1. FUNCTIONS

Synopsis :

1. A relation f from a set A into a set B is said to be a *function* or *mapping* from A into B if for each $x \in A$ there exists a unique $y \in B$ such that $(x, y) \in f$. It is denoted by $f : A \to B$.
2. If $f : A \to B$ is a function, then A is called *domain*, B is called *codomain* and $f(A) = \{f(x) : x \in A\}$ is called range of f.
3. If A, B are two finite sets, then the number of functions that can be defined from A in to B is $n(B)^{n(A)}$.
4. A function $f : A \to B$ is said to be one one *function* or *injection* from A into B if different elements in A have different f – images in B.
5. If $f : A \to B$ is one one and A, B are finite then $n(A) \leq n(B)$.
6. If A, B are two finite sets, then the number of one one functions that can be defined from A into B is ${}^{n(B)}P_{n(A)}$.
7. A function f $A \to B$ is said to be onto function or subjection from A onto B if $f(A) = B$. i.e., range = codomain.
8. A function $f : A \to B$ is onto if $y \in B \Rightarrow \exists\ x \in A \ni f(x) = y$.
9. If A, B are two finite sets and $f : A \to B$ is onto then $n(B) \leq n(A)$
10. If A, B are two finite sets and $n(B) = 2$, then the number of onto functions that can be defined from A onto B is $2^{n(A)} - 2$.
11. A function $f : A \to B$ is said to be *one one onto function* or *bijection* from A onto B if $f : A \to B$ is both one one function and onto function.
12. If A, B are two finite sets and $f : A \to B$ is a bijection, then $n(A) = n(B)$.
13. If A, B are two finite sets and $n(A) = n(B)$, then the number of bijections that can be defined from A onto B is $n(A)!$.
14. If $f : A \to B$, $g : B \to C$ are two functions then the function go $f : A \to C$ defined $(gof)(x) = g[f(x)]$, $\forall\ x \in A$ is called composite function of f and g.
15. If $f : A \to B$, $g : B \to C$ are two one one functions then go $f : A \to C$ is also one one.
16. If $f : A \to B$, $g : B \to C$ are two onto functions then go $f : A \to C$ is also onto.
17. If $f : A \to B$, $g : B \to C$ are two one one onto functions then go$f : A \to C$ is also one one onto.
18. If A is a set, then the function I on A defined by $I(x) = x$, $\forall\ x \in A$, is called *Identity function* on A. It is denoted by I_A.
19. If $f : A \to B$ and I_A, I_B are identity functions on A, B respectively then $foI_A = I_Bof = f$.
20. If $f : A \to B$ is bijection, then the inverse relation f^{-1} from B into A is also a bijection.
21. If $f : A \to B$ is a bijection, then the function

    $f^{-1} : B \to A$ defined by $f^{-1}(y) = x$ if $f(x) = y$, $\forall\ y \in B$ is called inverse function of f.
22. If $f : A \to B$, $g : B \to C$ are two bisections then $(gof)^{-1} = f^{-1}og^{-1}$

23. If $f : A \to B$, $g : B \to A$ are two functions such that go $f = I_A$ *and* $fog = I_B$ then $f : A \to B$ is a bijection and $f^{-1} = g$.
24. A function $f : A \to B$ is said to be a constant function if the range of f contains only one element i.e., $f(x) = k$, $\forall\ x \in A$ where k is a fixed element of B.
25. A function $f : A \to B$ is said to be a real variable function if $A \subseteq R$.
26. A function $f : A \to B$ is said to be a real valued function if $B \subseteq R$.
27. A function $f : A \to B$ is said to be a real function if $A \subseteq R$, $B \subseteq R$.
28. A function $f : A \to R$ is said to be an even function if $f(-x) = f(x)$, $\forall\ x \in A$.
29. A function $f : A \to R$ is said to be an odd function if $f(-x) = -f(x)$, $\forall\ x \in A$.
30. If $a \in R$, $a > 0$ then the function f; $R \to R$ defined as $f(x) = a^x$ is called an exponential function.
31. The function $f : R \to R$ defined as $f(x) = n$ where $n \in Z$ such that $n \le x < n + 1$, $\forall\ x \in R$ is called step function. It is denoted by $f(x) = [x]$.
32. The domains and ranges of some standard functions are given below

