8. COMPOUND ANGLES

Synopsis :

1. i) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

   ii) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

2. i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

   ii) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

3. i) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

   ii) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

4. i) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

   ii) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

5. $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$

   $= \cos^2 B - \cos^2 A$

6. $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$

   $= \cos^2 B - \sin^2 A$

7. i) $\tan\left(\dfrac{\pi}{4} + A\right) = \dfrac{1 + \tan A}{1 - \tan A}$

   $= \dfrac{\cos A + \sin A}{\cos A - \sin A}$

   ii) $\tan\left(\dfrac{\pi}{4} - A\right) = \dfrac{1 - \tan A}{1 + \tan A}$

   $= \dfrac{\cos A - \sin A}{\cos A + \sin A}$

8. i) $\sin(A + B + C) = \sum(\sin A \cos B \cos C) - \sin A \sin B$

   ii) $\cos(A + B + C) = \cos A \cos B \cos C - \sum(\sin A \sin B \cos C)$

   iii) $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

9. i) $\sin 15° + \cos 15° = \sqrt{3/2} = \sin 75° + \cos 75°$

   ii) $\cos 15° - \sin 15° = 1/\sqrt{2} = \sin 75° - \cos 75°$

10. i) $\tan 15° + \cot 15° = 4 = \tan 75° + \cot 75°$

    ii) $\cot 15° - \tan 15° = 2\sqrt{3} = \tan 75° - \cot 75°$

11. i) $\sin 105° = \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

    ii) $\cos 105° = -\sin 15° = \frac{1-\sqrt{3}}{2\sqrt{2}}$

12. i) $\tan 105° = -\cot 15° = -2-\sqrt{3}$

    ii) $\cot 105° = -\tan 15° = \sqrt{3}-2$

13. i) If $\tan\alpha = \frac{m}{m+1}$, $\tan\beta = \frac{1}{2m+1}$, then

    $\tan(\alpha+\beta) = 1$

    ii) If $\tan\alpha = \frac{m+1}{m}$, $\tan\beta = \frac{1}{2m+1}$, then $\tan(\alpha-\beta) = 1$.

14. i) $\cos\theta - \cos(60°+\theta) - \cos(60°-\theta) = 0$

    ii) $\cos\theta + \cos(120°+\theta) + \cos(120°-\theta) = 0$

    iii) $\cos\theta + \cos(240°+\theta) + \cos(240°-\theta) = 0$

    iv) $\sin\theta - \sin(60°+\theta) + \sin(60°-\theta) = 0$

    v) $\sin\theta + \sin(120°+\theta) - \sin(120°-\theta) = 0$

    vi) $\sin\theta + \sin(240°+\theta) - \sin(240°-\theta) = 0$

15. i) If $A + B = 45°$, then $(1+\tan A)(1+\tan B) = 2$

    ii) If $A + B = 135°$, then $(1-\tan A)(1-\tan B) = 2$

    iii) If $A + B = 225°$, then $(1+\tan A)(1+\tan B) = 2$

    iv) If $A + B = 45°$, then $(1-\cot A)(1-\cot B) = 2$

    v) If $A + B = 135°$, then $(1+\cot A)(1+\cot B) = 2$

    vi) If $A + B = 225°$, then $\frac{(1+\cot A)(1+\cot B)}{\cot A \cot B} = 2$

16. i) $\tan(45°+\theta)\tan(45°-\theta) = 1$

    ii) $\cot(45°+\theta)\cot(45°-\theta) = 1$