Gate for Mechanical Engineering:

Introduction to Mechanisms:

1 Physical Principles

This chapter introduces the basic physical principles behind mechanisms as well as basic concepts and principles required for this course.

1.1 Force and Torque

1.1.1 Force

Force: an agent or influence that, if applied to a free body results chiefly in an acceleration of the body and sometimes in elastic deformation and other effects.

Every day we deal with ***forces*** of one kind or another. A pressure is a force. The earth exerts a force of attraction for all bodies or objects on its surface. To study the forces acting on objects, we must know how the forces are applied, the direction of the forces and their value. Graphically, forces are often represented by a vector whose end represents the point of action.

A *mechanism* is what is responsible for any action or reaction. *Machines* are based on the idea of transmitting forces through a series of predetermined motions. These related concepts are the basis of dynamic movement.

1.1.2 Torque

Torque: *Something that produces or tends to produce rotation and whose effectiveness is measured by the product of the force and the perpendicular distance from the line of action of the force to the axis of rotation.*

Consider the lever shown in Figure 1-1. The lever is a bar that is free to turn about the fixed point, A, called the *fulcrum*; a weight acts on the one side of the lever, and a balancing force acts on the other side of the lever.

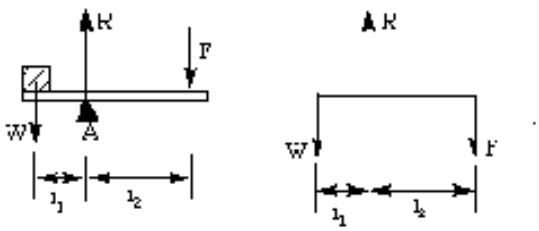


Figure 1-1 A lever with balanced forces

To analyze levers, we need to find the ***torques*** of the forces acting on the lever. To get the *torque* of force W about point A, multiply W by l_1, its distance from A. Similarly F x l_2 is the *torque* of F about fulcrum A.

1.2 Motion

Motion*: a change of position or orientation.*

1.2.1 Motion Along a Straight Path

We begin our study of motion with the simplest case, motion in a straight line.

1. Position and displacement along a line
   The first step in the study of motion is to describe the position of a moving object. Consider a car on an east-west stretch of straight highway. We can describe the *displacement* of the car by saying "the car is 5 kilometers west of the center town". In this description, we specified two factors, the original point of measure and the direction of the *displacement*.
2. Velocity
   We can define the *velocity* of an object moving steadily as its displacement per unit time:

$$v = \frac{x_2 - x_1}{t_2 - t_1} = \frac{d}{t}$$

(1-1)

where t = t2 - t1 is the time interval during which the displacement occurred. When velocity varies, we can let the time interval become infinitesimally small, thus

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

(1-2)

3. Acceleration
   Acceleration is the variation of the velocity in a unit time period. If the velocity changes in a constant rate, then we can describe the acceleration by

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

(1-3)

More generally, acceleration is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

(1-4)

1.2.2 Linear Motion in Space

The picture becomes more complicated when the motion is not merely along a straight line, but rather extends into a plane. Here we can describe the motion with a vector which includes the magnitude and the direction of movement.

1. *Position vector* and *displacement vector*
   The directed segment which describes the position of an object relative to an origin is the *position vector*, as d_1 and d_2 in Figure 1-2

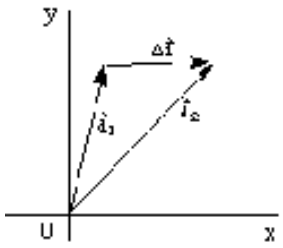


Figure 1-2 Position vector and displacement vector

If we wish to describe a motion from position $\mathbf{d}_1$ to position $\mathbf{d}_2$, for example, we can use vector $\mathbf{d}_1$, the vector starts at the point described by $\mathbf{d}_1$ and goes to the point described by $\mathbf{d}_2$, which is called the *displacement vector*.

$$\Delta \vec{d} = \vec{d}_2 - \vec{d}_1$$

(1-5)

2. *Velocity vector*
   For a displacement Δd occurring in a time interval Δt, the average velocity during the interval is

$$\vec{v}_{ave} = \frac{\Delta \vec{d}}{\Delta t}$$

(1-6)

Clearly $\mathbf{V}_{ave}$ has the direction of $\Delta \mathbf{d}$.

In the limit as delta t approaches zero, the instantaneous velocity is

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{d}}{\Delta t}$$

(1-7)

The direction of $\mathbf{V}$ is the direction of $\Delta \mathbf{d}$ for a very small displacement; it is therefore along, or tangent to, the path.

3. *Acceleration vector*
   The instantaneous acceleration is the limit of the ratio $\Delta V/\Delta t$ as Δt becomes very small:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

(1-8)

1.2.3 Motion of a Rigid Body in a Plane

The previous sections discuss the motion of particles. For a rigid body in a plane, its motion is often more complex than a particle because it is comprised of a linear motion and a rotary motion. Generally, this kind of motion can be decomposed into two motions (Figure 1-3), they are:

1. The linear motion of the center of the mass of the rigid body. In this part of the motion, the motion is the same as the motion of a particle on a plane.
2. The rotary motion of the rigid body relative to its *center of mass*.

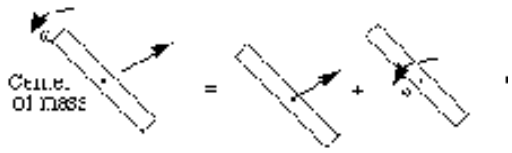


Figure 1-3 Motion of a rigid body in a plane

1.3 Newton's Law of Motion

1.3.1 Newton's First Law

When no force is exerted on a body, it stays at rest or moves in a straight line with constant speed. This principle of inertia is also known as ***Newton's first law.*** It is from this law that Newton was able to build up our present understanding of dynamics.

1.3.2 Newton's Second Law

From our daily life, we can can observe that:

1. When a force F is applied on an object, ΔV, the change of the velocity of the object, increases with the length of time delta t increases;
2. The greater the force F, the greater ΔV; and
3. The larger the body (object) is, the less easily accelerated by forces.

It is convenient to write the proportionality between $\mathbf{F}\Delta t$ and $\Delta \mathbf{V}$ in the form:

$$F\Delta t = m\Delta V$$

(1-9)

The proportionality constant *m* varies with the object. This constant *m* is refered to as the ***inertial mass*** of the body. The relationship above embodies ***Newton's law of motion*** (***Newton's second law***). As

$$\frac{\Delta V}{\Delta t} = a$$

(1-10)

in which *a* is the acceleration of the object. We have

$$F = ma$$

(1-11)

If m = 1 kg and a = 1m/sec^2, than F = 1 *newton*.

Forces and accelerations are vectors, and Newton's law can be written in vector form.

$$\vec{F} = m\vec{a}$$

(1-12)

1.4 Momentum and Conservation of Momentum

1.4.1 Impulse

Try to make a baseball and a cannon ball roll at the same speed. As you can guess, it is harder to get the cannon ball going. If you apply a constant force **F** for a time Δt, the change in velocity is given by Equation 1-9. So, to get the same $\Delta \mathbf{v}$, the product $\mathbf{F}\,\Delta t$ must be greater the greater the mass m you are trying to accelerate.

To throw a cannon ball from rest and give it the same final velocity as a baseball (also starting from rest), we must push either harder or longer. What counts is the product $\mathbf{F}\,\Delta t$. This product $\mathbf{F}\,\Delta t$ is the natural measure of how hard and how long we push to change a motion. It is called the ***impulse*** of the force.

1.4.2 Momentum

Suppose we apply the same *impulse* to a baseball and a cannon ball, both initially at rest. Since the initial value of the quantity $m\mathbf{v}$ is zero in each case, and since equal impulses are applied, the final values $m\mathbf{v}$ will be equal for the baseball and the cannon ball. Yet, because the mass of the cannon ball is much greater than the mass of the baseball, the velocity of the cannon ball will be much less than the velocity of the baseball. The product $m\mathbf{v}$, then, is quite a different measure of the motion than simply **v** alone. We call it the ***momentum*** **p** of the body, and measure it in kilogram-meters per second.

$$\vec{p} = m\vec{v}$$

(1-13)

Velocity and *momentum* are quite different concepts: velocity is a *kinematical* quantity, whereas momentum is a *dynamic* one, connected with the causes of changes in the motion of masses.

Because of its connection with the *impulse* which occurs naturally in Newton's law (Equation 1-9), we expect *momentum* to fit naturally into Newtonian dynamics. Newton did express his law of motion in terms of the *momentum*, which he called the *quantity of motion*. We can express Newton's law in terms of the change in *momentum* instead of change in *velocity*:

$$\vec{F}\Delta t = m\Delta\vec{v} = m(\vec{v}' - \vec{v})$$

(1-14)

where **v** and **v'** are the velocities before and after the impulse. The right-hand side of the last equation can be written as

$$m(\vec{v}' - \vec{v}) = m\vec{v}' - m\vec{v} = \vec{p}' - \vec{p} = \Delta\vec{p}$$

(1-15)

the change in the *momentum*. Therefore

$$\vec{F}\Delta t = \Delta\vec{p}$$

(1-16)

or, in other words, the *impulse* equals the change in *the momentum*.

1.4.3 Conservation of Momentum

In Figure 1-4 a moving billiard ball collides with a billiard ball at rest. The incident ball stops and the ball it hits goes off with the same velocity with which the incident ball came in. The two billiard balls have the same mass. Therefore, the *momentum* of the second ball after the collision is the same as that of the incident ball before collision. The incident ball has lost all its *momentum*, and the ball it struck has gained exactly the *momentum* which the incident ball lost.



Figure 1-4 Collision of billiard balls

This phenomenon is consistent with *the law of conservation of momentum* which says that the total *momentum* is constant when two bodies interact.

1.5 Work, Power and Energy

1.5.1 Work

Work is a force applied over a distance. If you drag an object along the floor you do *work* in overcoming the friction between the object and the floor. In lifting an object you do *wor*k against gravity which tends to pull the object toward the earth. Steam in a locomotive cylinder does *work* when it expands and moves the piston against the resisting forces. ***Work*** is the product of the resistance overcome and the distance through which it is overcome.

1.5.2 Power

Power *is the rate at which work is done.*

In the British system, power is expressed in foot-pounds per second. For larger measurements, the horsepower is used.

$$1\text{horsepower} = 550\text{ft} * \text{lb/s} = 33{,}000\text{ft} * \text{lb/min}$$

In SI units, power is measured in joules per second, also called the **watt** (W).

$$1\text{hp} = 746\text{ W} = 0.746\text{kW}$$

1.5.3 Energy

All object possess energy. This can come from having work done on it at some point in time. Generally, there are two kinds of energy in mechanical systems, **potential** and **kinetic**. Potential energy is due to the position of the object and kinetic energy is due to its movement.

For example, an object set in motion can overcome a certain amount of resistance before being brought to rest, and the energy which the object has on account of its motion is used up in overcoming the resistance, bring the object to rest. *Fly wheels* on engines both receive and give up *energy* and thus cause the *energy* to return more smoothly throughout the *stroke*.

Elevated weights have power to do *work* on account of their elevated position, as in various types of hammers, *etc*.

