14. HYPERBOLIC FUNCTIONS

Synopsis :

1. i) $\sinh x = \frac{e^x - e^{-x}}{2}$

   ii) $\cosh x = \frac{e^x + e^{-x}}{2}$

   iii) $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

   iv) $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

   v) $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

   vi) $\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$ are called hyperbolic functions.

   Note : $\sin(ix) = i\sinh x$, $\cos(ix) = \cosh x$, $\tan(ix) = i\tanh x$.

2. $\cosh^2 x - \sinh^2 x = 1$

3. $1 - \tanh^2 x = \operatorname{sech}^2 x$

4. $\coth^2 x - 1 = \operatorname{cosech}^2 x$

5. i) $\sinh(\alpha + \beta) = \sinh\alpha\cosh\beta + \cosh\alpha\sinh\beta$

   ii) $\sinh(\alpha - \beta) = \sinh\alpha\cosh\beta - \cosh\alpha\sinh\beta$

   iii) $\cosh(\alpha + \beta) = \cosh\alpha\cosh\beta + \sinh\alpha\sinh\beta$

   iv) $\cosh(\alpha - \beta) = \cosh\alpha\cosh\beta - \sinh\alpha\sinh\beta$

   v) $\tanh(\alpha + \beta) = \frac{\tanh\alpha + \tanh\beta}{1 + \tanh\alpha\tanh\beta}$

   vi) $\tanh(\alpha - \beta) = \frac{\tanh\alpha - \tanh\beta}{1 - \tanh\alpha\tanh\beta}$

6. i) $\sinh 2x = 2\sinh x\cosh x$

   ii) $\cosh 2x = \cosh^2 x + \sinh^2 x$

   iii) $\cosh 2x = 2\cosh^2 x - 1$ or $\cosh^2 x = \frac{1 + \cosh 2x}{2}$

   iv) $\cosh 2x = 1 + \sinh^2 x$ or $2\sinh^2 x = \frac{\cosh 2x - 1}{2}$

   v) $\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$

vi) $\sinh 3x = 3\sinh x + 4\sinh^3 x$

vii) $\cosh 3x = 4\cosh^3 x - 3\cosh x$

viii) $\tanh 3x = \dfrac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$

7. **Values of inverse hyperbolic functions as logarithms functions** :

i) $\text{Sinh}^{-1}x = \log_e(x + \sqrt{x^2 + 1})$

ii) $\text{Cosh}^{-1}x = \log(x + \sqrt{x^2 - 1}),\ x \geq 1$

iii) $\text{Tanh}^{-1}x = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right),\ x \in (-1,1)$

iv) $\text{Coth}^{-1}x = \dfrac{1}{2}\log\left(\dfrac{x+1}{x-1}\right),\ |x| > 1$

v) $\text{Sech}^{-1}x = \log\left(\dfrac{1 \pm \sqrt{1-x^2}}{x}\right),\ 0 < x \leq 1$

vi) $\text{Co sec h}^{-1}x = \log\left(\dfrac{1+\sqrt{1+x^2}}{x}\right),\ x > 0$ or

$\log\left(\dfrac{1-\sqrt{1+x^2}}{x}\right),\ x < 0$