STATEMENTS & SETS

Statement: A sentence which is either true or false but not both is called "statement".

- Truthfulness of a statement is denoted by 'T' whereas it's falsity is denoted by 'F'.
- Statements are denoted by lower case alphabets of English like p, q, r, s…etc.

Open Sentence: A sentence which is not a statement is called "open sentence". (or) A mathematical sentence which has atleast one variable is called "open sentence".

Eg: 1. **p : 2 × 5 = 7** is a false statement.
2. **q : 2 is even prime** is a true statement.
3. **r : x + y = 8** is an open sentence but not statement.

Connectives: The words that are used to combine the simple statements are called connectives.

Eg: **a**nd, or, if then, if and only if, not... etc.

Compound Statements: The statements that are composed of other simple statements with the connectives are called "compound statements".

Negation: If 'p' is a statement, then negation of p is denoted by "~p" and it is formed by using the word 'not' before 'p', read as "negation p".

Negation Truth Table:

p	~p
T	F
F	T

Eg: p: 3 is a prime (T)
p: 3 is not a prime (F)

Conjunction: The compound statement that is combined two simple statements with the connective "AND" is called "Conjunction".

- The symbol of conjunction is "$\wedge$"read as "and".

Conjunction Truth Table

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Note: The conjunction of two statements is true only when both the statements are true.

Eg: p : 3 × 4 = 8 (F)
q : 3 + 4 = 7 (T)
p$\wedge$q truth value is F.

Disjunction: The compound statement that is combined two simple statements with the connective "OR" is called "Disjunction".

- The symbol of disjunction is "$\vee$" read as "or".

Disjunction Truth Table

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Note: The disjunction of two statements is false only when both the statements are false.

Eg: p : 3 × 4 = 8 (F)
q : 3 + 4 = 7 (T)
p∨q truth value is T

Conditional or Implication: The compound statement that is combined two simple statements with the connective "if-then" is called "conditional".

- The symbol of conditional is "→" or "⇒" read as "if-then" or "implies".

Conditional Truth Table

p	q	p⇒q
T	T	T
T	F	F
F	T	T
F	F	T

Note: The conditional of two statements is false only when the first statement is true and second statement is false.

Eg: p : 4 × 5 = 20 (T)
q : 4 + 5 = 12 (F)
p⇒q truth value is F

Biconditional or Bi-implication: The compound statement that is combined two simple statements with the connective "IF AND ONLY IF" or "IFF" is called bi-conditional.

- The symbol of bi-conditional is "↔" or "⇔" read as if and only if or briefly "iff".

Bi-Conditional Truth Table

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

Note: Bi-conditional of two statements is true only when both the statements are either true or false.

Eg: p : 3 × 5 = 15 (T)
q : 3 + 4 = 12 (F)
p⇔q truth value is 'F'

Converse, Inverse, Contrapositive of a Conditional:

Conditional : $p \Rightarrow q$
Converse : $q \Rightarrow p$
Inverse : $(\sim p) \Rightarrow (\sim q)$
Contrapositive : $(\sim q) \Rightarrow (\sim p)$

Problem: Write the converse inverse and contrapositive of a conditional "If in a triangle ABC, AB > AC then $\angle C > \angle B$".

Sol:

Converse: If in a triangle ABC, $\angle C > \angle B$ then AB > AC

Inverse: If in a trangle ABC, $AB \not> AC$ then $\angle C \not> \angle B$

Contrapositive: If in a triangle ABC, $\angle C \not> \angle B$ then $AB \not> AC$

Problem: Write the truth table of $(q \wedge (\sim p)) \Rightarrow (\sim p)$

Sol:

p	q	~p	q∧(~p)	q∧(~p)⇒(~p)
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

Tautology: The compound statement which contains only "T" in the last column of it's truth table is called "Tautology".

Contradiction: The compound statement which contains only 'F' in the last column of it's truth table is called "Contradiction".

Eg: $p \vee (\sim p)$ is a tautology.
$p \wedge (\sim p)$ is a contradiction.

Problem: Show that $(p \wedge (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.

Sol:

p	q	~p	~q	p∧(~q)	(~p)∨q	(p∧(~q))∧((~p)∨q)
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Equivalent of Statements: Two compound statements are said to be logically equivalent if their truth tables are identical.

