1. COORDINATE SYSTEM

Synopsis :

1. The distance between the points A (x_1, y_1) and B (x_2, y_2) is AB = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
2. The point which divides the line segment joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ in the ratio l : m.

   i) internally is $\left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}\right)$ $(l + m \neq 0)$

   ii) externally is $\left(\frac{lx_2 - mx_1}{l-m}, \frac{ly_2 - my_1}{l-m}\right)$ $(l \neq m)$
3. Let A, B be two points. The points which divide $\overline{AB}$ in the ratio 1 : 2 and 2 : 1 are called points of trisection of $\overline{AB}$.
4. If P(h, k) lies on the line joining $A(x_1, y_1)$, $B(x_2, y_2)$ then $\frac{x_1 - h}{h - x_2} = \frac{y_1 - k}{k - y_2}$ and P divides $\overline{AB}$ in the ratio $x_1 - h : h - x_2$ or $y_1 - k : k - y_2$.
5. x-axis divides the line segment joining (x_1, y_1), (x_2, y_2) in the ratio $-y_1 : y_2$.
6. y-axis divides the line segment joining (x_1, y_1), (x_2, y_2) in the ratio $-x_1 : x_2$.
7. If D (x_1, y_1), E (x_2, y_2), F (x_3, y_3) are the midpoints of the sides $\overline{BC}, \overline{CA}, \overline{AB}$ of ABC, then $A = (x_2 + x_3 - x_1, y_2 + y_3 - y_1)$, $B(x_3 + x_1 - x_2, y_3 + y_1 - y_2)$, $C = (x_1 + x_2 - x_3, y_1 + y_2 - y_3)$
8. If (x_1, y_1), (x_2, y_2), (x_3, y_3) are three consecutive vertices of a parallelogram, then the fourth vertex is $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$.
9. If a point P divides the line segment joining the points A, B in the ratio l : m, then the point Q which divides AB in the ratio l : –m is called harmonic conjugate of P with respect to A and B.
10. The centroid of the triangle formed by the points
    $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.
11. The circumcentre of a right angled triangle is the midpoint of the hypotenuse.
12. The orthocentre of a right angled triangle is the vertex at the right angle.
13. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of ΔABC and if a = BC, b = CA, c = AB, then incentre

    $$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

    Excentre opposite to A is

    $$I_1 = \left(\frac{-ax_1 + bx_2 + cx_3}{-a+b+c}, \frac{-ay_1 + by_2 + cy_3}{-a+b+c}\right)$$

    Excentre opposite to B is

    $$I_2 = \left(\frac{ax_1 - bx_2 + cx_3}{a-b+c}, \frac{ay_1 - by_2 + cy_3}{a-b+c}\right)$$

Excentre opposite to C is

$I_3 = \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c} \right)$

14. Orthocentre O, centroid G, circumcentre S of a triangle are collinear and G divides OS in the ratio 2 : 1.

15. The area of the triangle formed by the points O (0, 0), A (x_1, y_1), B (x_2, y_2) is $\frac{1}{2}|x_1y_2 - x_2y_1|$.

16. The area of the triangle formed by the points (x_1, y_1), (x_2, y_2), (x_3, y_3) is $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$.

17. If G is the centroid of ΔABC, then area of ΔGAB = area of ΔGBC = area of ΔGCA = $\frac{1}{3}$[area of ΔABC]

18. If D, E, F are the mid points of the sides $\overline{BC}, \overline{CA}, \overline{AB}$ of ABC, then area of ΔDEF = area of ΔAEF = area of ΔBDF = area of ΔCDE = $\frac{1}{4}$[area of ΔABC]

19. Three points A, B, C are collinear if area of ΔABC is zero.

20. The area of the quadrilateral formed by the points (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4) taken in order is $\frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$.

21. $A_1(x_1, y_1)$, $A_2(x_2, y_2)$, …. $A_n(x_n, y_n)$ are the vertices of a polygon, then its area = $\frac{1}{2}\left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \ldots + \begin{vmatrix} x_{n-1} & y_{n-1} \\ x_n & y_n \end{vmatrix} + \begin{vmatrix} x_n & y_n \\ x_1 & y_1 \end{vmatrix}\right]$

22. The incentre of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is

$\left(\frac{a|b|}{|a| + |b| + \sqrt{a^2 + b^2}}, \frac{|a|b}{|a| + |b| + \sqrt{a^2 + b^2}} \right)$

23. If (x_1, y_1), (x_2, y_2) are two vertices of an equilateral triangle, then its third vertex is $\left(\frac{x_1 + x_2 \pm \sqrt{3}(y_1 - y_2)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_1 - x_2)}{2} \right)$

24. If P and Q are the points of trisections of the line joining the points A and B, then the mid point of AB = mid point PQ.

