5. VECTOR PRODUCT OF TWO VECTORS

Synopsis:

1. The cross product of two vectors $\bar{a}$ and $\bar{b}$ is a vector and is defined as $|\bar{a}|.|\bar{b}|\sin\theta.\hat{n}$, where $\hat{n}$ is the unit vector perpendicular to the plane containing $\bar{a}$ and $\bar{b}$ such that $\bar{a}, \bar{b}, \hat{n}$ form a right handed system.
2. Let $\vec{F}$ be a force directed along a line. Let O be a point (origin). Let $\overrightarrow{OP} = \bar{r}$ be the position vector of any point P on the line of action of $\vec{F}$. Then $\bar{r} \times \vec{F}$ gives the moment of the force $\vec{F}$ about the point O.
3. $\bar{a} \times \bar{b} = \bar{O} \Leftrightarrow$ either $\bar{a} = \bar{O}$ (or) $\bar{b} = \bar{O}$ (or) $\bar{a}$ is parallel to $\bar{b}$.
4. If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$, $\bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$, then $\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
5. If $\bar{a} \times \bar{b} \neq \bar{b} \times \bar{a}$ but $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$ (i.e., cross product is anti commutative)
6. If $\bar{i}, \bar{j}, \bar{k}$ is a orthonormal triad of unit vectors forming a right handed system then $\bar{i} \times \bar{j} = -\bar{j} \times \bar{i}$; $\bar{j} \times \bar{k} = -\bar{k} \times \bar{j} = \bar{i}$; $\bar{k} \times \bar{i} = -\bar{i} \times \bar{k} = \bar{j}$ and $\bar{i} \times \bar{i} = \bar{j} \times \bar{j} = \bar{k} \times \bar{k} = 0$.
7. Unit vector perpendicular to the both the vectors $\bar{a}$ and $\bar{b}$ is $\pm \dfrac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}$.
8. If $\bar{a}$ and $\bar{b}$ are the adjacent sides of a parallelogram then vector area of the parallelogram is $\bar{a} \times \bar{b}$ and area = $|\bar{a} \times \bar{b}|$ sq.units.
9. If $\bar{a}$ and $\bar{b}$ are two adjacent sides of a triangle then area of the triangle is $\frac{1}{2}|\bar{a} \times \bar{b}|$ sq.units.
10. If $\bar{a}, \bar{b}, \bar{c}$ are the vertices of a triangle then the area of the triangle is $\frac{1}{2}|\bar{b} \times \bar{c} + \bar{c} \times \bar{a} + \bar{a} \times \bar{b}|$ sq.units.
11. The points $\bar{a}, \bar{b}, \bar{c}$ are collinear if $\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a} = \bar{0}$.
12. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C then the perpendicular distance from C to the straight line AB is $\dfrac{|\bar{b} \times \bar{c} + \bar{c} \times \bar{a} + \bar{a} \times \bar{b}|}{|\bar{b} - \bar{a}|}$ or $\dfrac{|\overrightarrow{AC} \times \overrightarrow{AB}|}{|\overrightarrow{AB}|}$.
13. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction ratios of a straight line then $\cos\theta = \dfrac{(l_1l_2 + m_1m_2 + n_1n_2)}{\sqrt{\sum l_1^2}\sqrt{\sum l_2^2}}$.
14. For every vector
    i) $\bar{a} \times \bar{a} = \bar{0}$
    ii) $|\bar{a} \times \bar{b}| = |\bar{b} \times \bar{a}|$
    iii) $\bar{a} \times (\bar{b} + \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$ cross product is left distributive over vector addition
    iv) $(\bar{a} + \bar{b}) \times \bar{c} = \bar{a} \times \bar{c} + \bar{b} \times \bar{c}$ cross product is right distributive over vector addition
15. For any two vectors $\bar{a}$ and $\bar{b}$, $(\bar{a} \times \bar{b})^2 + (\bar{a}.\bar{b})^2 = |\bar{a}|^2|\bar{b}|^2$