11. DIFFERENTIATION

Synopsis :

1. Let f be a function defined in a neighbourhood of a real number a. Then f is said to be differentiable or derivable at a if $\underset{x\to a}{Lt}\frac{f(x)-f(a)}{x-a}$ exists. The limit is called the derivative or differential coefficient of f at a. It is denoted by $f^{I}(a)$.
2. If f is differentiable at a, then $f^{I}(a) = \underset{h\to 0}{Lt}\frac{f(a+h)-f(a)}{h}$.
3. Let f be a function defined on a neighbourhood of a real number a. Then f is said to be right differentiable at a if $\underset{x\to a^{+}}{Lt}\frac{f(x)-f(a)}{x-a}$ exists. The limit is called the right derivative of f at a. It is denoted by $f^{I}(a+)$.
4. Similarly the left derivative of a function f at a is defined as $f^{I}(a-) = \underset{x\to a^{-}}{Lt}\frac{f(x)-f(a)}{x-a}$.
5. Let f be a function defined on [a, b]. Then f is said to be differentiable on [a, b] if
   i) f is differentiable at c where $c \in (a, b)$
   ii) f is right differentiable at a
   iii) f is left differentiable at b.
6. If a function f is differentiable at a, then f is continuous at a.
7. If c is a constant then $\frac{d}{dx}\{c\} = 0$.
8. $\frac{d}{dx}\{x\} = 1$.
9. $\frac{d}{dx}\{x^{n}\} = nx^{n-1}$.
10. $\frac{d}{dx}\{\sqrt{x}\} = \frac{1}{2\sqrt{x}}$.

f(x)	**$f^{I}(x)$**
$c \in R$	0
x	1
$x^{n}; n \in N$	nx^{n-1}
$x^{n}; n \in R$	nx^{n-1}
e^{x}	e^{x}
$a^{x}, a \in R^{+}$	$a^{x}\log a$
logx	1/x
|x|	|x|/x, x≠0
log|x|	1/x
x^{x}	$x^{x}(1 + \log x)$
sinx	cosx
cosx	−sinx
tanx	$\sec^{2}x$
cotx	$-\operatorname{cosec}^{2}x$
secx	secxtanx

cosecx	$-\text{cosecxcotx}$
$\text{Sin}^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$\text{Cos}^{-1}\,x$	$\frac{-1}{\sqrt{1-x^2}}$
$\text{Tan}^{-1}\,x$	$\frac{1}{1+x^2}$
$\text{Cot}^{-1}\,x$	$\frac{-1}{1+x^2}$
$\text{Sec}^{-1}\,x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
$\text{Cosec}^{-1}\,x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$
Sinhx	coshx
Coshx	sinhx
Tanhx	sech^2x
Cothx	$-\text{cosech}^2x$
Sechx	$-\text{sechxtanhx}$
Cosechx	$-\text{cosechxcothx}$
$\text{Sinh}^{-1}\,x$	$\frac{1}{\sqrt{1+x^2}}$
$\text{Cosh}^{-1}\,x$	$\frac{1}{\sqrt{x^2-1}}$
$\text{Tanh}^{-1}\,x$	$\frac{1}{1-x^2}$ $(\lvert x\rvert<1)$
$\text{Coth}^{-1}\,x$	$\frac{1}{1-x^2}$ $(\lvert x\rvert>1)$
$\text{Sech}^{-1}\,x$	$\frac{-1}{\lvert x\rvert\sqrt{1-x^2}}$
$\text{Cosech}^{-1}\,x$	$\frac{-1}{\lvert x\rvert\sqrt{x^2+1}}$

11. $\frac{d}{dx}\left\{f(x)^{g(x)}\right\}=f(x)^{g(x)}\left[g(x)\frac{f'(x)}{f(x)}+g'(x)\log f(x)\right]$

12. $\frac{d}{dx}\left\{f(x)^{f(x)}\right\}=f(x)^{f(x)}\left[1+\log f(x)\right]f'(x)$

13. If $y=f(x)^y$, then $\frac{dy}{dx}=\frac{y^2f'(x)}{f(x)[1-y\log f(x)]}=\frac{y^2f'(x)}{f(x)[1-\log y]}$

14. If $y = \sqrt{f(x) + y}$, then $\frac{dy}{dx} = \frac{f'(x)}{2y - 1}$.

15. If $y = f(x) + \frac{1}{y}$, then $\frac{dy}{dx} = \frac{y^2 f'(x)}{y^2 + 1}$

16. If f(x, y) = c is an implicit function, then $\frac{dy}{dx} = -\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}$.

17. If f(x + y) = f(x)f(y), $\forall$ x,y $\in$ R and $f(x) \neq 0$, f(a) = k, $f'(0)$ exists, then $f'(a) = kf'(0)$.

18. If $y = \sqrt{f(x)\sqrt{f(x)\sqrt{f(x)....}}}$, then $\frac{dy}{dx} = f'(x)$.

19. If $x^m y^n = (x + y)^{m+n}$, then $\frac{dy}{dx} = \frac{y}{x}$.