Introduction to Mechanisms and Simple Machines:

2 Mechanisms and Simple Machines

Mechanism*: the fundamental physical or chemical processes involved in or responsible for an action, reaction or other natural phenomenon.*

Machine*: an assemblage of parts that transmit forces, motion and energy in a predetermined manner.*

Simple Machine*: any of various elementary mechanisms having the elements of which all machines are composed. Included in this category are the lever, wheel and axle, pulley, inclined plane, wedge and the screw.*

The word ***mechanism*** has many meanings. In *kinematics*, a mechanism is a means of transmitting, controlling, or constraining relative movement (Hunt 78). Movements which are electrically, magnetically, pneumatically operated are excluded from the concept of mechanism. The central theme for mechanisms is rigid bodies connected together by joints.

A ***machine*** is a combination of rigid or resistant bodies, formed and connected do that they move with definite relative motions and transmit force from the source of power to the resistance to be overcome. A machine has two functions: transmitting definite relative motion and transmitting force. These functions require strength and rigidity to transmit the forces.

The term ***mechanism*** is applied to the combination of geometrical bodies which constitute a machine or part of a machine. A ***mechanism*** may therefore be defined as a combination of rigid or resistant bodies, formed and connected so that they move with definite relative motions with respect to one another (Ham *et al.* 58).

Although a truly *rigid body* does not exist, many engineering components are rigid because their deformations and distortions are negligible in comparison with their relative movements.

The *similarity* between *machines* and *mechanisms* is that

- they are both combinations of rigid bodies
- the relative motion among the rigid bodies are definite.

The *difference* between *machine* and *mechanism* is that machines transform energy to do work, while mechanisms so not necessarily perform this function. The term ***machinery*** generally means machines and mechanisms. Figure 2-1 shows a picture of the main part of a diesel engine. The mechanism of its cylinder-link-crank parts is a *slider-crank mechanism*, as shown in Figure 2-2.

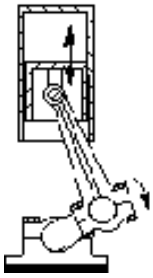


Figure 2-1 Cross section of a power cylinder in a diesel engine

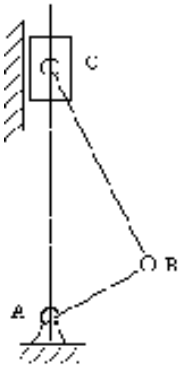


Figure 2-2 Skeleton outline

2.1 The Inclined Plane

Figure 2-3a shows an ***inclined plane***, AB is the base, BC is the height and AC the *inclined plane*. With the use of the inclined plane a given resistance can be overcome with a smaller force than if the plane is not used. For example, in Figure 2-3b, suppose we wish to raise a weight of 1000 lb. through the vertical distance BC = 2 ft. If this weight were raised vertically and without the use of the inclined plane the force 1000 lb. would have to be exerted through the distance BC. If, however, the inclined plane is used and the weight is moved over its inclined plane AC, a force of only 2/3 of 1000 lb. or 667 lb. is necessary, although this force is exerted through a distance AC which is greater than distance BC.

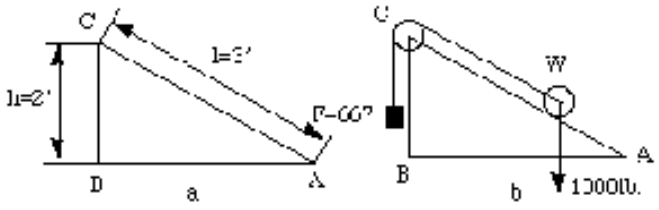


Figure 2-3 Inclined plane

Using an *inclined plane* requires a smaller force exerted through a greater distance to do a certain amount of work.

Letting **F** represent the force required to raise a given weight on the inclined plane, and W the weight to be raised, we have the proportion:

(2-1)

2.1.1 Screw Jack

One of the most common application of the principle of the *inclined plane* is in the ***screw jack*** which is used to overcome a heavy pressure or raise a heavy weight of *W* by a much smaller force *F* applied at the handle. *R* represents the length of the handle and *P* the *pitch* of the screw, or the distance advances in one complete turn.

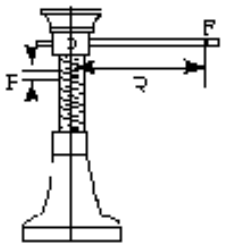


Figure 2-4 The screw jack

Neglecting the friction the following rule is used: The force *F* multiplied by the distance through which it moves in one complete turn is equal to the weight lifted times the distance through which it is lifted in the same time. In one complete turn the end of the handle describes a circle

of circumference 2 *R*. This is the distance through which the force *F* is exerted.

Therefore from the rule above

(2-2)

and

(2-3)

Suppose *R* equals 18 in., *P* equals 1/8 in. and the weight to be lifted equals 100,000 lb., then the force required at *F* is then 110 lb. This means that, neglecting friction, 110 lb. at *F* will raise 100,000 lb. at *W*, but the weight lifted moves much slower than the force applied at *F*.

2.2 Gears

A gear, or toothed wheel, when in operation, may actually be considered as a lever with the additional feature that it can be rotated continuously, instead of rocking back and forth through a short distance. One of the basic relationships for a gear is the number of teeth, the diameter, and the rotary velocity of gears. Figure 2-5 shows the ends of two shafts A and B connected by 2 gears of 24 and 48 teeth respectively. Notice that the larger gear will make only one-half turn while the smaller makes a complete turn. That is, the ratio of speeds (velocity ratio) of the large to the smaller is as 1 to 2.

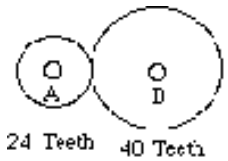


Figure 2-5 Gears

The gear that is closer to the source of power is called the *driver*, and the gear that receives power from the driver is called the *driven gear*.

2.2.1 Gear Trains

A *gear train* may have several drivers and several driven gears.

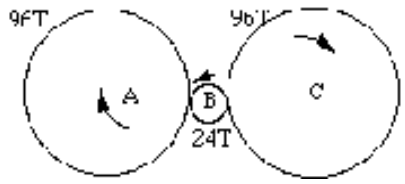


Figure 2-6 Gear train

When gear A turns once clockwise, gear B turns 4 times counter-clockwise and gear C turns once clockwise. Hence gear B does not change the speed of C from what it would have been if geared directly to gear A, but it changes its direction from counterclockwise to clockwise.

The velocity ratio of the first and last gears in a train of simple gears dose not changed by putting any number of gears between them.

Figure 2-7 shows *compound gears* in which two gears are on the middle shaft. Gears B and D rotate at the same speed since they are keyed (fixed) to the same shaft. The number of teeth on each gear is given in the figure. Given these numbers, if gear A rotates at 100 r.p.m. clockwise, gear B turns 400 r.p.m. (rotations per minute) counterclockwise and gear C turns 1200 r.p.m. clockwise.

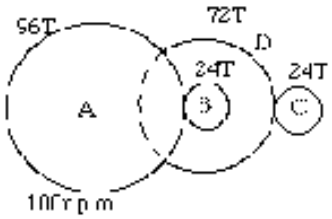


Figure 2-7 Compound gears

2.2.2 Gear Ratios

It is important when working with gears to know what number of teeth the gears should have so that they can mesh properly in a gear train. The size of the teeth for connecting gears must be match properly.

2.3 Belts and Pulleys

Belts and ***pulleys*** are an important part of most machines. ***Pulleys*** are nothing but gears without teeth and instead of running together directly they are made to drive one another by cords, ropes, cables, or belting of some kinds.

As with gears, the velocities of pulleys are inversely proportional to their diameters.



2 Mechanisms and Simple Machines

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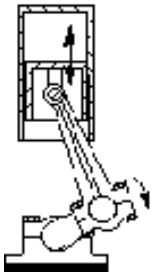


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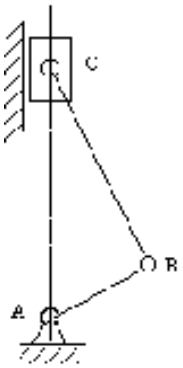


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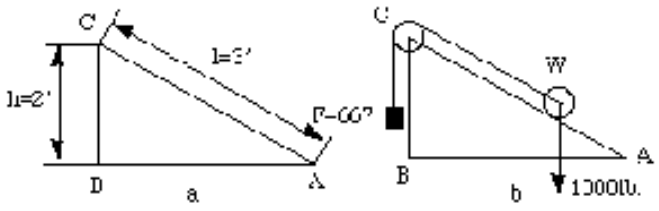


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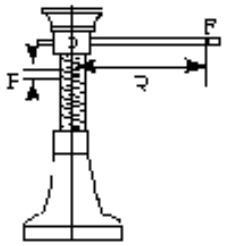


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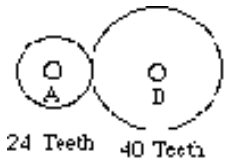


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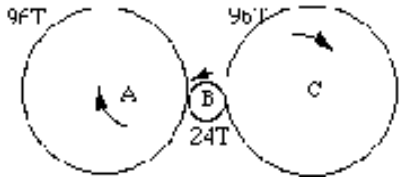


Figure 2-6 Gear train

When gear A turns once clockwise, gear B turns 4 times counter-clockwise and gear C turns once clockwise. Hence gear B does not change the speed of C from what it would have been if geared directly to gear A, but it changes its direction from counterclockwise to clockwise.

The velocity ratio of the first and last gears in a train of simple gears dose not changed by putting any number of gears between them.

Figure 2-7 shows *compound gears* in which two gears are on the middle shaft. Gears B and D rotate at the same speed since they are keyed (fixed) to the same shaft. The number of teeth on each gear is given in the figure. Given these numbers, if gear A rotates at 100 r.p.m. clockwise, gear B turns 400 r.p.m. (rotations per minute) counterclockwise and gear C turns 1200 r.p.m. clockwise.

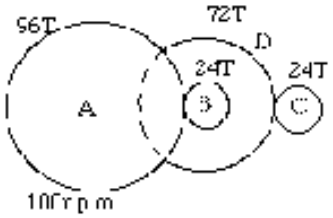


Figure 2-7 Compound gears

2.2.2 Gear Ratios

It is important when working with gears to know what number of teeth the gears should have so that they can mesh properly in a gear train. The size of the teeth for connecting gears must be match properly.

2.3 Belts and Pulleys

Belts and ***pulleys*** are an important part of most machines. ***Pulleys*** are nothing but gears without teeth and instead of running together directly they are made to drive one another by cords, ropes, cables, or belting of some kinds.

As with gears, the velocities of pulleys are inversely proportional to their diameters.



2 Mechanisms and Simple Machines

Mechanism*: the fundamental physical or chemical processes involved in or responsible for an action, reaction or other natural phenomenon.*

Machine*: an assemblage of parts that transmit forces, motion and energy in a predetermined manner.*

Simple Machine*: any of various elementary mechanisms having the elements of which all machines are composed. Included in this category are the lever, wheel and axle, pulley, inclined plane, wedge and the screw.*

The word ***mechanism*** has many meanings. In *kinematics*, a mechanism is a means of transmitting, controlling, or constraining relative movement (Hunt 78). Movements which are electrically, magnetically, pneumatically operated are excluded from the concept of mechanism. The central theme for mechanisms is rigid bodies connected together by joints.

A ***machine*** is a combination of rigid or resistant bodies, formed and connected do that they move with definite relative motions and transmit force from the source of power to the resistance to be overcome. A machine has two functions: transmitting definite relative motion and transmitting force. These functions require strength and rigidity to transmit the forces.

