10. TRANSFORMATIONS

Synopsis :

1. i) $\sin (A + B) + \sin (A - B) = 2 \sin A \cos B$

   ii) $\sin (A + B) - \sin (A - B) = 2 \cos A \sin B$

   iii) $\cos (A + B) + \cos (A - B) = 2 \cos A \cos B$

   iv) $\cos (A - B) - \cos (A + B) = 2 \sin A \sin B$

2. i) $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

   ii) $\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

   iii) $\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

   iv) $\cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

   $= 2\sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right)$

3. i) $2 \sin A \cos B = \sin (A + B) + \sin (A - B)$

   ii) $2 \cos A \sin B = \sin (A + B) - \sin (A - B)$

   iii) $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

   iv) $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

4. If $\cos x + \cos y = a$, $\sin x + \sin y = b$, then

   i) $\tan \frac{x+y}{2} = \frac{b}{a}$ ii) $\sin (x + y) = \frac{2ab}{a^2 + b^2}$

   iii) $\cos (x + y) = \frac{a^2 - b^2}{a^2 + b^2}$ iv) $\tan (x + y) = \frac{2ab}{a^2 - b^2}$

5. If $\cos x - \cos y = a$, $\sin x - \sin y = b$, then

   i) $\tan \frac{x+y}{2} = -\frac{a}{b}$

ii) $\sin(x + y) = -\frac{2ab}{a^2 + b^2}$

iii) $\cos(x + y) = \frac{b^2 - a^2}{b^2 + a^2}$

iv) $\tan(x + y) = -\frac{2ab}{a^2 - b^2}$

6. If $\tan x + \tan y = a$, $\cot x + \cot y = b$, then

i) $\tan(x + y) = \frac{ab}{b - a}$

ii) $\cot(x + y) = \frac{b - a}{ab}$

7. If $\tan x - \tan y = a$, $\cot x - \cot y = b$, then

i) $\tan(x - y) = \frac{ab}{a + b}$

ii) $\cot(x - y) = \frac{a + b}{ab}$

8. If $A + B + C = 180°$, then

i) $\sin(A + B) = \sin C$ ii) $\cos(A + B) = -\cos C$

iii) $\tan(A + B) = -\tan C$ iv) $\cot(A + B) = -\cot C$

v) $\sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$ vi) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

vii) $\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$ viii) $\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

If $A + B + C = 180°$

9. $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

10. $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

11. $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

12. $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

13. $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

14. $\cos 2A + \cos 2B + \cos 2C =$

$$-1 - 4 \cos A \cos B \cos C$$

15. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$

16. $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$

17. $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

18. $\cos A + \cos B + \cos C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} + 1$