18. DEMOVIRE'S THEOREM AND TRIGONOMETRIC EXPANSION

Synopsis :

1. **De moivre's Theorem for integral index:** If n is an integer, then
   $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
2. $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$
3. $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$
4. $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$
5. **nth roots of a complex number :** The nth roots of a complex number
   $z = r\cos\theta$ are $r^{1/n}\,\text{cis}\left(\frac{2k\pi+\theta}{n}\right)$ where $k = 0,1,2,.....n-1$
6. **nth roots of unity :** The nth roots of unity are $1,\ \alpha, \alpha^2, ...\alpha^{n-1}$ where $\alpha = \text{cis}\frac{2\pi}{n}$
7. The nth roots of a unity form a G.P.
8. The sum of nth roots of unity = 0
9. The product of nth roots unity = $(-1)^{n-1}$
10. **DeMoivre's theorem for rational index :** If $\frac{p}{q}(q>1)$ is a rational number, then
    $\cos\frac{p\theta}{q} + i\sin\frac{p\theta}{q}$ is one of the qth roots of $(\cos\theta + i\sin\theta)^p$
11. $(\text{cis}\alpha)(\text{cis}\beta) = \text{cis}(\alpha+\beta)$
12. $(\text{cis}\alpha)/(\text{cis}\beta) = \text{cis}(\alpha-\beta)$
13. Expansion of trigonometric functions :

    i) $\cos n\theta = \cos^n\theta - C(n,2)\cos^{n-2}\theta.\sin^2\theta +$
    $C(n,4)\cos^{n-4}\theta.\sin^4\theta + \ldots\ldots\ldots$

    ii) $\sin n\theta = C(n,1)\cos^{n-1}\theta.\sin\theta + C(n,3)\cos^{n-3}\theta.\sin 3\theta + C(n,5)\cos^{n-5}\theta.\sin 5\theta + \ldots\ldots$

    iii) $\tan n\theta = \frac{C(n,1)\tan\theta - C(n,3)\tan^3\theta + C(n,5)\tan^5\theta - ....}{1 - C(n,2)\tan^2\theta + C(n,4)\tan^4\theta - ....}$

    iv) $\cos^n\theta = \frac{1}{2^{n-1}}[\cos n\theta + C(n,1)\cos(n-2)\theta + C(n,2)\cos(n-4)\theta + \ldots\ldots]$

    v) $\sin^n\theta = \frac{1}{2^{n-1}}[\cos n\theta + C(n,1)\cos(n-2)\theta + C(n,2)\cos(n-4)\theta + \ldots\ldots]$ if n is even.

    vi) $\sin^n\theta = \frac{1}{(2i)^{n-1}}[\cos n\theta + C(n,1)\cos(n-2)\theta + C(n,2)\cos(n-4)\theta + \ldots.]$ if n is odd