2. MATHEMATICAL INDUCTION

Synopsis:

1. **Induction theorem :** If A is a subset of N such that
   i) $1 \in A$ and ii) $k \in A \Rightarrow k + 1 \in A$, then A = N.
2. **Principle of finite Mathematical Induction :** Let S(n) be a statement for each $n \in N$. If i) S(1) is true ii) S(k) is true $\Rightarrow$ S(k + 1) is true, then S(n) is true for all $n \in N$.
3. **Principle of complete Mathematical Induction :** Let S(n) be a statement for each $n \in N$. If i) S(1) is true ii) S(1), S(2) … S(k) are true $\Rightarrow$ S(k + 1) is true then S(n) is true for all $n \in N$.
4. Division algorithm : If $0 \neq a, b \in Z$ then $\exists$ q, $r \in Z$ uniquely $\in$ b = aq + r where $0 \leq r < |a|$.
5. The sum of first n natural numbers = $\sum n = \frac{n(n+1)}{2}$.
6. The sum of squares of first n natural numbers = $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$.
7. The sum of cubes of first n natural numbers = $\sum n^3 = \frac{n^2(n+1)^2}{4} = (\sum n)^2$.
8. i) The sum of first n even positive integers = $\sum 2n = n(n + 1)$.
   ii) The sum of first n odd positive integers = $\sum (2n-1) = n^2$
   iii) $\sum n(n+1) = \frac{n(n+1)(n+2)}{3}$
   iv) $\sum n(n+1)(n+2) = \frac{(n+1)(n+2)(n+3)}{4}$
9. $a + (a + d) + (a + 2d) + \ldots\ldots\ldots\ldots + (a + (n-1)d) = \frac{n}{2}[2a + (n-1)d]$
10. $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$
11. For all + ve integral values of n, $x^n - y^n$ is divisible by x – y.
12. n is any odd integer then $n(n^2 - 1)$ is divisible by 24.
13. The sum of the cubes of three successive natural numbers is divisible by 9.
14. The product of r consecutive integers are divisible by r!