7. DIRECTION RATIOS AND DIRECTION COSINES

Synopsis :

1. If a directed line (ray) makes angle a, b, g with positive directions of the axes of x,y and z respectively the cosa, cosa, cosg are called the *direction cosines* (d.c's) of the line and these are usually denoted by l,m,n.

2. A line in a space can be extended in two directions, it has two sets of direction cosines. If (l,m,n) is one set then other set is . Here $(-l,-m,-n)$

i) $l^2+m^2+n^2=1$

ii) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

iii) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

3. Direction cosines of

i) x – axis are (1,0,0)

ii) y – axis are (0,1,0)

iii) z – axis are (0,0,1)

4. If (l, m, n) are direction cosines of a line OP where 'O' is the origin and OP = r then the coordinates P are *(lr, mr, nr).*

5. If the coordinates of a point P are (x,y, z) and OP=r then the direction Cosines of OP are (x/r, y/r/ z/r).

6. If $A=(x_1,y_1,z_1)$ and $B=(x_2,y_2,z_2)$ then AB d.cs $\frac{x_1-x_2}{AB},\frac{y_1-y_2}{AB},\frac{z_1-z_2}{AB}$ are

7. If θ is the angle between the two lines whose d.cs are l_1,m_1,n_1 and l_2,m_2,n_2 then

i) $\cos\theta=l_1l_2+m_1m_2+n_1n_2$

ii) $\sin\theta=\sqrt{\Sigma\left[l_1m_2-l_2m_1\right]^2}$

iii) if the lines are perpendicular $l_1l_2+m_1m_2+n_1n_2=0$ then

iv) If the lines are parallel then $\frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$.

8. If are the d.cs of the line AB and a,b,c are three numbers such that $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$ then are called the direction ratios (d.rs) of the line AB.

9. If a,b,c are the d.rs of the line AB then the d.cs of the line AB are

$$\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}$$

(or)

$$\frac{-a}{\sqrt{a^2+b^2+c^2}},\frac{-b}{\sqrt{a^2+b^2+c^2}},\frac{-c}{\sqrt{a^2+b^2+c^2}}$$

10. If $A=(x_1,y_1,z_1)$ and $B=(x_2,y_2,z_2)$ then the d.rs of AB are $x_1-x_2,\ y_1-y_2,\ z_1-z_2$

11. If θ is the angle between the two lines whose d.rs are a_1,b_1,c_1 and a_2,b_2,c_2 then

i) $\cos\theta=\dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$

ii) $\sin\theta=\dfrac{\sqrt{\Sigma(a_1b_2-a_2b_1)^2}}{\sqrt{a_1^2+b_1^2+c_1^2}\ \sqrt{a_2^2+b_2^2+c_2^2}}$

iii) if the lines are perpendicular $a_1a_2+b_1b_2+c_1c_2=0$ then

iv) if the lines are parallel then $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$ (or) $(a_1,b_1,c_1)=k(a_2,b_2,c_2)$ where k is a constant. Where $k\in R-\{0\}$

12. If a_1,b_1,c_1 and a_2,b_2,c_2 are the d.rs of the lines AB and AC then the d.rs of the line perpendicular to both AB and AC are $b_1c_2-b_2c_1,\ c_1a_2-c_2a_1,\ a_1b_2-a_2b_1$.

13. If l_1,m_1,n_1 and l_2,m_2,n_2 are the d.cs of the lines AB and AC then the d.rs of the bisectors of the angles between AB and AC are $l_1+l_2,\ m_1+m_2,\ n_1+n_2$ and $l_1-l_2,\ m_1-m_2,\ n_1-n_2$

14. If θ is the angle between two lines whose d.cs are l_1,m_1,n_1 and l_2,m_2,n_2 then the d.cs of their angular bisectors are $\dfrac{l_1+l_2}{2\cos\left(\frac{\theta}{2}\right)},\ \dfrac{m_1+m_2}{2\cos\left(\frac{\theta}{2}\right)},\ \dfrac{n_1+n_2}{2\cos\left(\frac{\theta}{2}\right)}$