The term ***mechanism*** is applied to the combination of geometrical bodies which constitute a machine or part of a machine. A ***mechanism*** may therefore be defined as a combination of rigid

or resistant bodies, formed and connected so that they move with definite relative motions with respect to one another (Ham *et al.* 58).

Although a truly *rigid body* does not exist, many engineering components are rigid because their deformations and distortions are negligible in comparison with their relative movements.

The *similarity* between *machines* and *mechanisms* is that

- they are both combinations of rigid bodies
- the relative motion among the rigid bodies are definite.

The *difference* between *machine* and *mechanism* is that machines transform energy to do work, while mechanisms so not necessarily perform this function. The term ***machinery*** generally means machines and mechanisms. Figure 2-1 shows a picture of the main part of a diesel engine. The mechanism of its cylinder-link-crank parts is a *slider-crank mechanism*, as shown in Figure 2-2.

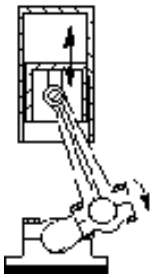


Figure 2-1 Cross section of a power cylinder in a diesel engine

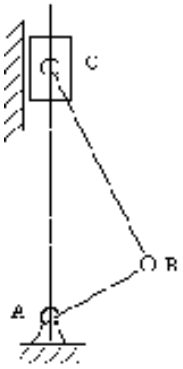


Figure 2-2 Skeleton outline

2.1 The Inclined Plane

Figure 2-3a shows an ***inclined plane***, AB is the base, BC is the height and AC the *inclined plane*. With the use of the inclined plane a given resistance can be overcome with a smaller force than if the plane is not used. For example, in Figure 2-3b, suppose we wish to raise a weight of 1000 lb. through the vertical distance BC = 2 ft. If this weight were raised vertically and without the use

of the inclined plane the force 1000 lb. would have to be exerted through the distance BC. If, however, the inclined plane is used and the weight is moved over its inclined plane AC, a force of only 2/3 of 1000 lb. or 667 lb. is necessary, although this force is exerted through a distance AC which is greater than distance BC.

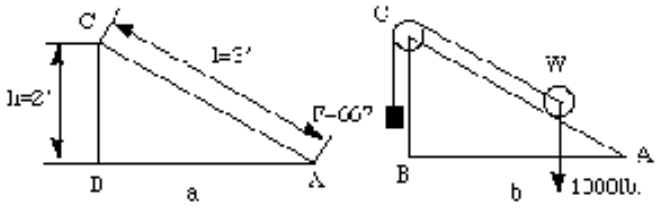


Figure 2-3 Inclined plane

Using an *inclined plane* requires a smaller force exerted through a greater distance to do a certain amount of work.

Letting **F** represent the force required to raise a given weight on the inclined plane, and W the weight to be raised, we have the proportion:

(2-1)

2.1.1 Screw Jack

One of the most common application of the principle of the *inclined plane* is in the ***screw jack*** which is used to overcome a heavy pressure or raise a heavy weight of *W* by a much smaller force *F* applied at the handle. *R* represents the length of the handle and *P* the *pitch* of the screw, or the distance advances in one complete turn.

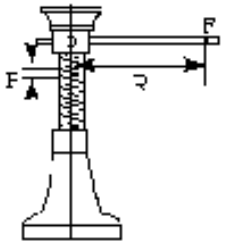


Figure 2-4 The screw jack

Neglecting the friction the following rule is used: The force *F* multiplied by the distance through which it moves in one complete turn is equal to the weight lifted times the distance through which it is lifted in the same time. In one complete turn the end of the handle describes a circle

of circumference 2 *R*. This is the distance through which the force *F* is exerted.

Therefore from the rule above

Suppose *R* equals 18 in., *P* equals 1/8 in. and the weight to be lifted equals 100,000 lb., then the force required at *F* is then 110 lb. This means that, neglecting friction, 110 lb. at *F* will raise 100,000 lb. at *W*, but the weight lifted moves much slower than the force applied at *F*.

2.2 Gears

A gear, or toothed wheel, when in operation, may actually be considered as a lever with the additional feature that it can be rotated continuously, instead of rocking back and forth through a short distance. One of the basic relationships for a gear is the number of teeth, the diameter, and the rotary velocity of gears. Figure 2-5 shows the ends of two shafts A and B connected by 2 gears of 24 and 48 teeth respectively. Notice that the larger gear will make only one-half turn while the smaller makes a complete turn. That is, the ratio of speeds (velocity ratio) of the large to the smaller is as 1 to 2.

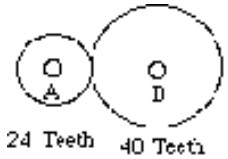


Figure 2-5 Gears

The gear that is closer to the source of power is called the *driver*, and the gear that receives power from the driver is called the *driven gear*.

2.2.1 Gear Trains

A *gear train* may have several drivers and several driven gears.

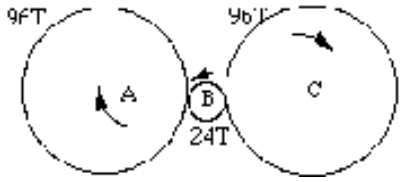


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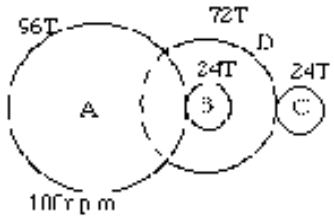


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More on Machines:

3 More on Machines and Mechanisms

3.1 Planar and Spatial Mechanisms

Mechanisms can be divided into ***planar mechanisms*** and ***spatial mechanisms***, according to the relative motion of the rigid bodies. In a ***planar mechanisms***, all of the relative motions of the rigid bodies are in one plane or in parallel planes. If there is any relative motion that is not in the same plane or in parallel planes, the mechanism is called the ***spatial mechanism***. In other words,

planar mechanisms are essentially two dimensional while *spatial mechanisms* are three dimensional. This tutorial only covers planar mechanisms.

3.2 Kinematics and Dynamics of Mechanisms

Kinematics of mechanisms is concerned with the motion of the parts without considering how the influencing factors (force and mass) affect the motion. Therefore, kinematics deals with the fundamental concepts of space and time and the quantities velocity and acceleration derived there from.

Kinetics deals with action of forces on bodies. This is where the the effects of gravity come into play.

Dynamics is the combination of *kinematics* and *kinetics*.

Dynamics of mechanisms concerns the forces that act on the parts -- both balanced and unbalanced forces, taking into account the masses and accelerations of the parts as well as the external forces.

3.3 Links, Frames and Kinematic Chains

A ***link*** is defined as a rigid body having two or more pairing elements which connect it to other bodies for the purpose of transmitting force or motion (Ham *et al.* 58).

In every machine, at least one link either occupies a fixed position relative to the earth or carries the machine as a whole along with it during motion. This link is the ***frame*** of the machine and is called the *fixed link*.

The combination of links and pairs without a fixed link is not a mechanism but a ***kinematic chain.***

3.4 Skeleton Outline

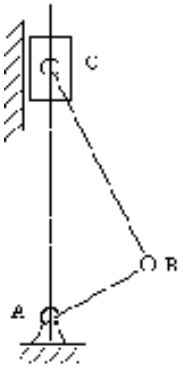


Figure 3-1 Skeleton outline

For the purpose of kinematic analysis, a mechanism may be represented in an abbreviated, or skeleton, form called the *skeleton outline* of the mechanism. The skeleton outline gives all the geometrical information necessary for determining the relative motions of the links. In Figure 3-1, the skeleton outline has been drawn for the engine shown in Figure 2-1. This skeleton contains all necessary information to determine the relative motions of the main links, namely, the length AB of the crank; the length BC of the connecting rod; A the location of the axis of the main bearing; and the path AC of point C, which represents the wrist-pin axis.

3.5 Pairs, Higher Pairs, Lower Pairs and Linkages

A ***pair*** is a joint between the surfaces of two rigid bodies that keeps them in contact and relatively movable. For example, in Figure 3-2, a door jointed to the frame with hinges makes ***revolute joint*** (***pin joint***), allowing the door to be turned around its axis. Figure 3-2b and c show skeletons of a revolute joint. Figure 3-2b is used when both links joined by the pair can turn. Figure 3-2c is used when one of the link jointed by the pair is the frame.

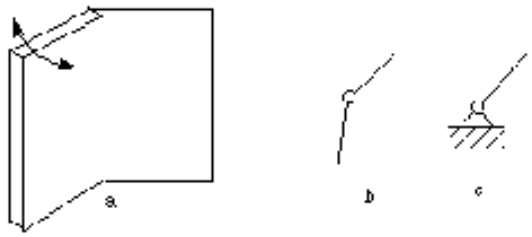


Figure 3-2 Revolute pair

In Figure 3-3a a sash window can be translated relative to the sash. This kind of relative motion is called a ***prismatic pair***. Its skeleton outlines are shown in *b*, *c* and *d*. *c* and *d* are used when one of the links is the *frame*.

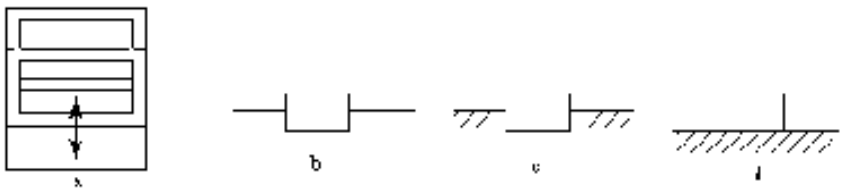


Figure 3-3 Prismatic pair

Generally, there are two kinds of *pairs* in mechanisms, ***lower pairs*** and ***higher pairs***. What differentiates them is the type of contact between the two bodies of the pair. Surface-contact pairs are called ***lower pairs***. In planar (2D) mechanisms, there are two subcategories of lower pairs -- *revolute pairs* and *prismatic pairs*, as shown in Figures 3-2 and 3-3, respectively. Point-, line-, or curve-contact pairs are called ***higher pairs***. Figure 3-4 shows some examples of *higher pairs* Mechanisms composed of rigid bodies and lower pairs are called ***linkages***.

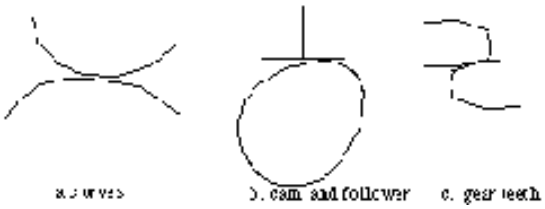


Figure 3-4 Higher pairs

3.6 Kinematic Analysis and Synthesis

In ***kinematic analysis***, a particular given mechanism is investigated based on the mechanism geometry plus other known characteristics (such as input angular velocity, angular acceleration, *etc.*). ***Kinematic synthesis***, on the other hand, is the process of designing a mechanism to accomplish a desired task. Here, both choosing the types as well as the dimensions of the new mechanism can be part of kinematic synthesis. (Sandor & Erdman 84)

Basic Kinematics of Constrained Rigid Bodies:

4 Basic Kinematics of Constrained Rigid Bodies

4.1 Degrees of Freedom of a Rigid Body

4.1.1 Degrees of Freedom of a Rigid Body in a Plane

The ***degrees of freedom*** (DOF) of a rigid body is defined as the number of independent movements it has. Figure 4-1 shows a rigid body in a plane. To determine the DOF of this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be *translated* along the *x* axis, translated along the *y* axis, and *rotated* about its centroid.

