) $\forall$ $x_1, x_2 \in R$, $f(x_1) = f(x_2)$

$\Rightarrow$ $2x_1 + 1 = 2x_2 + 1$

$\Rightarrow$ $2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one - one ..... (1)

ii) f(x) = 2x+1

Let y = f(x) = 2x+1

$\Rightarrow$ y = 2x+1

$\Rightarrow$ y – 1 = 2x

$\Rightarrow \frac{y-1}{2} = x$

Here $\forall$ $y \in R$, $\exists$ $x \in R$ Such that

f(x) = y.

$\therefore$ f is onto .....(2)

from (1) & (2)

$\therefore$ f is bijection

$\therefore$ f^{-1} exists.

Let y = f(x) = 2x+1

y = f(x)

$\Rightarrow f^{-1}(y) = x$ .....(3)

y = 2x+1

$\Rightarrow$ y – 1 = 2x

$\Rightarrow \frac{y-1}{2} = x$.....(4)

from (3) & (4)

$f^{-1}y = \frac{y-1}{2}$

$\Rightarrow f^{-1}(x) = \frac{x-1}{2}$

2. f: R $-\{3\}\to$ R and $f(x)=\frac{x+3}{x-3}$ then show that $f\left(\frac{3x+3}{x-1}\right)=x$

Sol:

f: R– {3} $\to$ R

$f(x)=\frac{x+3}{x-3}$

Consider LHS $f\left(\frac{3x+3}{x-1}\right)=\frac{\frac{3x+3}{x-1}+3}{\frac{3x+3}{x-1}-3}$

$$=\frac{\frac{3x+3+3(x-1)}{\cancel{x-1}}}{\frac{3x+3-3(x-1)}{\cancel{x-1}}}$$

$$=\frac{3x+\cancel{3}+3x-\cancel{3}}{\cancel{3x}+3-\cancel{3x}+3}$$

$$=\frac{6x}{6}=x=\text{RHS}$$

3. f(x) = x+2, g(x) = x^2– x – 2, find $\frac{g(1)+g(2)+g(3)}{f(-4)+f(-2)+f(2)}$

Sol:

f(x) = x + 2, g(x) = x^2– x – 2

f (x) = x + 2

f (- 4) = - 4 + 2 = - 2

f (- 2) = - 2 + 2 = 0

f (2) = 2 + 2 = 4

g (x) = x^2 - x - 2

g (1) = 1^2 - 1 - 2 = - 2

g (2) = 2^2 - 2 - 2 = 0

g (3) = 3^2 - 3 - 2 = 4

consider $\frac{g(1)+g(2)+g(3)}{f(-4)+f(-2)+f(2)}$

$$=\frac{-2+0+4}{-2+0+4}=\frac{2}{2}=1$$

4. f(x) $x^2 + 2x + 3$, find $\frac{f(x+h)-f(x)}{h}, h \neq 0$

Sol:

$f(x) = x^2 + 2x + 3$

$f(x+h) = (x+h)^2 + 2(x+h)+3$

$= x^2 + 2hx + h^2 + 2x + 2h + 3$

Consider $\frac{f(x+h)-f(x)}{h}$

$$= \frac{\left[x^2 + 2hx + h^2 + 2x + 2h + 3\right] - \left(x^2 + 2x + 3\right)}{h}$$

$$= \frac{x^2 + 2hx + h^2 + 2x + 2h + 3 - x^2 - 2x - 3}{h}$$

$$= \frac{2hx + 2h + h^2}{h}$$

$$= \frac{h(2x + 2 + h)}{h}$$

$= 2x+h+2$

Constant Function: f: A→B is said to be a constant function if $\forall\ x \in A$, f(x) = k when $k \in B$.
Note: Range of a constant function is a singleton set.

e.g. f: R→R, f(x) = 100 and f is a constant function then f(2) = 100

Identity function: I: A→A is said to be an Identity function, $\forall\ x \in A$, I(x) = x.

Note: Identity function is it's own inverse.
e.g. f: R→R, f is identity function then $f^{-1}(7) = 7$

Equal Functions: f and g are said to be equal functions if i) They are defined on the same domain and ii) f(x) = g(x)

e.g. 1. f: R – {1} → R, f(x) = x+1

g: R – {1} → R, $g(x) = \frac{x^2 - 1}{x - 1}$

Here f = g.

