CENTRE OF MASS

Synopsis :

1. Every particle is attracted towards the centre of the earth by the force of gravity and the **centre of gravity** of a body is the point where the resultant force of attraction of the weight of the body acts.
2. **Centre of mass** : If a system of parallel forces proportional to the masses of the various particles of a body are assumed to act on it, their resultant passes through a fixed point, irrespective of the direction of the parallel forces and that point is called centre of mass.
3. In a uniform gravitational field, the centre of mass of the body and the centre of gravity coincide with each other.
4. For small bodies (compared to the size of the earth) the centre of gravity and the centre of mass coincide.
5. If two particles of masses m_1 and m_2 are at distances x_1 and x_2 from the origin and x_{cm} is the position of the centre of mass of the system, then

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

6. For a system of two bodies of masses m_1 and m_2 separated by a distance d.
   a) Centre of mass lies on the line joining the centers of mass of two bodies.
   b) Distances of center of mass from first body $r_1 = \frac{m_2r}{m_1 + m_2}$.
   c) Distances of center of mass from second body $r_2 = \frac{m_1r}{m_1 + m_2}$.
   d) Centre of mass lies nearer to heavy mass.
7. If the position coordinates of particles of masses m_1, m_1, m_1... are (x_1, y_1), (x_2, y_2), (x_3,y_3)...The mass of the system is M. The position coordinates of the centre of mass are (x,y).

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3...}{m_1 + m_2 + m_3 + ...} = \frac{m_1x_1 + m_2x_2 + ...}{M}$$

$$y = \frac{m_1y_1 + m_2y_2 + m_3y_3...}{m_1 + m_2 + m_3 + ...} = \frac{m_1y_1 + m_2y_2 + ...}{M}$$

8. The distance of centre of mass from the origin is $\sqrt{x^2 + y^2}$.
9. If the two masses are equal, the centre of mass lies midway between the particles.
10. When a large number of particles are distributed in space, the centre of mass of the system is given by the coordinates.

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} ; y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} ; z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i}$$

11. a) The position of center of mass depends on the shape of the body and distribution of mass

    b) In symmetrical bodies in which distribution of mass is homogeneous. Centre of mass coincides with geometrical center.

    c) Centre of mass of regular bodies.

    1) Uniform rod : middle point of the rod

    2) Cubical box: Point of intersection of diagonals

    3) Circular ring : centre of the ring

    4) circular disc : centre of the disc

    5) Sphere : centre of the sphere

12. In vector notation, each particle of the system can be described by a position vector $\vec{r}$ and the centre of mass can be located by the position vector $\vec{r}_{cm}$.

    $\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$ where $\vec{r}_i = \hat{i}x_i + \hat{j}y_i + \hat{k}z_i$

    $\vec{r}_{cm} = \hat{i}x_{cm} + \hat{j}y_{cm} + \hat{k}z_{cm}$

13. Velocity of the centre of mass $(\vec{v}_{cm}) = \frac{\sum m\vec{V}}{\sum m} = \frac{\text{Total momentum}}{\text{Total mass}}$

14. If two particles masses m_1 and m_2 are moving with velocities $\bar{v}_1$ and $\bar{v}_2$ at right angles to each other, then the velocity of their centre of mass is given by

    $V_{cm} = \sqrt{\frac{m_1^2 v_1^2 + m_2^2 v_2^2}{(m_1 + m_2)}}$

15. The velocity of centre of mass of an isolated system remains constant as long as no external force acts on the system. Its acceleration is zero.

16. The momentum of centre of mass of the system is equal to the sum of individual momenta of n particles of the system.

17. If an external force is acting on the system of particles, centre of mass behaves as if total force is acting only at that point.

18. The centre of mass of the system is static under the action of internal forces.

19. When a body rotates, vibrates or moves linearly, the centre of mass moves in the same way as a single particle subjected to the same force.

20. Acceleration of the centre of mass = $\frac{\sum \vec{F}_{ext}}{\sum m}$.

21. **Characteristics of centre of mass** :

    i. The position of centre of mass depends upon the shape of the body and the distribution of mass.

    ii. If the origin is at the centre of mass, then sum of moments of mass ($\Sigma m_i x_i$) of the system about the centre of mass is zero.

    iii. Centre of mass may be within the body or on or outside the material of the body.

    iv. It is not necessary that mass should be present at centre of mass. In the case of uniform ring, centre of mass is outside the material where no mass is present.

    v. The location of the centre of mass is independent of the reference frame used to locate it.

    vi. The centre of mass of a system of particles depends only on the masses of the particles and their relative positions.

22. **Laws of motion of the centre of mass** :

    i. The centre of mass of a system of particles moves as though total mass of the system is concentrated at a point and external forces were applied at that point.

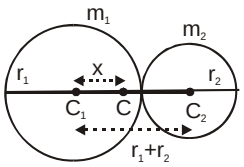
    ii. The motion of the centre of mass of the body is called the translational motion of the body.

    iii. The internal forces will not effect the motion of the centre of mass.

    iv. If no external force acts on a system, the acceleration of centre of mass is zero, the velocity and momentum of the centre of mass remains constant, though velocity and momentum of individual particles vary.

    v. The motion of the centre of mass can be studied using Newton's laws of motion.

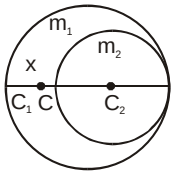
23. When two solid spheres of same materials but with radii r_1 and r_2 are kept in contact, the center of mass of the system is at a distance of x from the center of bigger sphere, where $x = \frac{r_2^3(r_1 + r_2)}{r_1^3 + r_2^3}$.



24. When a portion of m_2 is removed from a body of mass m_1 then shift in the position of center of mass =

$$\frac{\left[\begin{array}{c}\text{distance between cm of the} \\ \text{body and removed part}\end{array}\right] \times \text{mass of removed part}}{\text{mass of remaining part}}$$

25. When two circular discs of same material and thickness and radii r_1 and r_2 are kept in contact, then the center of mass of the system is at a distance of x from the center of first (large) disc of mass m_1 where $x = \frac{r_2^2(r_1 + r_2)}{r_1^2 + r_2^2}$.



26. A circular portion of radius r_2 is removed from a circular disc of radius r_1 from one edge. Then the shift in the center of mass of the disc is $x = \frac{r_2^2}{r_1 + r_2}$.

27. From a circular disc of radius r, a circle of diameter r is removed. The shift in the center of mass of the remainder is x=r/6.

28. When as sphere of radius r_2 is removed from a solid sphere of radius r_1 from its edge, then the shift in its centre of mass is $x = \frac{r_2^3(r_1 - r_2)}{(r_1^3 - r_2^3)}$.

29. When a person walks on a boat in still water, centre of mass of person, boat system is not displaced.

    a) If the man walks a distance L on the boat, the boat is displaced in the opposite direction relative to shore or water by a distance $x = \frac{mL}{M+m}$.

    b) Distance walked by the mass relative to shore or water is (L – x).

30. Two masses starting from rest move under mutual force of attraction towards each other, they meet at their centre of mass.

    a) In the above case V_{cm} and $a_{cm} = 0$

    b) If the two particles are m_1 and m_2 and their velocities are v_1 and v_2, then $m_1v_1 = -m_1v_2$

c) If the two particles have accelerations a_1 and a_2.

$m_1a_1 = – m_2a_2$

d) If s_1 and s_2 are the distances traveled before they meet

$m_1s_1 = m_2s_2$