3. CHANGE OF AXES

Synopsis :

1. If the coordinates (x, y) of a point are transformed to (X, Y), when the axes are translated to the point (h, k), then $x = X + h$, $y = Y + k$.
2. If the axes are translated to the point (h, k) then the equation $f(x, y) = 0$ of a curve is transformed to $f(X + h, Y + k) = 0$.
3. If $f(X, Y) = 0$ is the transformed equation of a curve when the axes are translated to the point (h, k), then the original equation of the curve is $f(x - h, y - k) = 0$.
4. The point to which the origin has to be shifted to eliminate x, y terms is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$, where $(h^2 \neq ab)$
5. The point to which the origin has to be shifted to eliminate x and y terms in the equation $a(x + \alpha)^2 + b(y + \beta)^2 = c$ is $(-\alpha, -\beta)$.
6. If the coordinates (x, y) of a point are transformed to (X, Y), when the axes are rotated through an angle θ about the origin then $x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta \Rightarrow X = x\cos\theta + y\sin\theta$, $Y = -x\sin\theta + y\cos\theta$.
7. If the axes are rotated through an angle θ then the equation $f(x, y) = 0$ of a curve is transformed to $f(X\cos\theta - Y\sin\theta, X\sin\theta + Y\cos\theta) = 0$.
8. If $f(X, Y) = 0$ is the transformed equation of a curve when the axes are rotated through an angle θ then the original equation of the curve is $f(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta) = 0$.
9. The angle of rotation of the axes to eliminate xy terms in the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{1}{2}\text{Tan}^{-1}\left(\frac{2h}{a - b}\right)$, where $(a \neq b)$
10. The angle of rotation of the axes to eliminate xy term in the equation $ax^2 + 2hxy + ay^2 + 2gx + 2fy + c = 0$ is $\pi/4$.
11. The condition that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to take the form $aX^2 + 2hXY + bY^2 = 0$, when the axes are translated, is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.
12. If x, y terms are eliminated from $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ by shifting the origin to (α, β) then the transformed equation is $ax^2 + 2hxy + by^2 + g\alpha + f\beta + c = 0$.
13. A curve $f(x, y) = 0$ is symmetric about
    i) x-axis if $f(x, -y) = f(x, y)$
    ii) y-axis if $f(-x, y) = f(x, y)$
    iii) origin if $f(-x, -y) = f(x, y)$.