8. PLANES

Synopsis :

1. Let 'P' and 'Q' be two points on a surface, if every point on the line 'PQ' lies on the surface then that surface is called a plane.

2. If a,b,c, are the d.rs of a normal to the plane passing through the point (x_1, y_1, z_1) then the equation of that plane is ax +by+cz = $ax_1 + by_1 + cz_1$

CARTESIAN EQUATION OF A PLANE PASSING THROUGH THREE POINTS:

3. Equation of the plane passing through the points (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_3, y_3, z_3)

is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

GENERAL EQUATION OF A PLANE:

4. The equation ax + by + cz + d = 0 represents a plane. Here a, b,c are the d.rs of a normal to the plane

5. The equation of a plane passing through the line of intersection of the planes u = 0 and v = 0 is $u + \lambda v = 0$ where 'λ' is a variable

6. Equations of the planes passing through the point (a,b,c) and parallel to the yz – plane, zx – plane, xy plane are x = a, y =b, z = c respectively

7. Equations ax+by+r=0, by+cz+p=0, cz+ax+q=0 represents the planes perpendicular to XY, YZ, ZX planes respectively.

8. The foot of the perpendicular of the point $P(x_1, y_1, z_1)$ on the plane $ax + by + cz + d = 0$ is Q (h, k, l) then $\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{l-z_1}{c} = \frac{-(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$

9. If Q (*h, k, l*) is the image of the point $p(x_1, y_1, z_1)$ w.r.t plane $ax + by + cz + d = 0$ then $\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{l-z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$

10. Equations of two planes bisecting the angles between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are $\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$

NOTE : Make the constants d_1, d_2 positive

Condition Acute Obtuse

i) $a_1a_2 + b_1b_2 + c_1c_2 > 0$ – +

ii) $a_1a_2 + b_1b_2 + c_1c_2 < 0$ + –

Note : i) Bisectors are perpendicular to each other

ii) Positive sign bisector is the bisector containing the origin.

11. Equation of plane parallel to the plane $ax+by+cz+d=0$

is given by $ax+by+cz+k=0$

12. The equation of the plane, mid way between the parallel planes $Ax+By+Cz+D_1=0$ and $Ax+By+Cz+D_2=0$ is $Ax+By+Cz+\left(\frac{D_1+D_2}{2}\right)=0$

13. The equation of the plane parallel to

I) x – axis is of the form by + cz =d

II) y – aixs is of the form ax + cz = d

III) z – axis is of the form ax + by = d

14. The equation of the plane passing through (x_1,y_1,z_1) and parallel to

I) yz – plane is $x=x_1$

II) zx – plane is $y=y_1$

III) xy – plane is $z=z_1$

15. The ratio in which the plane ax + by + cz + d = 0 divides the line segment joining (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{-(ax_1+by_1+cz_1+d)}{(ax_2+by_2+cz_2+d)}$

i)If $ax_1+by_1+cz_1+d$ and $ax_2+by_2+cz_2+d$ have the same sign then (x_1,y_1,z_1) and (x_2,y_2,z_2) lie on the same side of the plane.

ii)If $ax_1+by_1+cz_1+d$ and $ax_2+by_2+cz_2+d$ have opposite signs, then (x_1,y_1,z_1) and (x_2,y_2,z_2) lie on opposite sides of the plane.

Normal Form of a Plane:

16. Let OM be the perpendicular from O(0,0,0) to a plane. If OM = P and *l*,m,n are the d.cs of OM then the equation of that plane in the Normal form is *l*x + my + nz = P

17. The perpendicular distance from (x_1,y_1,z_1) to the plane ax + by + cz + d = 0 is $\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

18. The perpendicular distance of the palne ax+by+cz+d=0 from the origin is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$.

19. The distance between the parallel planes ax + by + cz + d_1 = 0 and ax + by + cz + d_2 = 0 is $\left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|$

Intercept Form of a Plane:

20. If a plane intersects the x, y, z axes at A,B,C respectively and O = (0,0,0) then OA, OB,OC are called the x – intercept, y – intercept, z – intercept of the plane respectively

21. The x,y,z intercepts of the plane ax + by + cz + d = 0 are $\frac{-d}{a}, \frac{-d}{b}, \frac{-d}{c}$ respectively

22. If a, b, c are the intercepts of a plane then the equation of the plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

23. The equation of the plane whose intercepts are 'K" times the intercepts made by the plane $Ax+By+Cz+D=0\ (D\neq 0)$ is $Ax+By+Cz+KD=0$

24. Area of the triangle formed by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ with

    i) x – axis , y –axis is $\frac{1}{2}|ab|$ Sq. units ii) y – axis, z – axis is $\frac{1}{2}|bc|$ Sq. units

    iii) z – axis, x – axis is $\frac{1}{2}|ca|$ Sq. units

25. If a plane meets the coordinate axes in A,B,C such that the centroid of the triangle ABC is the point (p,q,r) then the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$

Angle between Two Planes:

26. The angle between two planes is equal to the angle between the perpendiculars from the origin to the planes.

27. If 'θ' is the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ then $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

28. If the above two planes are parallel then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

    If the above two planes are perpendicular then $a_1a_2+b_1b_2+c_1c_2=0$