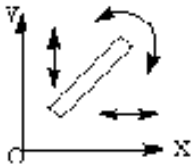


Figure 4-1 Degrees of freedom of a rigid body in a plane

4.1.2 Degrees of Freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions along the *x*, *y* and *z* axes and three rotary motions around the *x*, *y* and *z* axes respectively.

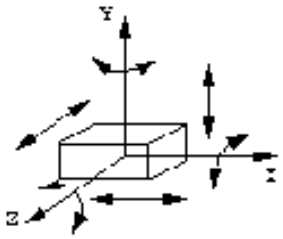


Figure 4-2 Degrees of freedom of a rigid body in space

4.2 Kinematic Constraints

Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with ***kinematic constraints***. *Kinematic constraints* are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

The term kinematic pairs actually refers to *kinematic constraints* between rigid bodies. The kinematic pairs are divided into lower pairs and higher pairs, depending on how the two bodies are in contact.

4.2.1 Lower Pairs in Planar Mechanisms

There are two kinds of lower pairs in planar mechanisms: revolute pairs and prismatic pairs.

A rigid body in a plane has only three independent motions -- two translational and one rotary -- so introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.

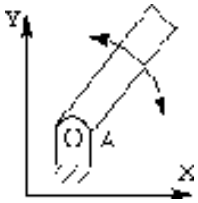


Figure 4-3 A planar revolute pair (R-pair)

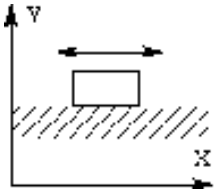


Figure 4-4 A planar prismatic pair (P-pair)

4.2.2 Lower Pairs in Spatial Mechanisms

There are six kinds of lower pairs under the category of spatial mechanisms. The types are: spherical pair, plane pair, cylindrical pair, revolute pair, prismatic pair, and screw pair.

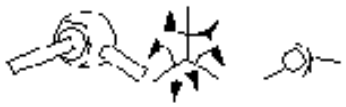


Figure 4-5 A spherical pair (S-pair)

A *spherical pair* keeps two spherical centers together. Two rigid bodies connected by this constraint will be able to *rotate* relatively around *x*, *y* and *z* axes, but there will be no relative translation along any of these axes. Therefore, a spherical pair removes three degrees of freedom in spatial mechanism. **DOF = 3**.

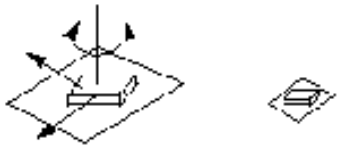


Figure 4-6 A planar pair (E-pair)

A *plane pair* keeps the surfaces of two rigid bodies together. To visualize this, imagine a book lying on a table where is can move in any direction except off the table. Two rigid bodies connected by this kind of pair will have two independent translational motions in the plane, and a rotary motion around the axis that is perpendicular to the plane. Therefore, a plane pair removes three degrees of freedom in spatial mechanism. In our example, the book would not be able to raise off the table or to rotate into the table. **DOF = 3.**

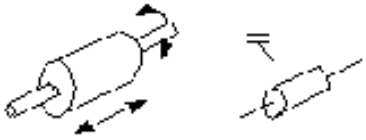


Figure 4-7 A cylindrical pair (C-pair)

A *cylindrical pair* keeps two axes of two rigid bodies aligned. Two rigid bodies that are part of this kind of system will have an independent translational motion along the axis and a relative rotary motion around the axis. Therefore, a cylindrical pair removes four degrees of freedom from spatial mechanism. **DOF = 2.**



Figure 4-8 A revolute pair (R-pair)

A *revolute pair* keeps the axes of two rigid bodies together. Two rigid bodies constrained by a revolute pair have an independent rotary motion around their common axis. Therefore, a revolute pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**

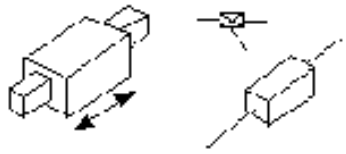


Figure 4-9 A prismatic pair (P-pair)

A *prismatic pair* keeps two axes of two rigid bodies align and allow no relative rotation. Two rigid bodies constrained by this kind of constraint will be able to have an independent translational motion along the axis. Therefore, a prismatic pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**



Figure 4-10 A screw pair (H-pair)

The *screw pair* keeps two axes of two rigid bodies aligned and allows a relative screw motion. Two rigid bodies constrained by a screw pair a motion which is a composition of a translational motion along the axis and a corresponding rotary motion around the axis. Therefore, a screw pair removes five degrees of freedom in spatial mechanism.

4.3 Constrained Rigid Bodies

Rigid bodies and kinematic constraints are the basic components of mechanisms. A constrained rigid body system can be a kinematic chain, a mechanism, a structure, or none of these. The influence of kinematic constraints in the motion of rigid bodies has two intrinsic aspects, which are the geometrical and physical aspects. In other words, we can analyze the motion of the constrained rigid bodies from their geometrical relationships or using Newton's Second Law.

A mechanism is a constrained rigid body system in which one of the bodies is the frame. The degrees of freedom are important when considering a constrained rigid body system that is a

mechanism. It is less crucial when the system is a structure or when it does not have definite motion.

Calculating the degrees of freedom of a rigid body system is straight forward. Any unconstrained rigid body has six degrees of freedom in space and three degrees of freedom in a plane. Adding kinematic constraints between rigid bodies will correspondingly decrease the degrees of freedom of the rigid body system. We will discuss more on this topic for planar mechanisms in the next section.

4.4 Degrees of Freedom of Planar Mechanisms

4.4.1 Gruebler's Equation

The definition of the ***degrees of freedom*** of a mechanism is the number of independent relative motions among the rigid bodies. For example, Figure 4-11 shows several cases of a rigid body constrained by different kinds of pairs.

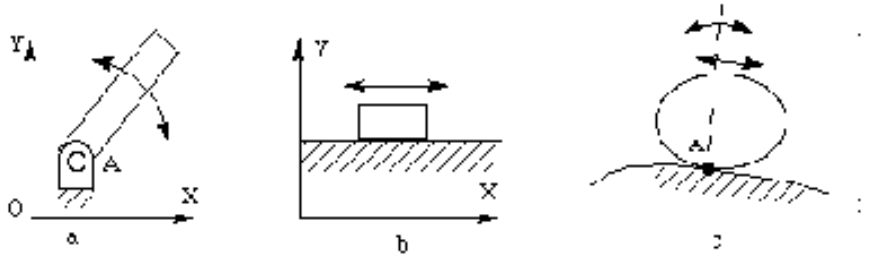


Figure 4-11 Rigid bodies constrained by different kinds of planar pairs

In Figure 4-11a, a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the *x* and *y* axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure 4-11b, a rigid body is constrained by a prismatic pair which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the *x* axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the *y* axis.

In Figure 4-11c, a rigid body is constrained by a higher pair. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a,b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.

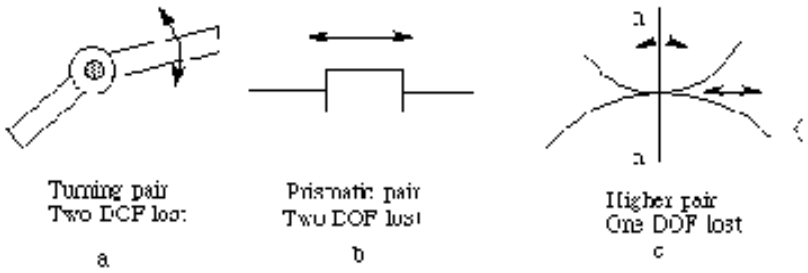


Figure 4-12 Kinematic Pairs in Planar Mechanisms

Therefore, we can write the following equation:

$$F = 3(n-1) - 2l - h$$

(4-1)

Where

F = total degrees of freedom in the mechanism

n = number of links (including the frame)

l = number of lower pairs (one degree of freedom)

h = number of higher pairs (two degrees of freedom)

This equation is also known as ***Gruebler's equation***.

Example 1

Look at the transom above the door in Figure 4-13a. The opening and closing mechanism is shown in Figure 4-13b. Let's calculate its degree of freedom.

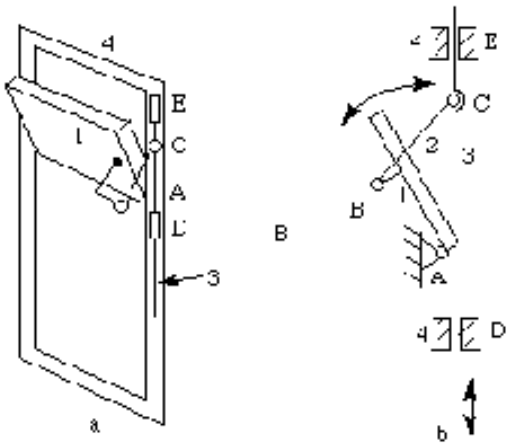


Figure 4-13 Transom mechanism

n = 4 (link 1,3,3 and frame 4), l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$

(4-2)

Note: D and E function as a same prismatic pair, so they only count as one lower pair.

Example 2

Calculate the degrees of freedom of the mechanisms shown in Figure 4-14b. Figure 4-14a is an application of the mechanism.

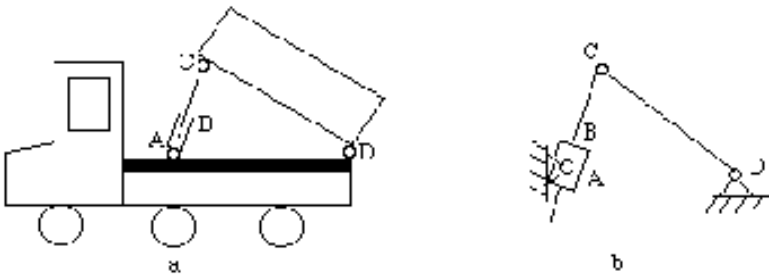


Figure 4-14 Dump truck

n = 4, l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$

(4-3)

Example 3

Calculate the degrees of freedom of the mechanisms shown in Figure 4-15.

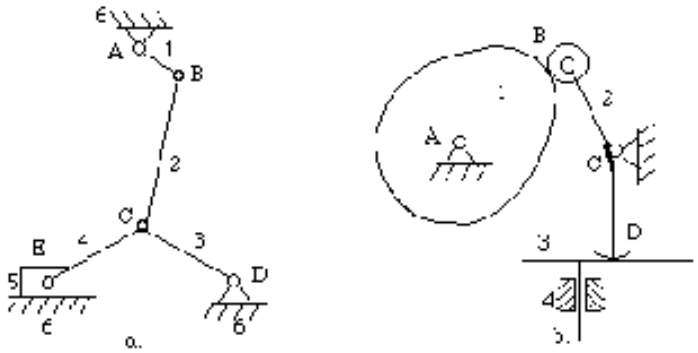


Figure 4-15 Degrees of freedom calculation

For the mechanism in Figure 4-15a

n = 6, l = 7, h = 0

$$F = 3(6-1) - 2\times 7 - 1\times 0 = 1$$

(4-4)

For the mechanism in Figure 4-15b

n = 4, l = 3, h = 2

$$F = 3(4-1) - 2\times 3 - 1\times 2 = 1$$

(4-5)

Note: The rotation of the roller does not influence the relationship of the input and output motion of the mechanism. Hence, the freedom of the roller will not be considered; It is called a ***passive*** or ***redundant*** degree of freedom. Imagine that the roller is welded to link 2 when counting the degrees of freedom for the mechanism.

