6. THREE DIMENSIONAL GEOMETRY

Synopsis :

1. The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.
2. If A = (x_1, y_1, z_1), B = (x_2, y_2, z_2), the co-ordinates of point which divides AB in the ratio l:m internally is $\left(\frac{lx_2+mx_1}{l+m}, \frac{ly_2+my_1}{l+m}, \frac{lz_2+mz_1}{l+m}\right)$.
3. The co-ordinates of the point which divides AB in the ratio l:m externally is $\left(\frac{lx_2-mx_1}{l-m}, \frac{ly_2-my_1}{l-m}, \frac{lz_2-mz_1}{l-m}\right)$.
4. Mid point of AB = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.
5. The centroid of the triangle formed by the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.
6. The centroid of the tetrahedron formed by the points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) is $\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$.
7. The line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is divided by
   i) YZ plane in the ratio $x_1:x_2$
   ii) ZX plane in the ratio $y_1:y_2$
   iii) XY plane in the ratio $z_1:z_2$
8. If A = (x_1, y_1, z_1), B = (x_2, y_2, z_2), P = (x_3, y_3, z_3) are the vertices of a triangle and BC = a, CA = b, AB = c, then the incentre of the triangle ABC is $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$.
9. If a directed line (ray) makes angle α, β, γ with positive directions of the axes of x,y and z respectively the cosα, cosα, cosγ are called the *direction cosines* (d.c's) of the line and these are usually denoted by l,m,n.
10. Direction cosines of i) x – axis are (1,0,0) ii) y – axis are (0,1,0) iii) z – axis are (0,0,1)
11. If (l, m, n) are direction cosines of a ling $\overline{OP}$ and OP = r then the coordinates p are *(lr, mr, nr).*
12. If the coordinates of a point P are (x,y, z) and OP=r then the direction Cosines of OP are (x/r, y/r/ z/r).

13. If α, β, γ are the angles, made by a line with the positive direction of coordinate axes then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

14. If α, β, γ are the angles, made by a directed line with the positive directions of the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

15. If (a, b, c) are direction ratio of a line than

$\left(\frac{a}{\sqrt{a^2+b+c^2}}, \frac{b}{\sqrt{a+b^2+c_2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$ are direction cosines of the line.

16. Direction ratios of the line joining the points A (x_1, y_1, z_1), B (x_2, y_2, z_2) are $(x_2-x_1, y_2-y_1, z_2-z_1)$.

17. If l_1, m_1, n_1 and l_2, m_2, n_2 are real numbers then
$(l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1l_2 + m_1m_2 + n_1n_2)^2 = (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1) + (l_1m_2 - l_2m_2)^2$

18. If (l_1, m_1, n_1) and (l_2, m_2, n_2) are direction cosines of two lines and θ is an angle between then, then $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$.

19. If a line makes equal angles with co-ordinate axes, then d.c.'s of a vector are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$ and number of such lines passing through the origin is 4.

20. If (a_1, b_1, c_1) and (a_2, b_2, c_2) are d.c.'s of two lines and θ is the angle between the lines, then $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$.

21. If two lines are perpendicular, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

22. If l, m, n be the direction cosines of a line and $P = (x_1, y_1, z_1)$, $Q = (x_2, y_2, z_2)$. The projection of PQ on a line is $|l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$.

23. If $P = (x_1, y_1, z_1)$, $Q = (x_2, y_2, z_2)$, then
    i) The projection of PQ on x-axis is $|x_2 - x_1|$.
    ii) The projection of PQ on y-axis is $|y_2 - y_1|$.
    iii) The projection of PQ on z-axis is $|z_2 - z_1|$.

24. If l_1, m_1, n_1 and l_2, m_2, n_2 are d.R.'s of a two lines, then the d.R.'s of a line perpendicular to these two lines are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

25. If (l_1, m_1, n_1), (l_2, m_2, n_2), (l_3, m_3, n_3) are d.c.'s of three mutually perpendicular lines, then d.R.'s of the lines which makes equal angles with the three lines are $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ and each angle is $\cos^{-1}(1/\sqrt{3})$.

26. Angle between any two diagonals of a cube is $\text{Cos}^{-1}(1/3)$.

27. Angle between one diagonal of a cube and a diagonal of one face is $\text{Cos}^{-1}(\sqrt{2/3})$.

28. If l, m, n are d.c.'s of a line then the maximum value of lmn is $1/3\sqrt{3}$.

29. If $\alpha, \beta, \gamma, \delta$ are angles made by a line with diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$.