# 9. MULTIPLE AND SUBMULTIPLE ANGLES

***Synopsis :***

1. i) $\sin 2A = 2 \sin A \cos A$

$= \dfrac{2\tan A}{1+\tan^2 A}$

ii) $\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin 2A$

$= 2\cos 2A - 1$

$= \dfrac{1-\tan^2 A}{1+\tan^2 A}$

iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

2. i) $\sin 3A = 3\sin A - 4\sin^3 A$

ii) $\cos 3A = 4\cos^3 A - 3\cos A$

iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

3. The values of Trigonometric functions of some standard angles :

| $\theta$ | 18° | 36° | 54° | 72° |
|---|---|---|---|---|
| sin | $\dfrac{\sqrt{5}-1}{4}$ | $\dfrac{\sqrt{10-2\sqrt{5}}}{4}$ | $\dfrac{\sqrt{5}+1}{4}$ | $\dfrac{\sqrt{10+2\sqrt{5}}}{4}$ |
| cos | $\dfrac{\sqrt{10+2\sqrt{5}}}{4}$ | $\dfrac{\sqrt{5}+1}{4}$ | $\dfrac{\sqrt{10-2\sqrt{5}}}{4}$ | $\dfrac{\sqrt{5}-1}{4}$ |

4. i) $\sin A \cdot \sin(60° - A) \sin(60° + A) = \dfrac{1}{4}\sin 3A$

ii) $\sin A \cdot \sin(120° - A) \sin(120° + A) = \dfrac{1}{4}\sin 3A$

5. i) $\cos A \cdot \cos(60° - A) \cos(60° + A) = \dfrac{1}{4}\cos 3A$

ii) $\cos A \cdot \cos(120° - A) \cos(120° + A) = \dfrac{1}{4}\cos 3A$

6. i) $\tan A \cdot \tan(60° - A) \tan(60° + A) = \tan 3A$

ii) $\tan A \cdot \tan(120° - A) \tan(120° + A) = \tan 3A$

7. i) $\cot A \cot(60° - A) \cot(60° + A) = \cot 3A$

ii) $\cot A \cot(120° - A) \cot(120° + A) = \cot 3A$

8. i) $\sin^2\theta + \sin^2(60° + \theta) + \sin^2(60° - \theta) = 3/2$

   ii) $\cos^2\theta + \cos^2(60° + \theta) + \cos^2(60° - \theta) = 3/2$

   iii) $\sin^2\theta + \sin^2(120° + \theta) + \sin^2(120° - \theta) = 3/2$

   iv) $\cos^2\theta + \cos^2(120° + \theta) + \cos^2(120° - \theta) = 3/2$

9. i) If $\tan A + \tan B + \tan C = \tan A \tan B \tan C$,

   then $A + B + C = n\pi$, $n \in Z$.

   ii) If $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$,

   then $A + B + C = (2n + 1)\,\pi/2$, $n \in Z$.

10. $\cos x \cos 2x \cos 4x \ldots \cos(2^n x) = \dfrac{\sin(2^{n+1}x)}{2^{n+1}\sin x}$.