4.4.2 Kutzbach Criterion

The number of degrees of freedom of a mechanism is also called the *mobility* of the device. The ***mobility*** is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The ***Kutzbach criterion***, which is similar to Gruebler's equation, calculates the *mobility*.

In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism. For example, the transom in Figure 4-13a has a single degree of freedom, so it needs one independent input motion to open or close the window. That is, you just push or pull rod 3 to operate the window.

To see another example, the mechanism in Figure 4-15a also has 1 degree of freedom. If an independent input is applied to link 1 (*e.g.*, a motor is mounted on joint A to drive link 1), the mechanism will have the a prescribed motion.

4.5 Finite Transformation

Finite transformation is used to describe the motion of a point on rigid body and the motion of the rigid body itself.

4.5.1 Finite Planar Rotational Transformation

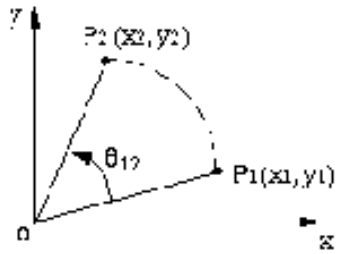


Figure 4-16 Point on a planar rigid body rotated through an angle

Suppose that a point *P* on a rigid body goes through a rotation describing a circular path from P_1 to P_2 around the origin of a coordinate system. We can describe this motion with a ***rotation operator*** R_{12}:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-6)

where

$$R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-7)

4.5.2 Finite Planar Translational Transformation

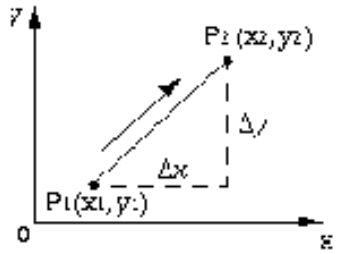


Figure 4-17 Point on a planar rigid body translated through a distance

Suppose that a point *P* on a rigid body goes through a translation describing a straight path from P_1 to P_2 with a change of coordinates of (Δx, Δy). We can describe this motion with a ***translation operator*** T_{12}:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

where

$$T_{12} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

4.5.3 Concatenation of Finite Planar Displacements

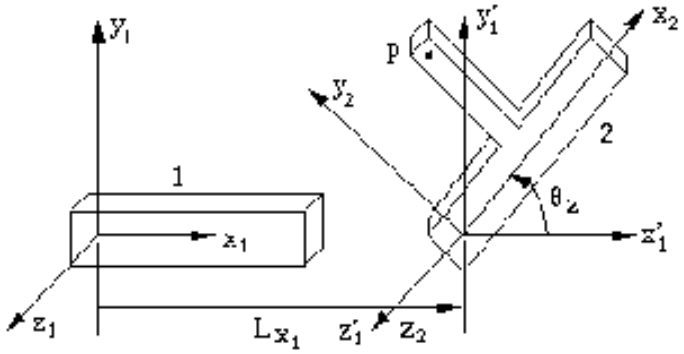


Figure 4-18 Concatenation of finite planar displacements in space

Suppose that a point *P* on a rigid body goes through a rotation describing a circular path from P_1 to P_2' around the origin of a coordinate system, then a translation describing a straight path from P_2' to P_2. We can represent these two steps by

$$\begin{bmatrix} X_2' \\ Y_2' \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-10)

and

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} X_2' \\ Y_2' \\ 1 \end{bmatrix}$$

(4-11)

We can concatenate these motions to get

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = D_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-12)

where D_{12} is the ***planar general displacement operator*** :

$$D_{12} = T_{12} R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-13)

4.5.4 Planar Rigid-Body Transformation

We have discussed various transformations to describe the displacements of a point on rigid body. Can these operators be applied to the displacements of a system of points such as a rigid body?

We used a 3 x 1 homogeneous column matrix to describe a vector representing a single point. A beneficial feature of the planar 3 x 3 translational, rotational, and general displacement matrix operators is that they can easily be programmed on a computer to manipulate a 3 x n matrix of n column vectors representing n points of a rigid body. Since the distance of each particle of a rigid body from every other point of the rigid body is constant, the vectors locating each point of a rigid body must undergo the same transformation when the rigid body moves and the proper axis, angle, and/or translation is specified to represent its motion. (Sandor & Erdman 84). For example, the general planar transformation for the three points *A, B, C* on a rigid body can be represented by

$$D_{12} = T_{12} R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & \Delta x \\ \sin\theta_{12} & \cos\theta_{12} & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

(4-14)

4.5.5 Spatial Rotational Transformation

We can describe a spatial rotation operator for the rotational transformation of a point about an unit axis **u** passing through the origin of the coordinate system. Suppose the rotational angle of the point about **u** is ϕ, the ***rotation operator*** will be expressed by

$$R_{\theta,u} = \begin{bmatrix} u_x^2 v\theta + c\theta & u_x u_y v\theta - u_z s\theta & u_x u_z v\theta + u_y s\theta & 0 \\ u_x u_y v\theta + u_z s\theta & u_y^2 v\theta + c\theta & u_y u_z v\theta - u_x s\theta & 0 \\ u_x u_z v\theta - u_y s\theta & u_y u_z v\theta + u_x s\theta & u_z^2 v\theta + c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-15)

where

u_x, u_y, u_z are the othographical projection of the unit axis u on *x*, *y*, and *z* axes, respectively.

$s_\phi = sin_\phi$

$c_\phi = cos_\phi$

$v_\phi = 1 - cos_\phi$

4.5.6 Spatial Translational Transformation

Suppose that a point *P* on a rigid body goes through a translation describing a straight path from P_1 to P_2 with a change of coordinates of (Δx, Δy, Δz), we can describe this motion with a ***translation operator*** *T*:

$$T = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-16)

4.5.7 Spatial Translation and Rotation Matrix for Axis Through the Origin

Suppose a point *P* on a rigid body rotates with an angular displacement about an unit axis **u** passing through the origin of the coordinate system at first, and then followed by a translation *D***u** along **u**. This composition of this rotational transformation and this translational transformation is a screw motion. Its corresponding matrix operator, the ***screw operator***, is a concatenation of the translation operator in Equation 4-7 and the rotation operator in Equation 4-9.

$$S = \begin{bmatrix} u_x^2 v\theta + c\theta & u_x u_y v\theta - u_z s\theta & u_x u_z v\theta + u_y s\theta & 0 \\ u_x u_y v\theta + u_z s\theta & u_y^2 v\theta + c\theta & u_y u_z v\theta - u_x s\theta & 0 \\ u_x u_z v\theta - u_y s\theta & u_y u_z v\theta + u_x s\theta & u_z^2 v\theta + c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-17)

4.6 Transformation Matrix Between Rigid Bodies

4.6.1 Transformation Matrix Between two Arbitray Rigid Bodies

For a system of rigid bodies, we can establish a local Cartesian coordinate system for each rigid body. Transformation matrices are used to describe the relative motion between rigid bodies.

For example, two rigid bodies in a space each have local coordinate systems $x_1y_1z_1$ and $x_2y_2z_2$. Let point *P* be attached to body 2 at location (x_2, y_2, z_2) in body 2's local coordinate system. To find the location of *P* with respect to body 1's local coordinate system, we know that that the point $x_2y_2z_2$ can be obtained from $x_1y_1z_1$ by combining translation L_{x1} along the *x* axis and rotation θz about z axis. We can derive the transformation matrix as follows:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Lx_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & Lx_1 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

(4-18)

If rigid body 1 is fixed as a frame, a global coordinate system can be created on this body. Therefore, the above transformation can be used to map the local coordinates of a point into the global coordinates.

4.6.2 Kinematic Constraints Between Two Rigid Bodies

The transformation matrix above is a specific example for two unconstrained rigid bodies. The transformation matrix depends on the relative position of the two rigid bodies. If we connect two rigid bodies with a kinematic constraint, their degrees of freedom will be decreased. In other words, their relative motion will be specified in some extent.

Suppose we constrain the two rigid bodies above with a revolute pair as shown in Figure 4-19. We can still write the transformation matrix in the same form as Equation 4-18.

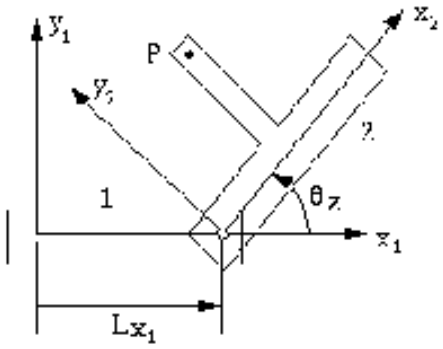


Figure 4-19 Relative position of points on constrained bodies

The difference is that the L_{x1} is a constant now, because the revolute pair fixes the origin of coordinate system $x_2y_2z_2$ with respect to coordinate system $x_1y_1z_1$. However, the rotation θz is still a variable. Therefore, kinematic constraints specify the transformation matrix to some extent.

4.6.3 Denavit-Hartenberg Notation

Denavit-Hartenberg notation (Denavit & Hartenberg 55) is widely used in the transformation of coordinate systems of linkages and robot mechanisms. It can be used to represent the transformation matrix between links as shown in the Figure 4-20.

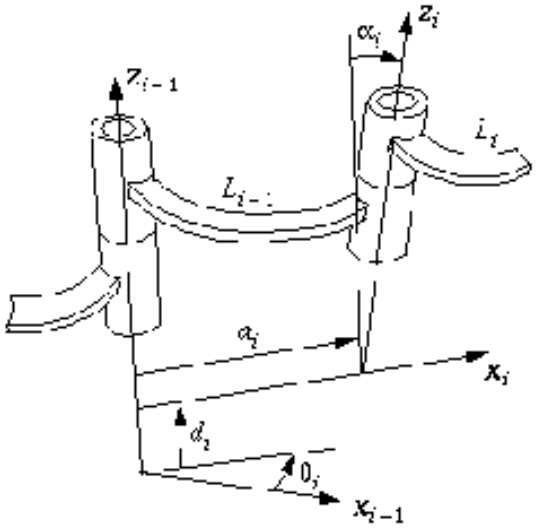


Figure 4-20 Denavit-Hartenberg Notation

In this figure,

- z_{i-1} and z_i are the axes of two revolute pairs;
- θ_i is the included angle of axes x_{i-1} and x_i;

- d_i is the distance between the origin of the coordinate system $x_{i-1}y_{i-1}z_{i-1}$ and the foot of the common perpendicular;
- a_i is the distance between two feet of the common perpendicular;
- α_i is the included angle of axes z_{i-1} and z_i;

The transformation matrix will be $T_{(i-1)i}$

$$T_{(i-1)i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-19)

The above transformation matrix can be denoted as $T(a_i, \alpha_i, \theta_i, d_i)$ for convenience.

4.6.4 Application of Transformation Matrices to Linkages

A linkage is composed of several constrained rigid bodies. Like a mechanism, a linkage should have a frame. The matrix method can be used to derive the kinematic equations of the linkage. If all the links form a closed loop, the concatenation of all of the transformation matrices will be an identity matrix. If the mechanism has *n* links, we will have:

$$T_{12}T_{23}...T_{(n-1)n} = I$$

4 Basic Kinematics of Constrained Rigid Bodies

4.1 Degrees of Freedom of a Rigid Body

4.1.1 Degrees of Freedom of a Rigid Body in a Plane

The ***degrees of freedom*** (DOF) of a rigid body is defined as the number of independent movements it has. Figure 4-1 shows a rigid body in a plane. To determine the DOF of this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be *translated* along the *x* axis, translated along the *y* axis, and *rotated* about its centroid.