SNO	Functions	Domain	Range
1.	a^x	R	$(0, \infty)$
2.	$\log_a x$	$(0, \infty)$	R
3.	$[x]$	R	Z
4.	$\lvert x \rvert$	R	$[0, \infty)$
5.	$\sqrt{x}$	$[0, \infty)$	$[0, \infty)$
6.	$\sin x$	R	$[-1, 1]$
7.	$\cos x$	R	$[-1, 1]$
8.	$\tan x$	$R-\{(2n+1)\frac{\pi}{2} : n \in Z\}$	R
9.	$\cot x$	$R - [n\pi : n \in Z\}$	R
10.	$\sec x$	$R - \{(2n + 1)\frac{\pi}{2} : n \in Z\}$	$(-\infty,-1]\cup[1, \infty)$
11.	$\text{Sin}^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
12.	$\text{Cos}^{-1} x$	$[-1, 1]$	$[0, \pi]$
13.	$\text{Tan}^{-1} x$	R	$(-\pi/2, \pi/2)$
14.	$\text{Cot}^{-1} x$	R	$(0, \pi)$

SNO	Functions	Domain	Range
15.	Sec^{-1} x	$(-\infty,-1]\cup[1,\infty)$	$[0, \pi/2)\cup(\pi/2, \pi]$
16.	$Cose^{-1}$ x	$(-\infty,-1]\cup[1,\infty)$	$[-\pi/2, 0)\cup(0, \pi/2]$
17.	Sinh x x	R	R
18.	Cosh x	R	$[1,\infty)$
19.	tanh x	R	$(-1, 1)$
20.	coth x	$(-\infty, 0)\cup(0,\infty)$	$(-\infty,-1)\cup(1,\infty)$
21.	sech x	R	$(0, 1]$
22.	cosech x	$(-\infty, 0)\cup(0,\infty)$	$(-\infty, 0)\cup(0,\infty)$
23.	$Sinh^{-1}$ x	R	R
24.	$Cosh^{-1}$ x	$[1,\infty)$	$[0,\infty)$
25.	$Tanh^{-1}$ x	$(-1, 1)$	R
26.	$Coth^{-1}$x	$(-\infty, -1)\cup(1,\infty)$	$(-\infty, 0)\cup(0,\infty)$
27.	$Sech^{-1}$ x	$(0, 1]$	$[0,\infty)$
28	$Cosech^{-1}$x	$(-\infty, 0)\cup(0,\infty)$	$(-\infty, 0)\cup(0,\infty)$

33. Signum Function : The signum function is defined as $\text{sgn } f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$.

34. f(x) is a polynomial function such that f(x), $f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$. Then $f(x) = x^n + 1$ or $-x^n + 1$.

35. f(x) is a function such that f(x + y) = f(x), f(y). Then $f(x) = a^x$.

36. f(x) is a function such that f(xy) = f(x) + f(y). Then $f(x) = \log_a x$.

37. If f(x + y) = f(x) + f(y) $\forall$ x, y, then f(n) = nf(1).

38. If $y = f(x) = \frac{ax+b}{cx-a}$, then f(y) = x or (fof)(x) = x.

39. If $f(x) = \frac{ax+b}{cx+d}$, then $f^{-1}(x) = -\frac{b-dx}{a-cx}$.

40. The range of function $f(x) = a\sin\theta + b\cos\theta + c$ is $\left[c-\sqrt{a^2+b^2}, c+\sqrt{a^2+b^2}\right]$.

41. The range of the function $f(x) = \sqrt{a^2-x^2}$ is [0, a].

42. If $f(x).f(y)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right] = 0$, then $f(x) = \cos(\log x)$.