25. If (x_1, y_1), (x_2, y_2) are extremities of the hypotenuse of a right angled isosceles triangle, then its third vertex is $\left(\frac{x_1 + x_2 \pm (y_1 - y_2)}{2}, \frac{y_1 + y_2 \mp (x_1 - x_2)}{2} \right)$.

26. The orthocentre of the triangle with vertices (0, 0), (x_1, y_1), (x_2, y_2) is $(-k(y_1 - y_2), k(x_1 - x_2))$ where $k = \frac{x_1x_2 + y_1y_2}{x_1y_2 - x_2y_1}$.

27. In ΔABC, if AD is the median drawn to BC, then
$AB^2 + AC^2 = 2(AD^2 + CD^2) = 2(AD^2 + BD^2)$

28. If G is the centroid of ΔABC, then $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.
29. If P is a point in ΔABC and G is the centroid, then $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3PG^2$.
30. The coordinates of the vertices of an equilateral triangle are not all rationals.
31. If Q is the harmonic conjugate of P w.r.t. A, B then AP, AB, AQ are in H.P.
32. **Nine point circle :** Let L, M, N be the feet of the altitudes and D, E, F be the midpoints of the sides of ΔABC. Let X, Y, Z be the midpoints of AO, BO, CO where O is the orthocentre of the triangle. Then L, M, N; D, E, F; X, Y, Z lie on a circle called nine point circle of the triangle.
33. The radius of the nine point circle of ABC is R/2 where R is the circumradius.
34. If N is the center of the nine point circle, O is the orthocentre, S is the circumcentre and G is the centroid of ΔABC, then ON : NG = 3 : 1, ON : NS = 1 : 1, NG : GS = 1 : 2.
35. Area of the triangle formed by (x_1, y_1), (x_2, y_2), (x_3, y_3) is $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$.
36. If θ is the angle subtended by the line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$ at the origin, then $\cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}$.
37. Three points which are such that only two x-coordinates are equal and only two y-coordinates are equal form a right angled triangle. Then its
    i) orthocentre = (repeated x-coordinate, repeated y-coordinate)
    ii) circumcentre = (sum of distinct x-coordinates/2, sum of distinct y-coordinates/2)

PREVIOUS EAMCET BITS

1. If, *l*, m, n are in arithmetic progression, then the straight line $lx + my + n = 0$ will pass through the point **[EAMCET 2008]**

   1) (–1, 2) 2) (1, –2) 3) (1, 2) 4) (2, 1)

   Ans: 2

Sol. *l*, m, n are in A.P $\Rightarrow m - l = n - m \Rightarrow l - 2m + n = 0 \Rightarrow (1, -2)$ lies on $lx + my + n = 0$

2. In the triangle with vertices at $A(6,3), B(-6,3)$ and $C(-6,-3)$, the median through A meets BC at P, the line AC meets the x-axis at Q, while R and S respectively denote the orthocentre and centroid of the triangle. Then the correct matching of the coordinates of points in List – I to List – II is **[EAMCET 2007]**

List – I	**List – II**
i) P	A) (0, 0)
ii) Q	B) (6, 0)
iii) R	C) (–2, 1)
iv) S	D) (–6, 0)
	E) (–6, –3)

F) (–6, 3)

	i	ii	iii	iv		i	ii	iii	iv
1)	D	A	E	C	2)	D	B	E	C
3)	D	A	F	C	4)	B	A	F	C

Ans: 3

Sol. i) P is midpoint of BC = (–6, 0) = D

ii) Midpoint of AC is $(0, 0) \Rightarrow$ AC meets x-axis at Q(0, 0) = A

iii) ΔABC is right angled at B. Orthocentre = R = (–6, 3) = F

iv) Centroid = S = (–2, 1) = C

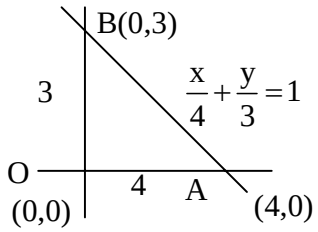
3. The area (in square units) of the triangle formed by the lines x = 0, y = 0 and 3x + 4y = 12 is **[EAMCET 2005]**