2. f: R→R, $f(x) = x^3$, g: N→R, $g(x) = x^3$
Here f ≠ g because they are not defined on the same domains.

5. f: R→R, f(x) = x+2 and f has a domain {x/2 ≤ x ≤ 5}. Find f^{-1} and it's domain and range.

Sol:

f: R→R

f(x) = x + 2 is clearly bijection

$\Rightarrow f^{-1}$ exists

Let y = f(x) = x+2

y = f(x)

$f^{-1}(y) = x$ .....(1)

y = x + 2

y – 2 = x .....(2)

from (1) & (2)

$f^{-1}(y) = y - 2$

$f^{-1}(x) = x - 2$

Domain of f = {x/2 ≤ x ≤ 5}

Range of f = {x+2/ 2 ≤ x ≤ 5}

= {f(x)/ 4 ≤ f(x) ≤ 7}

= {y/ 4 ≤ y ≤ 7}

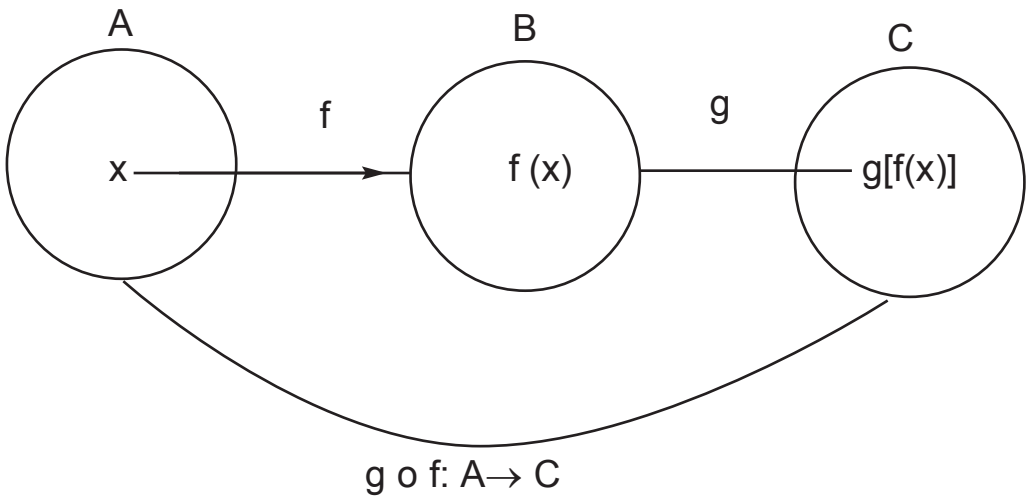
Here range of f = Domain of f^{-1}

= {y/ 4 ≤ y ≤ 7}

and Domain of f = Range of f^{-1} = {x/2 ≤ x ≤ 5}

Composite function or Product function:

If f:A→B and g: B→C then the composite function of f and g is denoted by g o f: A→C and defined as g o f (x) = g [f(x)]



Note:

1. Domain of g o f and f are same.
2. Co -domain of g o f and g is same.
3. Co-domain of f = domain of g

- An important property of Composite function when three functions are there is
  i) (hog)of = ho(gof)
  ii) (fog)oh = fo(goh)

6. If f={(1, 3), (2, 5), (3, 7)}; g={(3, 7), (5, 9), (7, 10)}, find i) (g o f) ii) f o g iii) What do you notice?

sol:

f={(1, 3), (2, 5), (3, 7)
g={(3, 7), (5, 9), (7, 10)}

i) g o f = g [f(x)]
g o f (1) = g [f(1)] = g [3] = 7
g o f (2) = g [f(2)] = g [5] = 9
g o f (3) = g [f(3)] = g [7] = 10

g o f = {(1, 7), (2, 9), (3, 10)}
f o g (x) = f [g(x)]
f o g (3) = f [g(3)] = f(7) doesn't exists.

$\therefore$ f o g is not defined.

