12. APPROXIMATIONS AND SMALL ERRORS

Synopsis :

1. Let y = f(x) be a function. If δx be the change in 'x', then the corresponding change in y is denoted by δy.

   $\delta y = \delta f = f(x + \delta x) - f(x)$

2. If y = f(x) is differentiable at x, then $f^{l}(x).\delta x$ is called the differential of the function. The differential of y = f(x) is denoted by dy or df.

   $\therefore\ dy = df = f^{l}(x).\delta x = \frac{dy}{dx}.\delta x$

3. $\delta y \cong dy$, $dy \cong f^{l}(x)\delta x$,

   $f(x + \delta x) \cong f^{l}(x)\delta x + f(x)$

4. i) δx is called the error in x.

   ii) $\frac{\delta x}{x}$ is called the relative error in x.

   iii) $\frac{\delta x}{x} x100$ is called the percentage error in x.

5. If y = f(x) is homogeneous function of degree n in x then

   i) The relative error in y is equal to n times the relative error in x

   ii) The percentage error in 'f' is equal to n times the percentage error in x.

6. Let r be the radius of a circle, then

   i) Circumference of a circle = $2\pi r$

   ii) Area of the circle = πr^2

7. Let r be the radius of a sphere, then

   i) Surface area of a sphere = $4\pi r^2$

   ii) Volume of a sphere = $\frac{4}{3}\pi r^3$

8. Let r be the radius and h be the altitude 'l' be the slant height of a right circular cone, then

   i) Surface area = πrl

   ii) Volume of the cone = $\frac{1}{3}\pi r^2 h$

   iii) $l^2 = r^2 + h^2$

   iv) $\sin\theta = \frac{r}{l}$

v) $\cos\theta = \frac{h}{l}$, where θ is semi-vertical angle

vi) $\tan\theta = \frac{r}{h}$

9. Let r be radius, h be height of the cylinder then

   i) Surface area = $2\pi rh$

   ii) Base area = πr^2

   iii) Volume = $\pi r^2 h$

10. Let r be the radius, θ be the vertex angle and 'l' be the arc length of a sector, then

    i) $l = r\theta$

    ii) Perimeter of a sector = $2r + r\theta$

    iii) Area of the sector = $\frac{1}{2}r^2\theta$

11. Let V be the volume, r be the radius, S be the surface area of a sphere;

    $V = \frac{4}{3}\pi r^3$

12. In a triangle if all the elements are changed slightly, then $\delta a.\sec A + \delta b.\sec B + \delta c.\sec C = 0$.