- Equivalence of statements is denoted by "≡"

Eg: Show that $p \equiv \sim(\sim p)$

Sol:

p	~p	~(~p)
T	F	T
F	T	F

Here p and, ~(~p) columns are identical

$\therefore\ p \equiv \sim(\sim p)$

Problem:
Prove that $\sim(p \Rightarrow q) \equiv p \wedge (\sim q)$
Sol:

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	$\sim q$	$p \wedge (\sim q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Here $\sim(p \Rightarrow q)$ and $p \wedge (\sim q)$ have identical truth values.
$\therefore\ \sim(p \Rightarrow q) \equiv p \wedge (\sim q)$

Problem:
Prove that converse of a conditional is equivalent to it's inverse.
Sol:

p	q	$q \Rightarrow p$	$\sim p$	$\sim q$	$(\sim p) \Rightarrow (\sim q)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

Here $q \Rightarrow p$ i.e. converse of a conditional and $(\sim p) \Rightarrow (\sim q)$ i.e. inverse of it have identical truth values.
$\therefore$ converse of a conditional is equivalent to it's inverse.

Quantifiers: The phrase that quantify the variable(s) in the open sentences are called 'quantifiers'.

Replacement set or Domain of the Variable: The set from which the variable values are taken to make the open sentence to become either true or false is called Replacement set or Domain of the variable.
Eg: N, W, Z, Q, R, etc.

- The phrases like "for all", "for every" are called Universal Quantifiers, denoted by "$\forall$".
- The phrases like "for some", "there exists atleast one" are called Existential quantifiers, denoted by "$\exists$".

Note: The quantifier "for no" can be written as 'for all' or 'for some' based on the situation.
Eg: The symbolic form of "No square of a real number is negative" is $\forall x,\ x \in R,\ x^2 \geq O$.

- The symbolic form of "there exists a real number x such that $x+2 = 7$ is $\exists x \in R,\ x+2 = 7$
- The symbolic form of "for all natural numbers n, n(n+1) (n+2) is divisible by 6 is $\forall n \in N$, 6 / n(n+1) (n+2)

Methods of proof:
1. Direct method of proof
2. Indirect method of proof

Problem:

Prove that "If x is even, then x^2 is even".

Proof:

Given : x is even

RTP : x^2 is even

Proof : The general form of even number is 2n, $n \in z$

Let the even number of x = 2n, $n \in z$

Squaring on both sides

$x^2 = (2n)^2$

$x^2 = 4n^2$

$x^2 = 2(2n^2)$

$x^2 = 2m$, where $m \in z$

$\therefore$ x^2 is even

Methods of Disproof:

1. Disproof by counter example.
2. Disproof by contradiction.

Counter example: An example which is used to disprove the given statement is called "counter example".

Eg: The counter example to disprove "all primes are odd" is 2.

Problem:

Disprove "the square of every even number is odd" by using the method of contradiction.

Sol:

Let the even number be 2n, $n \in z$

and odd number be 2m + 1, $m \in z$

given that $(2n)^2 = 2m + 1$

$\Rightarrow 4n^2 = 2m + 1$

Here $4n^2$ is divisible by 2 where as 2m+1 is not divisible by 2.

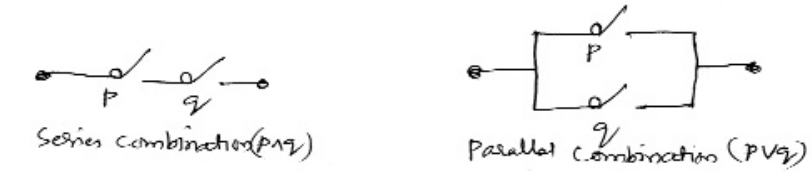
Hence L.H.S. and R.H.S do not represent the same number.

$\therefore$ The square of even number is odd is false.

Thus the given statement is disproved by contradiction.

Applications of truth tables to switching net works:

- Let p, q, ... denote electrical switches and let p, p^1 denote the switches that the property that one is on the other is off and vice versa.
- Switches can be combined in series and parallel combination.