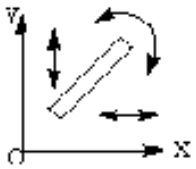


Figure 4-1 Degrees of freedom of a rigid body in a plane

4.1.2 Degrees of Freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions along the *x*, *y* and *z* axes and three rotary motions around the *x*, *y* and *z* axes respectively.

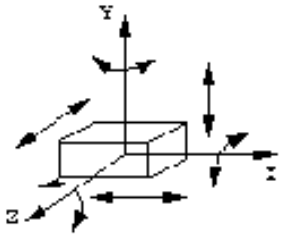


Figure 4-2 Degrees of freedom of a rigid body in space

4.2 Kinematic Constraints

Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with ***kinematic constraints***. *Kinematic constraints* are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

The term kinematic pairs actually refers to *kinematic constraints* between rigid bodies. The kinematic pairs are divided into lower pairs and higher pairs, depending on how the two bodies are in contact.

4.2.1 Lower Pairs in Planar Mechanisms

There are two kinds of lower pairs in planar mechanisms: revolute pairs and prismatic pairs.

A rigid body in a plane has only three independent motions -- two translational and one rotary -- so introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.

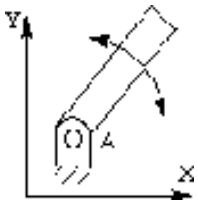


Figure 4-3 A planar revolute pair (R-pair)

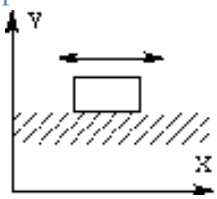


Figure 4-4 A planar prismatic pair (P-pair)

4.2.2 Lower Pairs in Spatial Mechanisms

There are six kinds of lower pairs under the category of spatial mechanisms. The types are: spherical pair, plane pair, cylindrical pair, revolute pair, prismatic pair, and screw pair.

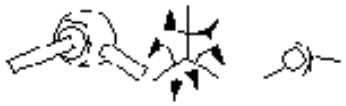


Figure 4-5 A spherical pair (S-pair)

A *spherical pair* keeps two spherical centers together. Two rigid bodies connected by this constraint will be able to *rotate* relatively around *x*, *y* and *z* axes, but there will be no relative translation along any of these axes. Therefore, a spherical pair removes three degrees of freedom in spatial mechanism. **DOF = 3**.

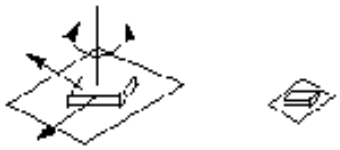


Figure 4-6 A planar pair (E-pair)

A *plane pair* keeps the surfaces of two rigid bodies together. To visualize this, imagine a book lying on a table where is can move in any direction except off the table. Two rigid bodies connected by this kind of pair will have two independent translational motions in the plane, and a rotary motion around the axis that is perpendicular to the plane. Therefore, a plane pair removes three degrees of freedom in spatial mechanism. In our example, the book would not be able to raise off the table or to rotate into the table. **DOF = 3.**

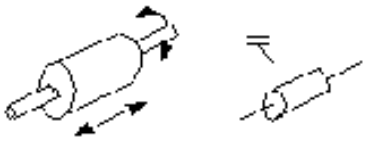


Figure 4-7 A cylindrical pair (C-pair)

A *cylindrical pair* keeps two axes of two rigid bodies aligned. Two rigid bodies that are part of this kind of system will have an independent translational motion along the axis and a relative rotary motion around the axis. Therefore, a cylindrical pair removes four degrees of freedom from spatial mechanism. **DOF = 2.**



Figure 4-8 A revolute pair (R-pair)

A *revolute pair* keeps the axes of two rigid bodies together. Two rigid bodies constrained by a revolute pair have an independent rotary motion around their common axis. Therefore, a revolute pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**

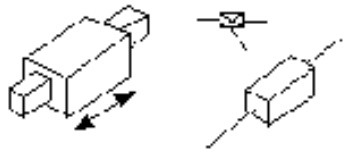


Figure 4-9 A prismatic pair (P-pair)

A *prismatic pair* keeps two axes of two rigid bodies align and allow no relative rotation. Two rigid bodies constrained by this kind of constraint will be able to have an independent translational motion along the axis. Therefore, a prismatic pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**



Figure 4-10 A screw pair (H-pair)

The *screw pair* keeps two axes of two rigid bodies aligned and allows a relative screw motion. Two rigid bodies constrained by a screw pair a motion which is a composition of a translational motion along the axis and a corresponding rotary motion around the axis. Therefore, a screw pair removes five degrees of freedom in spatial mechanism.

4.3 Constrained Rigid Bodies

Rigid bodies and kinematic constraints are the basic components of mechanisms. A constrained rigid body system can be a kinematic chain, a mechanism, a structure, or none of these. The influence of kinematic constraints in the motion of rigid bodies has two intrinsic aspects, which are the geometrical and physical aspects. In other words, we can analyze the motion of the constrained rigid bodies from their geometrical relationships or using Newton's Second Law.

A mechanism is a constrained rigid body system in which one of the bodies is the frame. The degrees of freedom are important when considering a constrained rigid body system that is a mechanism. It is less crucial when the system is a structure or when it does not have definite motion.

Calculating the degrees of freedom of a rigid body system is straight forward. Any unconstrained rigid body has six degrees of freedom in space and three degrees of freedom in a plane. Adding kinematic constraints between rigid bodies will correspondingly decrease the degrees of freedom of the rigid body system. We will discuss more on this topic for planar mechanisms in the next section.

4.4 Degrees of Freedom of Planar Mechanisms

4.4.1 Gruebler's Equation

The definition of the ***degrees of freedom*** of a mechanism is the number of independent relative motions among the rigid bodies. For example, Figure 4-11 shows several cases of a rigid body constrained by different kinds of pairs.

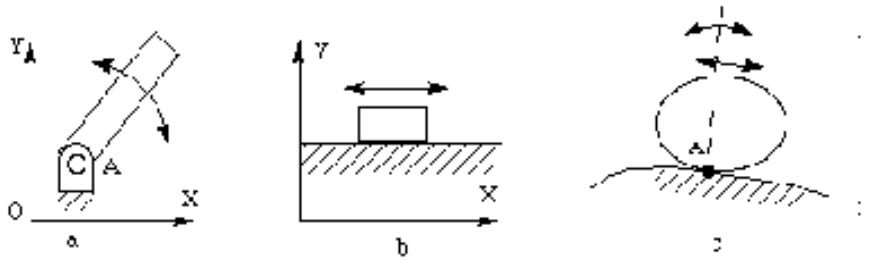


Figure 4-11 Rigid bodies constrained by different kinds of planar pairs

In Figure 4-11a, a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the *x* and *y* axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure 4-11b, a rigid body is constrained by a prismatic pair which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the *x* axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the *y* axis.

In Figure 4-11c, a rigid body is constrained by a higher pair. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a,b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.

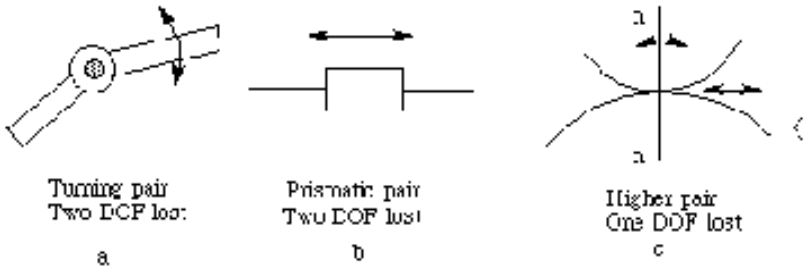


Figure 4-12 Kinematic Pairs in Planar Mechanisms

Therefore, we can write the following equation:

$$F = 3(n-1) - 2l - h$$

(4-1)

Where

F = total degrees of freedom in the mechanism

n = number of links (including the frame)

l = number of lower pairs (one degree of freedom)

h = number of higher pairs (two degrees of freedom)

This equation is also known as ***Gruebler's equation***.

Example 1

Look at the transom above the door in Figure 4-13a. The opening and closing mechanism is shown in Figure 4-13b. Let's calculate its degree of freedom.

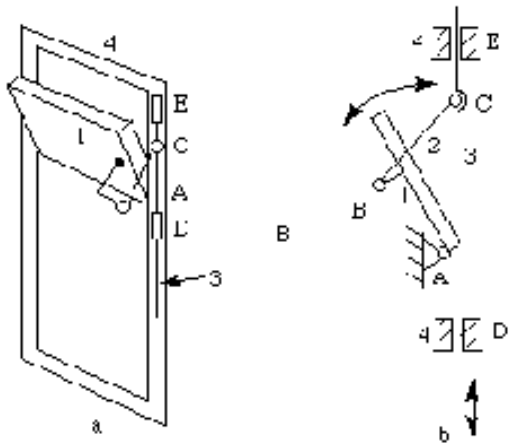


Figure 4-13 Transom mechanism

n = 4 (link 1,3,3 and frame 4), l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$

(4-2)

Note: D and E function as a same prismatic pair, so they only count as one lower pair.

Example 2

Calculate the degrees of freedom of the mechanisms shown in Figure 4-14b. Figure 4-14a is an application of the mechanism.

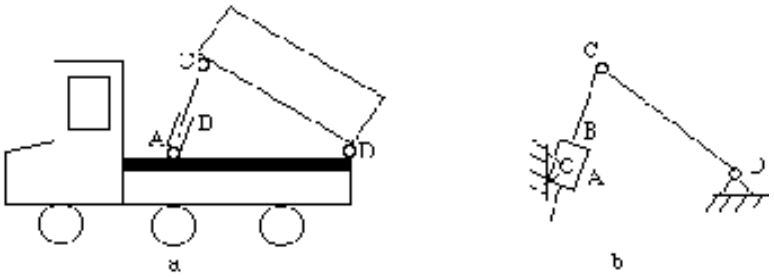


Figure 4-14 Dump truck

n = 4, l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$

(4-3)

Example 3

Calculate the degrees of freedom of the mechanisms shown in Figure 4-15.

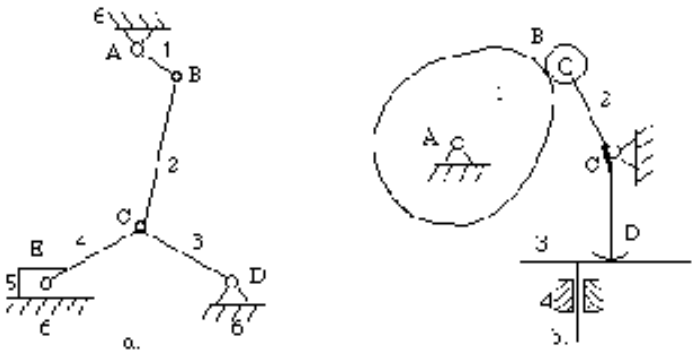


Figure 4-15 Degrees of freedom calculation

For the mechanism in Figure 4-15a

n = 6, l = 7, h = 0

$$F = 3(6-1) - 2\times 7 - 1\times 0 = 1$$

(4-4)

For the mechanism in Figure 4-15b

n = 4, l = 3, h = 2

$$F = 3(4-1) - 2\times 3 - 1\times 2 = 1$$

(4-5)

Note: The rotation of the roller does not influence the relationship of the input and output motion of the mechanism. Hence, the freedom of the roller will not be considered; It is called a ***passive*** or ***redundant*** degree of freedom. Imagine that the roller is welded to link 2 when counting the degrees of freedom for the mechanism.

