4. SCALAR (DOT) PRODUCT OF TWO VECTORS

1. The scalar product or (dot product) : The scalar product of two vectors $\bar{a}$ and $\bar{b}$ is a scalar and is defined as the product of the magnitude of $\bar{a}$ and $\bar{b}$ and the cosine of the angle θ between them. i.e., $\bar{a}.\bar{b} = |\bar{a}|.|\bar{b}|.\cos\theta,\ 0 \le \theta < \pi$.
2. Two non zero vectors $\bar{a}$ and $\bar{b}$ are perpendicular if and only if their dot product is zero.
3. Projection of $\bar{a}$ and $\bar{b}$ is $\frac{|\bar{a}.\bar{b}|}{|\bar{b}|}$ the orthogonal projection of $\bar{a}$ and $\bar{b}$ is $\frac{(\bar{a}.\bar{b})\bar{b}}{|\bar{b}|^2}$ or $(\bar{a}.\hat{b})\hat{b}$.

   The component vector of $\bar{a}$ along $\bar{b}$ is $\frac{(\bar{a}.\bar{b})\bar{b}}{|\bar{b}|^2}$ or $(\bar{a}.\hat{b})\hat{b}$.

   The component vector of $\bar{a}$ perpendicular to $\bar{b}$ is $\bar{a} - \frac{(\bar{a}.\bar{b})\bar{b}}{|\bar{b}|^2}$.

   Note : The component vector of $\bar{a}$ along $\bar{b}$ is equal to the orthogonal projection of $\bar{a}$ on $\bar{b}$.
4. If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$, $\bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$, then

   i) $\bar{a}.\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$

   ii) If θ is the angle between $\bar{a}$ and $\bar{b}$, then $\cos\theta = \frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{\sum a_1^2}.\sqrt{\sum b_1^2}}$

   iii) $\sin\theta = \sqrt{\frac{\sum (a_2b_3 - a_3b_2)^2}{\left(\sum a_1^2\right)\left(\sum b_1^2\right)}}$

   iv) $\bar{a}.\bar{b} = \bar{b}.\bar{a}$ (dot product is commutative)

   v) $\bar{a}.(-\bar{b}) = (-\bar{a}).\bar{b} = -(\bar{a}.\bar{b})$

   vi) $\lambda\bar{a}.\mu\bar{b} = \lambda\mu(\bar{a}.\bar{b}) = \mu\bar{a}.\lambda\bar{b}$

   vii) $\bar{a}.\bar{a} = |\bar{a}|^2 = \bar{a}^2$

   viii) $(\bar{a} \pm \bar{b})^2 = |\bar{a}|^2 + |\bar{b}|^2 \pm 2\bar{a}.\bar{b}$

   ix) $(\bar{a} + \bar{b})^2 = (\bar{a} - \bar{b})^2 = 2(\bar{a}^2 + \bar{b}^2)$ and
   $(\bar{a} + \bar{b})^2 - (\bar{a} - \bar{b})^2 = 4\bar{a}.\bar{b}$

   x) $(\bar{a} + \bar{b} + \bar{c})^2 =$
   $\bar{a}^2 + \bar{b}^2 + \bar{c}^2 + 2(\bar{a}.\bar{b}) + 2(\bar{b}.\bar{c}) + 2(\bar{c}.\bar{a})$

   xi) $\bar{a}^2 - \bar{b}^2 = (\bar{a} + \bar{b}).(\bar{a} - \bar{b})$

   xii) $\bar{a}.(\bar{b} + \bar{c}) = \bar{a}.\bar{b} + \bar{a}.\bar{c}$, dot product is left distributive over vector addition.

   xiii) $(\bar{a} + \bar{b}).\bar{c} = \bar{a}.\bar{c} + \bar{b}.\bar{c}$, dot product is right distributive over vector addition.
5. For the unit orthognormal vectors $\bar{i}, \bar{j}, \bar{k}$

   i) $\bar{i}.\bar{i} = \bar{j}.\bar{j} = \bar{k}.\bar{k} = 1$

   ii) $\bar{i}.\bar{j} = \bar{j}.\bar{k} = \bar{k}.\bar{i} = 0$
6. For any vector $\bar{r}, \bar{r} = (\bar{r}.\bar{i})\bar{i} + (\bar{r}.\bar{j})\bar{j} + (\bar{r}.\bar{k})\bar{k}$
7. Work done by a force F in displacing a particle from A to B is given by $W = \vec{F}.\overrightarrow{AB}$
8. If the length of the perpendicular from the origin be p (> 0) and $\hat{n}$ be the unit vector perpendicular to the plane then the equation of the plane is $\bar{r}.\hat{n} = p$.

9. The vector equation of a plane through a point $\bar{a}$ and perpendicular to a unit vector $\hat{n}$ is $(\bar{r} - \bar{a}).\hat{n} = 0$.

10. If π_1 and π_2 be two planes whose equations are $\bar{r}_1.\bar{m}_1 = q_1$, $\bar{r}_2.\bar{m}_2 = q_2$, then the angle between the planes is $\text{Cos}^{-1}\left(\frac{m_1.m_2}{|m_1|.|m_2|}\right)$.

11. Perpendicular distance from the origin to the plane $(\bar{r} - \bar{a}).\hat{n} = 0$ is $\bar{a}.\hat{n}$, where $\bar{a}$ is the position vector of a point in the plane and $\hat{n}$ is the unit vector perpendicular to the plane.