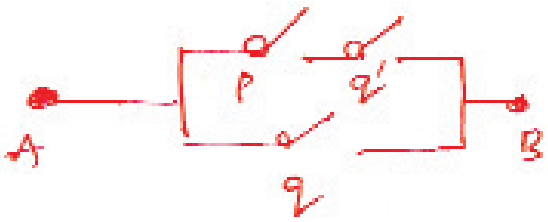


- A switch has two possibility i.e. (i) it is either open (F)
  Where there is no flow of current. or (ii) it is closed (T), where there is flow of current.
  Thus every switch has two truth values T or F.
- Series net work truth table is identical to the truth table of conjunction.

- Parallel net work truth table is identical to the truth table of disjunction.
- The switches P and P^1 truth table is identical to the truth table of Negation.

Problem:

Determine when current flows from A to B in the following net work.



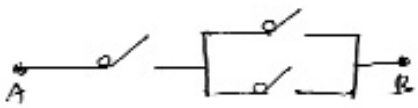
Sol:

p	q	q^1	$p \wedge q^1$	$(p \wedge q^1) \vee q$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	F

There is a flow of current from A to B in all the cases except the following case i.e. p is open, q is open.

Assignment:

1. Write the truth table of $(p \wedge (\sim q)) \Rightarrow q$
2. Write the converse, inverse and contrapositive of the conditional "If a polygon is a square then it is a rectangle".
3. Verify $((\sim p) \vee q) \wedge (p \vee (\sim q))$ is Tautology or contradiction?
4. Prove $p \vee (\sim(q \wedge r)) \equiv (p \vee (\sim q)) \vee (\sim r)$
5. Determine when current flows from A to B in the following net work.



6. Prove that "if a pair of alternate angles are equal then the two lines are parallel".

Fill in the blanks

1. Denial of a statement is called _____
2. The symbolic form of $7 \neq 10$ is _____
3. $\sim(p \vee q) \equiv$ _____ and it is _____ law
4. $\sim(p \Leftrightarrow q) \equiv$ _____ $\equiv$ _____
5. The symbol for existential quantifier is _____
6. "$\forall$" is _____ quantifier
7. The counter example to disprove $x^2 - 3x + 2 = 0$, $x \in R$ is _____
8. A open sentence contains at least _____ variable(s).
9. A conditional is equivalent to its _____
10. $\sim((\sim p) \Rightarrow (\sim q))$ = _____

SETS

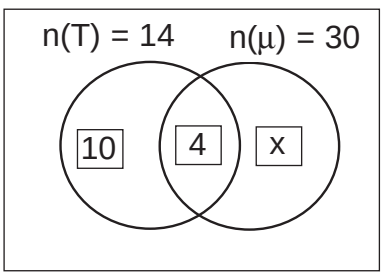
- This is a small topic, where we can score more marks. From this chapter, 4 Marks – 1Q, 2 Marks – 1Q, 1 Mark – 1Q and 5 – 6 objective bits (2½ - 3 marks), altogether 10 Marks, very easily can be scored. 4 Marks problems will be given from elementary wise proofs, 2 marks question also can be given from that area or from word problems. This topic helps us to score more marks with less hard work. The material provided below consists of some important questions for the public exam purpose.

- Set theory was developed by "George Cantor"
- **Set:** A well defined collection of distinct objects is called set.
- Sets are denoted by higher case alphabets of English, where as elements are denoted by lower case alphabets of English.
- The symbol for "is belongs to" is "$\in$" and "is doesn't belongs to" is "$\notin$".
- $A \subset B$ & $B \subset A \Leftrightarrow A = B$
- $A \cap B$ is the set containing only those elements that are common in A & B.
- $A \cup B$ = contains the elements that are either in A or in B or in both.
- $A \cap B = \phi$, then A & B are disjoint sets and $n(A \cap B) = 0$
- A & B are disjoint then $n(A \cup B) = n(A) + n(B)$
- i) $\phi = \{\ \}$ ii) $\phi \neq \{0\}$
- Every set is a subset of it self
- Null set is subset of every set.
- The set builder form of $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} ..... \frac{1}{8}\right\}$ is $A = \left\{\frac{1}{x} / x \in N, x \leq 8\right\}$
- The set builder for of B = {2, 3, 5, 7, 11, 13, 17, 19} is
  B = {x / x$\in$P, P is prime and x $\leq$ 19}
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A) = 20$, $n(B) = 44$, $n(A \cup B) = 51$ then $n(A \cap B) = 13$

Problem: 1

In a class of 30 students, 10 take tea but not coffee and 14 take tea, then how many take coffee but not tea?