4.4.2 Kutzbach Criterion

The number of degrees of freedom of a mechanism is also called the *mobility* of the device. The ***mobility*** is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The ***Kutzbach criterion***, which is similar to Gruebler's equation, calculates the *mobility*.

In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism. For example, the transom in Figure 4-13a has a single degree of freedom, so it needs one independent input motion to open or close the window. That is, you just push or pull rod 3 to operate the window.

To see another example, the mechanism in Figure 4-15a also has 1 degree of freedom. If an independent input is applied to link 1 (*e.g.*, a motor is mounted on joint A to drive link 1), the mechanism will have the a prescribed motion.

4.5 Finite Transformation

Finite transformation is used to describe the motion of a point on rigid body and the motion of the rigid body itself.

4.5.1 Finite Planar Rotational Transformation

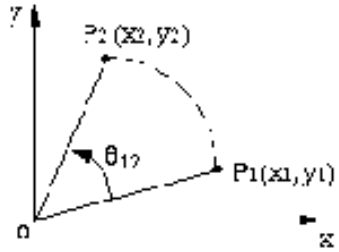


Figure 4-16 Point on a planar rigid body rotated through an angle

Suppose that a point P on a rigid body goes through a rotation describing a circular path from P_1 to P_2 around the origin of a coordinate system. We can describe this motion with a ***rotation operator*** R_{12}:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-6)

where

$$R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-7)

4.5.2 Finite Planar Translational Transformation

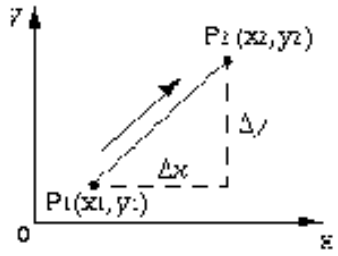


Figure 4-17 Point on a planar rigid body translated through a distance

Suppose that a point *P* on a rigid body goes through a translation describing a straight path from P_1 to P_2 with a change of coordinates of (Δx, Δy). We can describe this motion with a ***translation operator*** T_{12}:

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-8)

where

$$T_{12} = \begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

(4-9)

4.5.3 Concatenation of Finite Planar Displacements

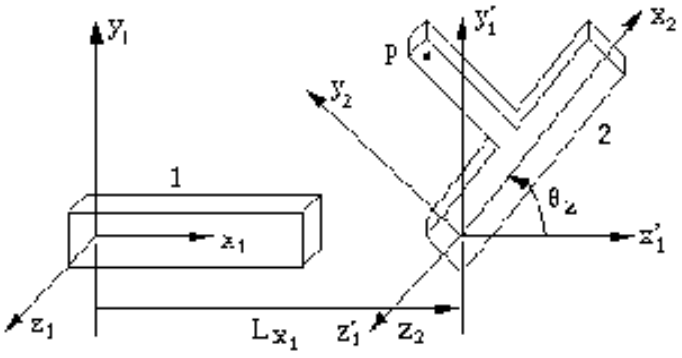


Figure 4-18 Concatenation of finite planar displacements in space

Suppose that a point *P* on a rigid body goes through a rotation describing a circular path from P_1 to P_2' around the origin of a coordinate system, then a translation describing a straight path from P_2' to P_2. We can represent these two steps by

$$\begin{bmatrix} X_2' \\ Y_2' \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-10)

and

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} X_2' \\ Y_2' \\ 1 \end{bmatrix}$$

(4-11)

We can concatenate these motions to get

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = D_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

(4-12)

where D_{12} is the ***planar general displacement operator*** :

$$D_{12} = T_{12} R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-13)

4.5.4 Planar Rigid-Body Transformation

We have discussed various transformations to describe the displacements of a point on rigid body. Can these operators be applied to the displacements of a system of points such as a rigid body?

We used a 3 x 1 homogeneous column matrix to describe a vector representing a single point. A beneficial feature of the planar 3 x 3 translational, rotational, and general displacement matrix operators is that they can easily be programmed on a computer to manipulate a 3 x n matrix of n column vectors representing n points of a rigid body. Since the distance of each particle of a rigid body from every other point of the rigid body is constant, the vectors locating each point of a rigid body must undergo the same transformation when the rigid body moves and the proper axis, angle, and/or translation is specified to represent its motion. (Sandor & Erdman 84). For

example, the general planar transformation for the three points *A, B, C* on a rigid body can be represented by

$$D_{12} = T_{12} R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & \Delta x \\ \sin\theta_{12} & \cos\theta_{12} & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$$

(4-14)

4.5.5 Spatial Rotational Transformation

We can describe a spatial rotation operator for the rotational transformation of a point about an unit axis **u** passing through the origin of the coordinate system. Suppose the rotational angle of the point about **u** is θ, the ***rotation operator*** will be expressed by

$$R_{\theta,u} = \begin{bmatrix} u_x^2 v\theta + c\theta & u_x u_y v\theta - u_z s\theta & u_x u_z v\theta + u_y s\theta & 0 \\ u_x u_y v\theta + u_z s\theta & u_y^2 v\theta + c\theta & u_y u_z v\theta - u_x s\theta & 0 \\ u_x u_z v\theta - u_y s\theta & u_y u_z v\theta + u_x s\theta & u_z^2 v\theta + c\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-15)

where

u_x, u_y, u_z are the othographical projection of the unit axis u on *x*, *y*, and *z* axes, respectively.

$s_\theta = \sin\theta$

$c_\theta = \cos\theta$

$v_\theta = 1 - \cos\theta$

4.5.6 Spatial Translational Transformation

Suppose that a point *P* on a rigid body goes through a translation describing a straight path from P_1 to P_2 with a change of coordinates of (Δx, Δy, Δz), we can describe this motion with a ***translation operator*** *T*:

$$T = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-16)

4.5.7 Spatial Translation and Rotation Matrix for Axis Through the Origin

Suppose a point *P* on a rigid body rotates with an angular displacement about an unit axis **u** passing through the origin of the coordinate system at first, and then followed by a translation *D***u** along **u**. This composition of this rotational transformation and this translational transformation is a screw motion. Its corresponding matrix operator, the ***screw operator***, is a concatenation of the translation operator in Equation 4-7 and the rotation operator in Equation 4-9.

$$S = \begin{bmatrix} u_x^2 v\theta + c\theta & u_x u_y v\theta - u_z s\theta & u_x u_z v\theta + u_y s\theta & L u_x \\ u_x u_y v\theta + u_z s\theta & u_y^2 v\theta + c\theta & u_y u_z v\theta - u_x s\theta & L u_y \\ u_x u_z v\theta - u_y s\theta & u_y u_z v\theta + u_x s\theta & u_z^2 v\theta + c\theta & L u_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-17)

4.6 Transformation Matrix Between Rigid Bodies

4.6.1 Transformation Matrix Between two Arbitray Rigid Bodies

For a system of rigid bodies, we can establish a local Cartesian coordinate system for each rigid body. Transformation matrices are used to describe the relative motion between rigid bodies.

For example, two rigid bodies in a space each have local coordinate systems $x_1y_1z_1$ and $x_2y_2z_2$. Let point *P* be attached to body 2 at location (x_2, y_2, z_2) in body 2's local coordinate system. To find the location of *P* with respect to body 1's local coordinate system, we know that that the point $x_2y_2z_2$ can be obtained from $x_1y_1z_1$ by combining translation L_{x1} along the *x* axis and rotation θz about z axis. We can derive the transformation matrix as follows:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = T_{12} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Lx \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & Lx_1 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

(4-18)

If rigid body 1 is fixed as a frame, a global coordinate system can be created on this body. Therefore, the above transformation can be used to map the local coordinates of a point into the global coordinates.

4.6.2 Kinematic Constraints Between Two Rigid Bodies

The transformation matrix above is a specific example for two unconstrained rigid bodies. The transformation matrix depends on the relative position of the two rigid bodies. If we connect two rigid bodies with a kinematic constraint, their degrees of freedom will be decreased. In other words, their relative motion will be specified in some extent.

Suppose we constrain the two rigid bodies above with a revolute pair as shown in Figure 4-19. We can still write the transformation matrix in the same form as Equation 4-18.

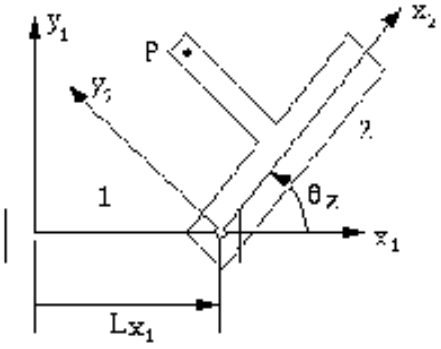


Figure 4-19 Relative position of points on constrained bodies

The difference is that the L_{x1} is a constant now, because the revolute pair fixes the origin of coordinate system $x_2y_2z_2$ with respect to coordinate system $x_1y_1z_1$. However, the rotation θz is still a variable. Therefore, kinematic constraints specify the transformation matrix to some extent.

4.6.3 Denavit-Hartenberg Notation

Denavit-Hartenberg notation (Denavit & Hartenberg 55) is widely used in the transformation of coordinate systems of linkages and robot mechanisms. It can be used to represent the transformation matrix between links as shown in the Figure 4-20.

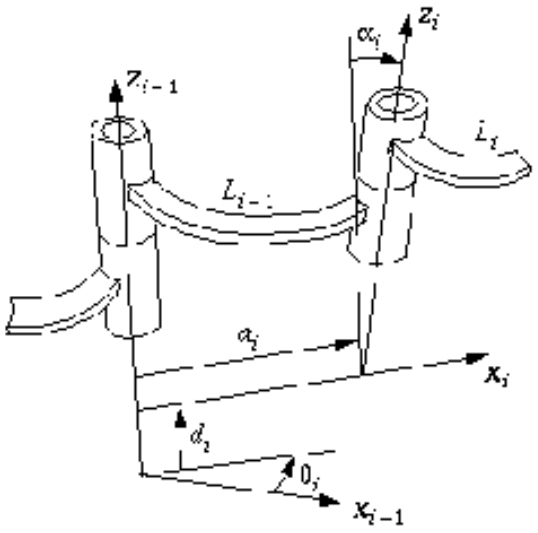


Figure 4-20 Denavit-Hartenberg Notation

In this figure,

- z_{i-1} and z_i are the axes of two revolute pairs;
- θ_i is the included angle of axes x_{i-1} and x_i;
- d_i is the distance between the origin of the coordinate system $x_{i-1}y_{i-1}z_{i-1}$ and the foot of the common perpendicular;
- a_i is the distance between two feet of the common perpendicular;
- α_i is the included angle of axes z_{i-1} and z_i;

The transformation matrix will be $T_{(i-1)i}$

$$T_{(i-1)i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4-19)

The above transformation matrix can be denoted as $T(a_i, \alpha_i, \theta_i, d_i)$ for convenience.