1) 3 2) 4 3) 6 4) 12

Ans: 3

Sol. Area of Δle OAB = $\frac{1}{2}$ base $\times$ height

$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6$



4. If PM is the perpendicular from P(2, 3) onto the line x + y = 3, then the coordinates of M are **[EAMCET 2005]**

1) (2, 1) 2) (–1, 4) 3) (1, 2) 4) (4, –1)

Ans: 3

Sol. P(2, 3), $\ell = x + y = 3$, slope = - 1 by verification product of slopes = –1

from (3) option slope PM = $\frac{3-2}{2-1} = 1$

1(–1) = – 1

5. The point P is equidistant from A(1, 3), B(–3, 5) and C(5, –1). Then PA - . **[EAMCET 2003]**

1) 5 2) $5\sqrt{5}$ 3) 25 4) $5\sqrt{10}$

Ans: 4

Sol. $PA^2 = PB^2 = PC^2$

$(x-1)^2 + (y-3)^2 = (x+3)^2 + (y-5)^2$

$= (x-5)^2 + (y+1)^2$

$\Rightarrow P(x, y) = (-8, -10)$

$\therefore\ PA = 5\sqrt{10}$

6. If (–2, 6) is the image of the point (4, 2) with respect to the line L = 0, then L = **[EAMCET 2002]**

1) $6x-4y-7$ 2) $2x+3y-5$ 3) $3x-2y+5$ 4) $3x-2y+10$

Ans: 3

Sol. L = 0 is perpendicular bisector of line segment joining the roots (–2, 6) (4, 2) L = 3x –2y+5

7. If the altitude of a triangle are in arithmetic progression, then the sides of the triangle are in .. progression **[EAMCET 2002]**

1) arithmetic 2) harmonic 3) geometric 4) arithmetico-geometric

Ans: 2

Sol. $\Delta=\frac{1}{2}P_1a\Rightarrow P_1=\frac{2\Delta}{a}$

$P_2=\frac{2\Delta}{b}$ $P_3=\frac{2\Delta}{c}$ P_1,P_2,P_3 are in A.P.

$\Rightarrow a,b,c$ are in H.P

8. The lines $2x+3y=6, 2x+3y=8$ cut the x-axis at A, B respectively. A line *l* drawn through the point (2, 2) meets the x-axis at C. In such a way that abscissae of A, B and C are in arithmetic progression. Then the equation of the line *l* is **[EAMCET 2001]**

1) $2x+3y=20$ 2) $3x+2y=10$ 3) $2x-3y=10$ 4) $3x-2y=10$

Ans: 1

Sol. The lines 2x + 3y = 6 and 2x + 3y = 8 cuts x-axis at A and B

$\therefore$ A(3, 0), B(4, 0)

The point 'C' lies on x-axis and the abscissae of the points A, B, C are in A.P.

$\therefore$C(5, 0)

$\therefore$ The equation of the line passing through (2, 2) and (5, 0) is 2x + 3y = 10

9. The incentre of the triangle formed by the lines x + y = 1 , x = 1, y = 1 is **[EAMCET 2001]**

1) $\left(1-\frac{1}{\sqrt{2}},1-\frac{1}{\sqrt{2}}\right)$ 2) $\left(1-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ 3) $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ 4) $\left(\frac{1}{\sqrt{2}},1-\frac{1}{\sqrt{2}}\right)$

Ans: 3

Sol. The vertices of the triangle are (1, 0) (0, 1), (1, 1) and lengths of the sides are 1, 1, $\sqrt{2}$

$\therefore$ Incentre $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

10. The vertices of a triangle are (6, 6), (0, 6) and (6, 0). The distance between the circumentre and centroid is **[EAMCET 2000]**

1) $2\sqrt{2}$ 2) 2 3) $\sqrt{2}$ 4) 1

Ans: 3

Sol. Circumcentre = S(3, 3)

Centroid = G(4,4) $\therefore$ $\overrightarrow{SG}=\sqrt{2}$

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