Sol:



$n(\mu) = 30$
$n(T) = 14$
only $n(T) = 10$
$n(T \cap C) = n(T) - \text{only } n(T)$
$= 14 - 10$
$= 4$
Let only $n(c) = x$
By Venn diagram
$10 + 4 + x = 30$
$14 + x = 30$
$x = 30 - 14$
$x = 16$
$\therefore$ Number of students who take coffee but not tea are 16.

Problem: 2

In a city there are three major newspapers A, B, C of which atleast two are read by 35% of the population. It is known that newspaper C is read by 45% of the population and the news paper A and B are read is 15% and all the three are read by 10%. What percent of population read only newspaper C?

Sol:

$n(A \cap B \cap C) = 10\%$

$n(A \cap B) = 15\%$

The news papers, at least two are read by 35%

i.e., $x\% + y\% + 5\% + 10\% = 35\%$

$\Rightarrow x\% + y\% + 15\% = 35\%$

$\Rightarrow x\% + y\% = 35\% - 15\%$

$\Rightarrow x\% + y\% = 20\%$ --------(1)

The population who read only newspaper C.

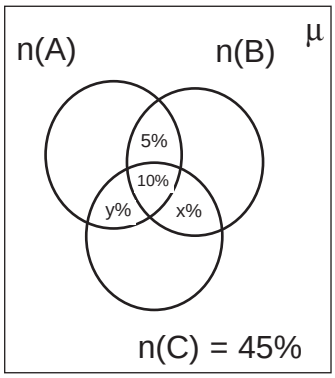
i.e., only $n(C) = n(C) - [x\% + y\% + 10\%]$

$= 45\% - [x\% + y\% + 10\%]$

$= 45\% - [20\% + 10\%]$ [from (1)]

$= 45\% - 30\%$

$= 15\%$



Problem: 3

Complaints about a student hostel mess fell into three categories. Complaints about (i) Mess (M) ii) Food (F) iii) Service (S) The total number of complaints received were 173 and were as follows. $n(M) = 110$, $n(F) = 55$, $n(S) = 67$, $n(M \cap F \cap S') = 20$, $n(M \cap S \cap F') = 11$, $n(F \cap S \cap m') = 16$. Determine the number of complaints about i) all the three ii) about two or more than two.

Sol:

$n(\mu) = 173$

$n(M) = 110$,

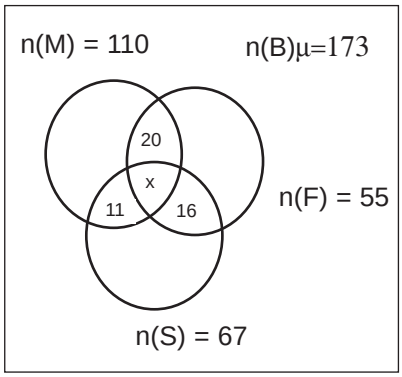
$n(F) = 55$

$n(S) = 67$,

$n(M \cap F \cap S') = 20$

$n(M \cap S \cap F') = 11$

$n(F \cap S \cap M') = 16$



i) Let the number of complaints about all the three be 'x'

by Venn diagram

Number of complaints about all three

i.e., $n(M \cup F \cup S) = n(M) + n(F) + n(S) - n(F \cap S) - n(M \cap S) - n(M \cap F) + n(M \cap F \cap S)$

$173 = 110 + 55 + 67 - (20 + x) - (16 + x) - (4 + x) + x$

$173 = 232 - 20 - x - 16 - x - 11 - x + x$

$173 = 232 - 47 - 2x$

$2x = 232 - 47 - 173$

$2x = 232 - 220$

$2x = 12$

$x = 6$

$\therefore$ No. of complaints about all the three are 6.

ii) No. of complaints received about two or morethan two = 20 + 16 + 11 + x
= 47 + 6
= 53

Basic Theorem:

