12. TRIGONOMETRIC EQUATIONS

Synopsis :

1. An equation involving trigonometric functions of a variable over a set of numbers or over a set of angles is called a trigonometric equation.
2. The set of values of the variable for which the equation is satisfied is the solution set of the equation.
3. The method of finding the solution set is called solving of an equation.
4. If the product of trigonometric functions is zero, then each factor should be equated to zero. The solutions of the resulting equations will give the solution set of the given equation.
5. If $\sin\theta = k$ where $-1 \le k \le 1$, then the value of θ lying in the interval $[-\pi/2, \pi/2]$ and satisfying the equation is called the principal solution.
6. If $\cos\theta = k$ where $-1 \le k \le 1$, then the value of θ lying in the interval $[0, \pi]$ and satisfying the equation is called the principal solution.
7. If $\tan\theta = k$ where $k \in R$, then the value of θ lying in the interval $(-\pi/2, \pi/2)$ and satisfying the equation is called the principal solution.
8. The general solution of $\sin\theta = k$ for $-1 \le k \le 1$ is $\theta = n\pi + (-1)^n\alpha$, $n \in Z$ where $\alpha \in [-\pi/2, \pi/2]$ is the principal value.
9. The general solution of $\cos\theta = k$ for $-1 \le k \le 1$ is $\theta = 2n\pi \pm \alpha$, $n \in Z$ where $\alpha \in (-\pi/2, \pi/2)$ is the principal value.
10. If $\sin\theta = 0$, then $\theta = n\pi, n \in Z$
11. If $\cos\theta = 0$, then $\theta = (2n+1)\pi/2, n \in Z$
12. If $\tan\theta = 0$, then $\theta = n\pi, n \in Z$
13. If (i) $\sin^2\theta = \sin^2\alpha$
    ii) $\cos^2\theta = \cos^2\alpha$ and
    iii) $\tan^2\theta = \tan^2\alpha$ then in each case $\theta = n\pi \pm \alpha, n \in Z$
14. The number of solutions of the equation $a\cos\theta + b\sin\theta = c$ is infinite if $|c| \le \sqrt{a^2 + b^2}$ (or) $c^2 \le a^2 + b^2$
15. The equation $a\cos\theta + b\sin\theta = c$ has no solution if $c^2 > a^2 + b^2$.