7. If f:R$\to$R, g: R$\to$R, h:R$\to$R, f(x)=x – 1, g (x) = x^2 – 2, h(x)=x^3–3. For x$\in$R. Find i) (fog)oh = f o (goh) iii) What do you notice?

sol:

f: R $\to$ R, g: R$\to$R, h:R$\to$R
f(x)=x – 1, g (x) = x^2 – 2, h(x)=x^3–3
i) (f o g) o h
f o g = f [g(x)]
= f (x^2 – 2)
= x^2 – 2 – 1
= x^2 – 3

(f o g) o h = (f o g) o h (x)
= (f o g) [h(x)]
= (f o g) (x^3 – 3)
= (x^3 – 3)^2 –3
= (x^3)^2 – 2x^3(3) + 3^2 – 3
= x^6 – 6x^3 + 9 – 3
= x^6 – 6x^3 + 6 .....(1)

ii) f o (g o h)
g o h = g o h (x)
= g [h(x)]

$= g(x^3 - 3)$
$= (x^3 - 3)^2 - 2$
$= (x^3)^2 - 2.x^3.3 + 3^2 - 2$
$= x^6 - 6x^3 + 9 - 2$
$= x^6 - 6x^3 + 7$

$f o(g o h) = f o (g o h)(x)$
$= f[g o h(x)]$
$= f[x^6 - 6x^3 + 7)$
$= x^6 - 6x^3 + 7 - 1$
$= x^6 - 6x^3 + 6$ .....(2)

iii) From (1) (2)
(f o g) o h = f o (goh)

Real Variable Function: f: A→B and A $\subseteq$ R then f is called Real Variable Function.

Real Valued Function: f: A→B and B $\subseteq$ R then f is called Real Valued Function.

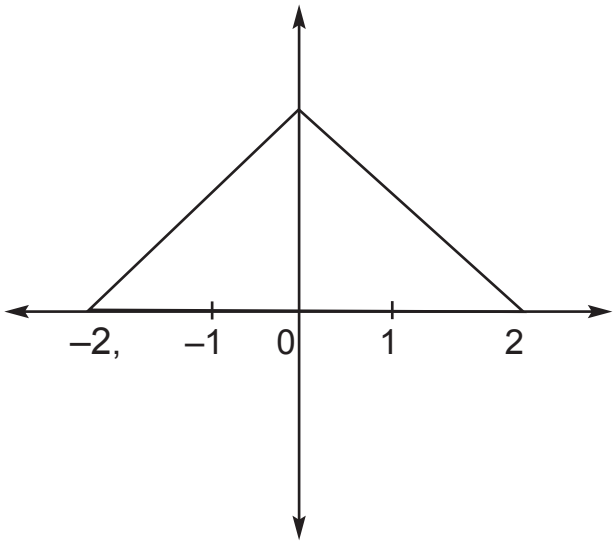
Note: f: A→B and A $\subseteq$ R and B $\subseteq$ R then f is called Real valued function with real variable or Real Function.

Zeros of a Function: f: A→R and for which $x \in A$, f(x) = 0 then x is called zeros of function.

Note:
1. For a function, zeros of functions may exists or may not.
2. If zeros of function exists for a function, it may not be unique.

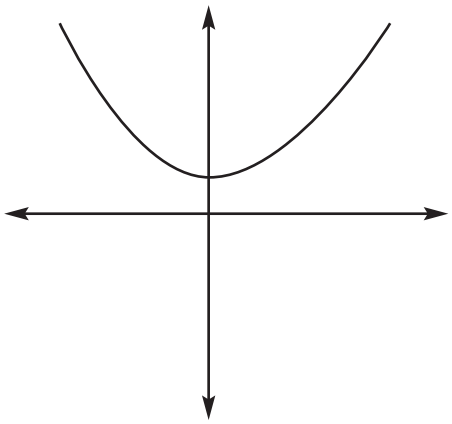
e.g.
1. The zeros of f(x) = 2x + 3 is $\underline{-3/2}$
2. The zeros of $f(x) = x^2 - 5x + 6$ is $\underline{2,3}$

3.



In the above figure zeros of function is $\underline{-2,2}$

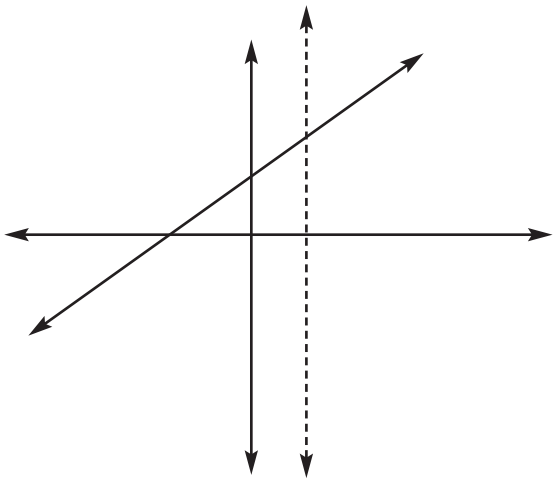
4.