X & Y are any two sets, $X \subset Y$ & $Y \subset X \Leftrightarrow X = Y$

$\rightarrow x \in A \cup B \Leftrightarrow x \in A \vee x \in B$

$\rightarrow x \notin A \cap B \Leftrightarrow x \notin A \wedge x \notin B$

$\rightarrow x \in A \cap B \Leftrightarrow x \in A \wedge x \in B$

$\rightarrow x \notin A \cap B \Leftrightarrow x \notin A \vee x \notin B$

$\rightarrow x \in A - B \Leftrightarrow x \in A \wedge x \notin B$

$\rightarrow x \notin A - B \Leftrightarrow x \notin A \vee x \in B$

$\rightarrow x \in A \Leftrightarrow x \notin A'$

$\rightarrow x \in A' \Leftrightarrow x \notin A$

$\rightarrow x \in A \Rightarrow x \in B \Leftrightarrow A \subset B$

Problem: 4

Prove that $A-(B \cup C) = (A - B) \cap (A - C)$ for any three sets, A, B, C

Proof: In order to prove $A-(B \cup C) = (A - B) \cap (A-C)$, according to BASIC THEOREM we have to prove that

i) $A- (B \cup C) \subset (A - B) \cap (A - C)$

ii) $A- (B \cap C) \subset (A - B) \cup (A - C)$

i) Let $x \in A-(B \cup C)$ $\Rightarrow x \in A \wedge x \notin (B \cup C)$

$\Rightarrow x \in A \wedge (x \notin B \wedge x \notin C)$

$\Rightarrow (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)$

$\Rightarrow x \in (A - B) \wedge x \in (A - C)$

$\Rightarrow x \in (A - B) \cap (A - C)$

$\therefore A-(B \cap C) \subset (A - B) \cap (A - C)$ ------(1)

ii) Let $x \in A-(B \cap C)$ $\Rightarrow x \in A \wedge x \notin (B \cap C)$

$\Rightarrow x \in A \wedge (x \notin B \vee x \notin C)$

$\Rightarrow (x \in A \wedge x \notin B) \vee (x \in A \wedge x \notin C)$

$\Rightarrow x \in (A - B) \vee x \in (A - C)$

$\Rightarrow x \in (A - B) \cup (A - C)$

$\therefore A-(B \cap C) \subset (A - B) \cup (A - C)$ ------(2)

From (1) & (2)

$A-(B \cup C) = (A - B) \cap (A - C)$

Problem: 5

Prove that $(A')' = A$

Proof: In order to prove $(A')' = A$, according to BASIC THEOREM. We have to prove that

(i) $(A')' \subset A$ ii) $A \subset (A')'$

i) Let $x \in (A')'$ $\Rightarrow x \notin A'$

$\Rightarrow x \in A$

$\therefore (A')' \subset A$ -------(1)

ii) Let $x \in A$ $\Rightarrow x \notin A'$
$\Rightarrow x \in (A')'$
$\therefore A \subset (A')'$ -------(2)

From (1) and (2)
$(A')' = A$

Problem: 6

Prove that $(A \cap B)' = A' \cup B'$ for any two sets A & B.

Proof: In order to prove $(A \cap B)' = A' \cup B'$,
According to "BASIC THEOREM" we have to prove that
i) $(A \cup B)' \subset A' \cup B'$ ii) $A' \cup B' \subset (A \cap B)'$

i) Let $x \in (A \cap B)'$ $\Rightarrow x \notin (A \cap B)$
$\Rightarrow x \notin A \vee x \notin B$
$\Rightarrow x \in A' \vee x \in B'$
$\Rightarrow x \in A' \cup B'$
$\therefore (A \cap B)' \subset A' \cup B'$ -------(1)

ii) Let $x \in A' \cup B'$ $\Rightarrow x \in A' \vee x \in B'$
$\Rightarrow x \notin A \vee x \notin B$
$\Rightarrow x \notin (A \cap B)$
$\Rightarrow x \in (A \cap B)'$
$A' \cup B' \subset (A \cap B)'$ --------(2)
From (1) and (2)
$(A \cap B)' = A' \cup B'$