And $\dfrac{l_1-l_2}{2\sin\left(\frac{\theta}{2}\right)},\dfrac{m_1-m_2}{2\sin\left(\frac{\theta}{2}\right)},\dfrac{n_1-n_2}{2\sin\left(\frac{\theta}{2}\right)}$

15. The projection of a point on a line is the foot of the perpendicular from that point to the line.

16. If $A=(x_1,y_1,z_1)$ and $B=(x_2,y_2,z_2)$ then the projection of AB on the line whose d.cs are l,m,n is $\left|l(x_1-x_2)+m(y_1-y_2)+n(z_1-z_2)\right|$

17. If $A=(x_1,y_1,z_1)$ and $B=(x_2,y_2,z_2)$ then

i) the projection of AB on the x-axis is $|x_1-x_2|$

ii) the projection of AB on the y-axis is $|y_1-y_2|$

iii) the projection of AB on the z-axis is $|z_1-z_2|$

18. If $A=(x_1,y_1,z_1)$ and $B=(x_2,y_2,z_2)$ then

i) the projection of AB on the yoz plane is $\sqrt{(y_1-y_2)^2+(z_1-z_2)^2}$

ii) the projection of AB on the zox plane is $\sqrt{(z_1-z_2)^2+(x_1-x_2)^2}$

iii) the projection of AB on the xoy plane is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

19. i) If the projections of a line of length 'd' on the coordinate axes are $d_1, d_2 \& d_3$ respectively, then $d_1^2+d_2^2+d_3^2=d^2$

ii) If the projections of a line of length 'd' on the coordinate planes are $d_1, d_2 \& d_3$ respectively, then $d_1^2+d_2^2+d_3^2=2d^2$

20. The d.rs of two lines are given by the equations $al+bm+cn=0$ and $fmn+gnl+hlm=0$. If the two lines are perpendicular then $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$. If the two lines are parallel then $\sqrt{af}+\sqrt{bg}+\sqrt{ch}=0$

21. The d.rs of two lines are given by the equations $al+bm+cn=0$ and $ul^2+vm^2+wn^2=0$. If the two lines are perpendicular then $\Sigma\left(a^2(v+w)\right)=0$. If the two lines are parallel then $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$

22. The equation of the line joining the points (x_1,y_1,z_1) and (x_2,y_2,z_2) is

$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}=t$ where t is a parameter. Here any point on the line is $\left[x_1+t(x_2-x_1),\ y_1+t(y_2-y_1),\ z_1+t(z_2-z_1)\right]$

23. If a,b,c are the d.rs of the line passing through the point (x_1,y_1,z_1) then the equation of that line is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

24. Lagrange's identity: If $l_1, m_1, n_1, l_2, m_2, n_2$ are real numbers then $\left(l_1^2+m_1^2+n_1^2\right)\left(l_2^2+m_2^2+n_2^2\right)$

$-\left(l_1l_2+m_1m_2+n_1n_2\right)^2$

$=\left(m_1n_2-m_2n_1\right)^2+\left(n_1l_2-n_2l_1\right)^2+\left(l_1m_2-l_2m_1\right)^2$

25. Angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

26. Angle between diagonal of a cube and diagonal of a face of a cube is

27. If $\alpha,\beta,\gamma,\ \delta$ are angles made by a ray with four diagonals of a cube, then $\cos^2\alpha+\cos^2\beta+\cos^2\ \ +\cos^2\ \ =\frac{4}{3}\gamma\qquad\qquad\delta$

28. If a line makes equal angles with the coordinate axes, then the line make an angle $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ or $\tan^{-1}\left(\sqrt{2}\right)$ or $\sin^{-1}\left(\sqrt{\frac{2}{3}}\right)$