10. CONTINUITY

Synopsis :

1. Let f be a function defined in a neighborhood of 'a'.

   i) If $\lim_{x\to a} f(x) = f(a)$ then we say that f(x) is continuous at x = a.

   ii) If $\lim_{x\to a^-} f(x) = f(a)$ then we say that f(x) is left continuous at x = a.

   iii) If $\lim_{x\to a^+} f(x) = f(a)$ then we say that f(x) is right continuous at x = a.

2. If f(x) is not continuous at x = a, then we can say that f(x) is discontinuous at x = a.

3. f(x) is discontinuous at x = a $\Leftrightarrow \begin{cases} \text{either } \lim_{x\to a} f(x) \text{ does not exists} \\ \text{or } \lim_{x\to a} f(x) \neq f(a) \end{cases}$

4. If f is continuous at every point in a set A, then we can say that f is continuous on the set A.

5. A function f is said to be continuous on [a b] if

   i) f is right continuous at x = a.

   ii) f is left continuous at x = b.

   iii) f is continuous at x = c where a < c < b.

6. If f(x) is continuous at x = a and g is continuous at f(a), then gof is continuous at x = a.

7. If f(x) and g(x) are continuous at x = a, then

   i) (f ±g)(x) and (f.g)(x) are continuous at x = a.

8. Every constant function is continuous on R.

9. The identify function is continuous on R.

10. Every polynomial function is continuous on R.

11. The function sinx and cosx are continuous on R.

12. The function tanx and secx are continuous on R– $\left\{x : x = (2n+1)\frac{\pi}{2},\ n \in Z\right\}$.

13. The function cotx and cosecx are continuous on R– $\{x : x = n\pi,\ n \in Z\}$

14. f(x) = [x] is continuous on R–Z and discontinuous on Z.

15. If f(x) is continuous at x=a, then |f(x)| is continuous at x = a. The converse need not be true.

16. The function $f(x) = e^x$ and $f(x) = a^x$ (a>0) are continuous on R