Problem: 7

Prove that $A \cap B = \phi$ implies $B \cap A' = B$,
According to BASIC THEOREM, we have to prove that
i) $B \cap A' \subset B$
ii) $B \subset B \cap A'$

i) Let $x \in B \cap A'$ $\Rightarrow x \in B \wedge x \in A'$
$\Rightarrow x \in B \wedge x \notin A$
$\Rightarrow x \in B$ $[A \cap B = \phi)$
$\therefore B \cap A' \subset B$ --------(1)
ii) Let $x \in B$ $\Rightarrow x \in B \wedge x \notin A$ $[A \cap B = \phi]$
$\Rightarrow x \in B \wedge x \in A'$
$\Rightarrow x \in B \cap A'$
$B \subset B \cap A'$ --------(2)

Note: 1) $A \subset B \Rightarrow$ i) $A \cup B = B$ ii) $A \cap B = A$
2) $A \subset B$ & $B \subset C$ then $A \subset C$ and it is transitive properly
3) $A \cup B = A \cap B \Leftrightarrow A = B$
4) $A \cup B = \phi \Rightarrow A = \phi$ and $B = \phi$
5) $A' - B' = B - A$

6) $A \cap B = A - B' = B - A'$

7) $A \cap B' = A - B$

Problem: 8

Prove that $A-(A - B) = A \cap B$

Proof: In order to prove $A-(A - B) = A \cap B$ according to BASIC THEOREM, we have to prove that

i) $A-(A - B) \subset A \cap B$

ii) $A \cap B \subset A-(A - B)$

i) Let $x \in A-(A - B)$

$\Rightarrow x \in A \wedge x \notin (A - B)$

$\Rightarrow x \in A \wedge (x \notin A \vee x \in B)$

$\Rightarrow (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$

$\Rightarrow (x \in A \wedge x \in A') \vee (x \in A \wedge x \in B)$

$\Rightarrow x \in (A \cap A') \vee x \in (A \cap B)$

$\Rightarrow x \in (A \cap A') \cup (A \cap B)$

$\Rightarrow x \in \phi \cup (A \cap B)$ $[A \cap A' = \phi]$

$\Rightarrow x \in A \cap B$ $[\phi \cup A = A]$

$\therefore A-(A - B) \subset A \cap B$ --------(1)

ii) Let $x \in A \cap B$

$\Rightarrow x \in \phi \cup (A \cap B)$ $[A = \phi \cup A]$

$\Rightarrow x \in \phi \vee x \in (A \cap B)$

$\Rightarrow x \in (A \cap A') \vee x \in (A \cap B)$ $[\phi = A \cap A']$

$\Rightarrow (x \in A \wedge x \in A') \vee (x \in A \wedge x \in B)$

$\Rightarrow (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B)$

$\Rightarrow x \in A \wedge (x \notin A \vee x \in B)$

$\Rightarrow x \in A \wedge x \notin (A - B)$

$\Rightarrow x \in A - (A - B)$

$\therefore A \cap B \subset A-(A - B)$ --------(2)

From (1) and (2)

$A-(A - B) = A \cap B$

Principle of duality : In any law of sets, if $\cup/\cap$, ϕ/μ are interchanged, then the equation so formed is also a law of sets. This property is called "principle of duality".

Eg: 1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \leftrightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2. $A \cup \mu = \mu \leftrightarrow A \cap \phi = \phi$

3. $A \cap \phi = \phi \leftrightarrow A \cup \mu = \mu$

4. $\phi' = \mu \leftrightarrow \mu' = \phi$

Assignment:

1. Prove that $A-(B \cap C) = (A - B) \cup (A - C)$ for any three sets A, B and C.
2. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
3. Prove that $(A \cup B)' = A' \cap B'$
4. Prove that $A \cap B' = A - B$

Objective Bits:

1. $A \cap (B \cup C)$ = _____ and it is _____ law.
2. The set builder form of A = {1, 8, 27, 64} is _____
3. A & B are disjoint sets then $n(A \cup B)$ = _____

4. $A \subset B$ and $B \subset A$ then _____
5. $A \cup B = \phi$ then _____
6. $A \subset B$, $B \subset C$ then _____ and it is _____ property.
7. The identity set over the union of sets is _____
8. $A - B$, $A \cap B$, $B - A$ are _____ sets.
9. $A \cup A' =$ _____
10. $n(A) = 3$ then $n(p[A]) =$ _____