4. STRAIGHT LINES

Synopsis :

1. If a line makes an angle θ ($0 \le \theta < \pi$) with x-axis in the positive direction, then θ is called **inclination** of the line and $\tan\theta$ is called **slope** of the line. Slope of a line is denoted by m.
2. If θ is an angle between two nonvertical lines having slopes m_1, m_2 then $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$.
3. If θ is the acute angle between the lines having slopes m_1, m_2 then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.
4. The equation of the horizontal line passing through (x_1, y_1) is $y = y_1$.
5. The equation of the vertical line passing through (x_1, y_1) is $x = x_1$.
6. **Slope point form :** The equation of the line passing through $A(x_1, y_1)$ and having slope m is $y - y_1 = m(x - x_1)$.
7. **Two points form :** The equation of the line passing through (x_1, y_1) and (x_2, y_2) $(x_1 \neq x_2)$ is $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.
8. If a line cuts x-axis at A(a, 0) and y-axis at B(0, b), then a is called x-intercept and b is called y-intercept of the line.
9. **Slope intercept form :** The equation of the line having slope m and y-intercept c is $y = mx + c$.
10. The equation of the line parallel to $ax + by + c = 0$ and passing through (x_1, y_1) is $a(x - x_1) + b(y - y_1) = 0$
11. The equation of the line perpendicular to $ax + by + c = 0$ and passing through (x_1, y_1) is $b(x - x_1) - a(y - y_1) = 0$.
12. **Intercepts form :** The equation of the line having x-intercept a and y-intercept b is $\frac{x}{a} + \frac{y}{b} = 1$.
13. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is $\frac{1}{2}|ab|$.
14. The area of the triangle formed by the line $ax + by + c = 0$ with the coordinate axes is $\frac{c^2}{2|ab|}$.
15. **Perpendicular form or normal form :** The equation of the line which is at a distance of p from the origin and A, $0 \le A < 360^o$ is the angle made by the perpendicular with positive direction of x-axis is $x\cos A + y\sin A = p$.
16. The equation of the line $ax + by + c = 0$ in the perpendicular form is

    $\frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{a^2 + b^2}} = \frac{-c}{\sqrt{a^2 + b^2}}$ if $c \le 0$;

    $\frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{a^2 + b^2}} = \frac{-c}{\sqrt{a^2 + b^2}}$ if $c > 0$.

17. The perpendicular distance from a point $P(x_1, y_1)$ to the line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.
18. The distance between the two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.
19. The equations $x = x_1 + r\cos\theta$, $y = y_1 + r\sin\theta$ are called parametric equations with parameter r of the line passing through $(x_1,y_1)$and having inclination θ.
20. The ratio in which the line $L = ax + by + c = 0$ divides the line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$ is $-L_{11} : L_{22}$ where $L_{11} = ax_1 + by_1 + c$, $L_{22} = ax_2 + by_2 + c$
21. The points A, B lie in the same side or opposite sides of the line $L = 0$ according as L_{11}, L_{22} have the same sign or opposite signs.
22. Let $L \equiv ax + by + c = 0$ be a line and $A(x_1, y_1)$ be a point. Then A lies in the origin side of $L = 0$ if $cL_{11} > 0$ and A lies in the opposite to the origin side of $L = 0$ if $cL_{11} < 0$.
23. The condition that the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_3x + b_3y + c_3 = 0$ to be concurrent is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.
24. If θ is the angle between the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, then $\cos\theta = \frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$.
25. If θ is an acute angle, then $\cos\theta = \frac{|a_1a_2 + b_1b_2|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$ and $\sin\theta = \frac{|a_1b_2 - a_2b_1|}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}$.
26. If θ ($\neq \pi/2$) is an angle between the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ then $\tan\theta = \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}$.
27. The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.
28. The lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, are perpendicular if $a_1a_2 + b_1b_2 = 0$.
29. The foot of the perpendicular from (x_1, y_1) to the line $ax + by + c = 0$ is $(ar + x_1, br + y_1)$ where $r = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$.
30. The image of (x_1, y_1) w.r.t the x-axis is $(x_1, -y_1)$.
31. The image of (x_1, y_1) w.r.t the y-axis is $(-x_1, y_1)$.
32. The image of (x_1, y_1) w.r.t the line $ax + by + c = 0$ is $(ar + x_1, br + y_1)$ where $r = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$.
33. The image of (x_1, y_1) w.r.t the line $x = y$ is (y_1, x_1).

34. i) The image of y = k w.r.t. x-axis is y = –k.
    ii) The image of x = k w.r.t y-axis is x = –k.

35. The area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2$, $y = m_3x + c_3$ is $\frac{1}{2}\left|\sum \frac{(c_1 - c_2)^2}{m_1 - m_2}\right|$.

36. If p is the altitude of an equilateral triangle then
    i) length of the side = $2p/\sqrt{3}$.
    ii) area of the triangle = $p^2/\sqrt{3}$.

37. The area of the parallelogram formed by $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + d_1 = 0$, $a_2x + b_2y + c_2 = 0$, $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(c_1 - d_1)(c_2 - d_2)}{a_1b_2 - a_2b_1}\right|$.

38. The area of the rhombus formed by $ax \pm by \pm c = 0$ is $\frac{2c^2}{|ab|}$.

39. Equation of the line passing through (x_1, y_1) and making an angle A with the line ax + by + c = 0 are given by $y - y_1 = m_1(x - x_1)$ where $m_1 = \frac{a + b\tan A}{a\tan A - b}$ and $y - y_1 = m_2(x - x_1)$ where $m_2 = \frac{b\tan A - a}{b + a\tan A}$.

40. The equation of the line whose portion between the coordinate axes is divided by the point (x_1, y_1) in the ratio m:n is $\frac{nx}{x_1} + \frac{my}{y_1} = m + n$.

41. The image of the line lx + my + n = 0 with respect to ax + by + c = 0 is $(a^2 + b^2)(lx + my + n) - 2(la + mb)(ax + by + c) = 0$.

42. The number of lines that can be drawn through the point P at a distance d units from the point Q is i) 0 if PQ < d,
    ii) 1 if PQ = d iii) 2 if PQ>d.

43. The distance of the point (x_1, y_1) from the line ax + by + c = 0 measured along a line making an angle θ with x-axis is $\left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$.

44. If P is the orthocentre of the ΔABC, then the orthocentre of the triangle formed by any three points P, A, B, C will be the remaining the point.

45. Area of the parallelogram or rhombus, equations of whose sides are given is $\frac{p_1p_2}{\sin\theta}$ where p_1, p_2 are distance between the parallel lines and θ is the angle between adjacent sides