In the adjacent figure zeros of function <u>does not exists</u>

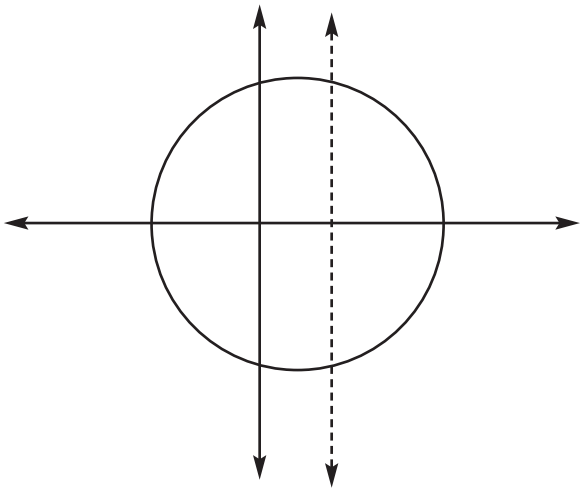
- The given graph of a relation is a function if any line drawn parallel to y-axis or perpendicular to x-axis should not cut the graph of more than one point. If so it is not a function.

e.g.



- In the adjacent figure, the given graph of relation is a function because the parallel line to y-axis cut the graph or only one point.

e.g.



- In the above figure, the given graph of a relation is not a function because the line drawn parallel to y-axis cuts the graph in two points.

8. Let $f(x) = x^3$, $g(x) = x^2 + 1$ $x \in R$, find f o g (x), g o f (x) and what do you notice?

Sol:

$f(x) = x^3$, $g(x) = x^2 + 1$

f o g (x) = f [g (x)]

$= f [x^2 + 1]$

$= (x^2 + 1)^3$ .....(1)

i) (gof) (x) = g [f (x)]

$g [(x)^3]$

$= (x^3)^2 + 1$

$= x^6 + 1$.....(2)

From (1) & (2) we noticed that f o g (x) $\neq$ g o f (x)

Objective bits

1. f = {x, 3)/ x $\in$ N} is _____ function.
2. f(x) = 2x + 3 then f (–3) = _____
3. f^{-1} (y) = y – 3, then f(x) = _____
4. f (2x – 5) = 5 is an identify function then x = _____
5. The range of f = {(1, 2), (1, 3), (2, 4)} is _____
6. f(x) = 2x – 1, g(x) = x the g o f (3) = _____
7. _____ function is it's own inverse.
8. f is a function and f^{-1} exists if and only if f is _____
9. If f(x) = 6x + 5, then f^{-1} (x) _____
10. $f(x) = \frac{1-x}{1+x}$ then f (3) _____

Answers:

1.Constant
2. – 3
3. x + 3
4. 5
5. {2, 3, 4}
6. 5
7. Identity
8. bijection
9. $\frac{x-5}{6}$
10. 1/2

Assignment:

1. $f(x)=\frac{x+1}{x-1}$, $x \neq 1$ prove that $f(x)+f\left(\frac{1}{x}\right)=0$
2. f: R – {2}$\rightarrow$ R and $f(x)=\frac{2x+1}{x-2}$ then S.T $f\left(\frac{2x+1}{x-2}\right)=x$
3. Define i) onto function ii) Zeros of function iii) Real valued function with examples.
4. f: R$\rightarrow$R f(x) = 2x + 3, find $f^{-1}(4)$, $\{f^{-1}(4)/\ 2 \leq x \leq 3\}$ $\{f^{-1}(x)/\ x \leq 5\}$
5. f, g and h are functions and f(x) = x + 2, g(x) = 3x – 1 and h(x) = 2x then show that ho(gof) = (h o g) o f.
6. f(x) = x + 2, g(x) = 2x + 3, h(x) = 3x + 4, x $\in$ R find
   i) (f o g) o h ii) f o (g o h) iii) What do you notice.