4.6.4 Application of Transformation Matrices to Linkages

A linkage is composed of several constrained rigid bodies. Like a mechanism, a linkage should have a frame. The matrix method can be used to derive the kinematic equations of the linkage. If all the links form a closed loop, the concatenation of all of the transformation matrices will be an identity matrix. If the mechanism has *n* links, we will have:

$$T_{12}T_{23}...T_{(n-1)n} = I$$

Planar Linkages:

5 Planar Linkages

5.1 Introduction

5.1.1 What are Linkage Mechanisms?

Have you ever wondered what kind of mechanism causes the wind shield wiper on the front widow of car to oscillate (Figure 5-1a)? The mechanism, shown in Figure 5-1b, transforms the rotary motion of the motor into an oscillating motion of the windshield wiper.

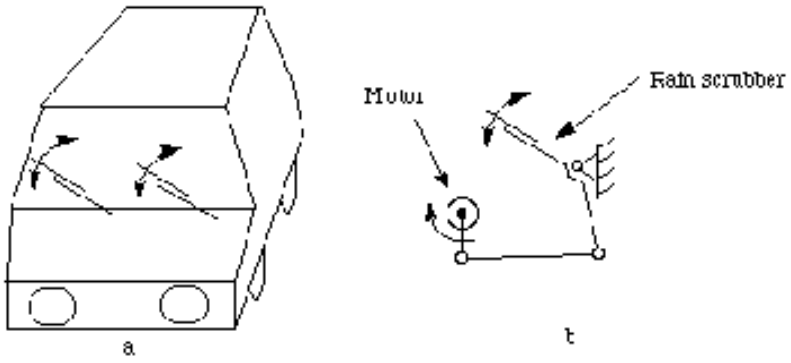


Figure 5-1 Windshield wiper

Let's make a simple mechanism with similar behavior. Take some cardboard and make four strips as shown in Figure 5-2a.

Take 4 pins and assemble them as shown in Figure 5-2b.

Now, hold the 6in. strip so it can't move and turn the 3in. strip. You will see that the 4in. strip oscillates.

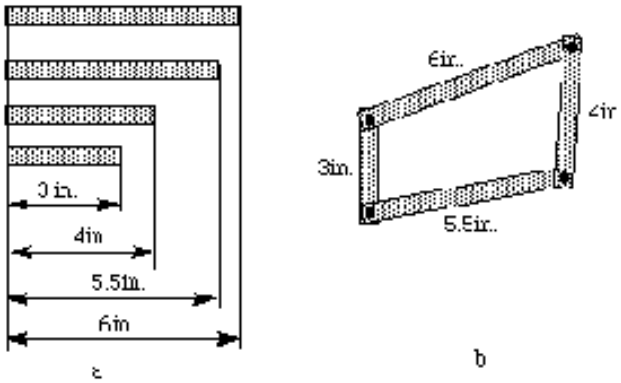


Figure 5-2 Do-it-yourself four bar linkage mechanism

The four bar linkage is the simplest and often times, the most useful mechanism. As we mentioned before, a mechanism composed of rigid bodies and lower pairs is called a linkage (Hunt 78). In planar mechanisms, there are only two kinds of lower pairs --- revolute pairs and prismatic pairs.

The simplest closed-loop linkage is the four bar linkage which has four members, three moving links, one fixed link and four pin joints. A linkage that has at least one fixed link is a mechanism. The following example of a four bar linkage was created in SimDesign in `simdesign/fourbar.sim`

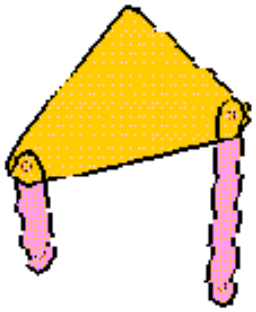


Figure 5-3 Four bar linkage in SimDesign

This mechanism has three moving links. Two of the links are pinned to the frame which is not shown in this picture. In SimDesign, links can be nailed to the background thereby making them into the frame.

How many DOF does this mechanism have? If we want it to have just one, we can impose one constraint on the linkage and it will have a definite motion. The four bar linkage is the simplest and the most useful mechanism.

Reminder: A mechanism is composed of rigid bodies and lower pairs called linkages (Hunt 78). In planar mechanisms there are only two kinds of lower pairs: turning pairs and prismatic pairs.

5.1.2 Functions of Linkages

The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or *vice versa* (Ham *et al.* 58). Stated more specifically linkages may be used to convert:

1. Continuous rotation into continuous rotation, with a constant or variable angular velocity ratio.
2. Continuous rotation into oscillation or reciprocation (or the reverse), with a constant or variable velocity ratio.
3. Oscillation into oscillation, or reciprocation into reciprocation, with a constant or variable velocity ratio.

Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

- Function generation: the relative motion between the links connected to the frame,
- Path generation: the path of a tracer point, or
- Motion generation: the motion of the coupler link.

5.2 Four Link Mechanisms

One of the simplest examples of a constrained linkage is the ***four-link mechanism***. A variety of useful mechanisms can be formed from a four-link mechanism through slight variations, such as changing the character of the pairs, proportions of links, *etc*. Furthermore, many complex link mechanisms are combinations of two or more such mechanisms. The majority of four-link mechanisms fall into one of the following two classes:

1. the four-bar linkage mechanism, and
2. the slider-crank mechanism.

5.2.1 Examples

Parallelogram Mechanism

In a parallelogram four-bar linkage, the orientation of the coupler does not change during the motion. The figure illustrates a loader. Obvioulsy the behavior of maintaining parallelism is important in a loader. The bucket should not rotate as it is raised and lowered. The corresponding SimDesign file is `simdesign/loader.sim`.

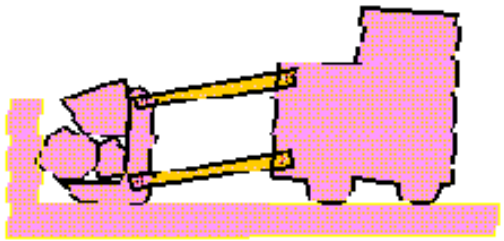


Figure 5-4 Front loader mechanism

Slider-Crank Mechanism

The four-bar mechanism has some special configurations created by making one or more links infinite in length. The slider-crank (or crank and slider) mechanism shown below is a four-bar linkage with the slider replacing an infinitely long output link. The corresponding SimDesign file is `simdesign/slider.crank.sim`.

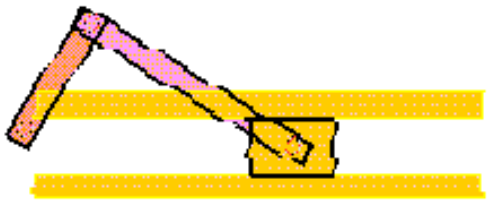


Figure 5-5 Crank and Slider Mechanism

This configuration translates a rotational motion into a translational one. Most mechanisms are driven by motors, and slider-cranks are often used to transform rotary motion into linear motion.

Crank and Piston

You can also use the slider as the input link and the crank as the output link. In this case, the mechanism transfers translational motion into rotary motion. The pistons and crank in an internal combustion engine are an example of this type of mechanism. The corresponding SimDesign file is `simdesign/combustion.sim`.

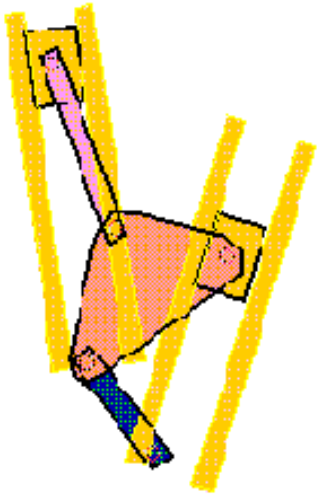


Figure 5-6 Crank and Piston

You might wonder why there is another slider and a link on the left. This mechanism has two dead points. The slider and link on the left help the mechanism to overcome these dead points.

Block Feeder

One interesting application of slider-crank is the block feeder. The SimDesign file can be found in `simdesign/block-feeder.sim`

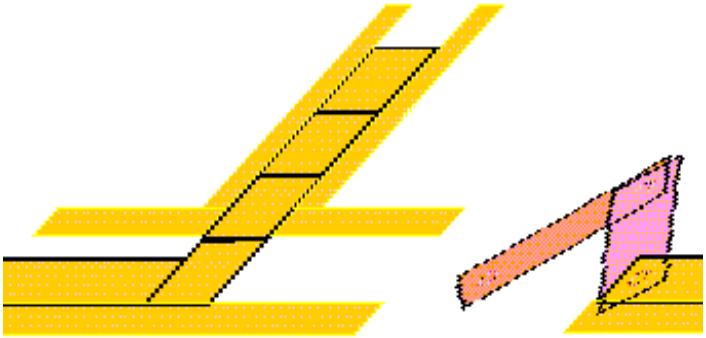


Figure 5-7 Block Feeder

5.2.2 Definitions

In the range of planar mechanisms, the simplest group of lower pair mechanisms are four bar linkages. A ***four bar linkage*** comprises four bar-shaped links and four turning pairs as shown in Figure 5-8.

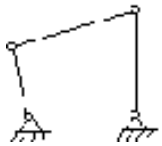


Figure 5-8 Four bar linkage

The link opposite the frame is called the **coupler link**, and the links which are hinged to the frame are called **side links**. A link which is free to rotate through 360 degree with respect to a second link will be said to **revolve** relative to the second link (not necessarily a frame). If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.

Some important concepts in link mechanisms are:

1. Crank: A side link which revolves relative to the frame is called a *crank*.
2. Rocker: Any link which does not revolve is called a *rocker*.
3. Crank-rocker mechanism: In a four bar linkage, if the shorter side link revolves and the other one rocks (*i.e.*, oscillates), it is called a *crank-rocker mechanism*.
4. Double-crank mechanism: In a four bar linkage, if both of the side links revolve, it is called a *double-crank mechanism*.
5. Double-rocker mechanism: In a four bar linkage, if both of the side links rock, it is called a *double-rocker mechanism*.

5.2.3 Classification

Before classifying four-bar linkages, we need to introduce some basic nomenclature.

In a four-bar linkage, we refer to the *line segment between hinges* on a given link as a **bar** where:

- *s* = length of shortest bar
- *l* = length of longest bar
- *p, q* = lengths of intermediate bar

Grashof's theorem states that a four-bar mechanism has *at least* one revolving link if

$$s + l <= p + q \tag{5-1}$$

and all three mobile links will rock if

$$s + l > p + q \tag{5-2}$$

The inequality 5-1 is **Grashof's criterion**.

All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

Case	l + s vers. p + q	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Doubl rocker
4	=	Any	Change point
5	>	Any	Double-rocker

Table 5-1 Classification of Four-Bar Mechanisms

From Table 5-1 we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links. However, this condition is necessary but not sufficient. Mechanisms satisfying this condition fall into the following three categories:

1. When the shortest link is a side link, the mechanism is a crank-rocker mechanism. The shortest link is the crank in the mechanism.
2. When the shortest link is the frame of the mechanism, the mechanism is a double-crank mechanism.

3. When the shortest link is the coupler link, the mechanism is a double-rocker mechanism.

5.2.4 Transmission Angle

In Figure 5-11, if *AB* is the input link, the force applied to the output link, *CD*, is transmitted through the coupler link *BC*. (That is, pushing on the link *CD* imposes a force on the link *AB*, which is transmitted through the link *BC*.) For sufficiently slow motions (negligible inertia forces), the force in the coupler link is pure tension or compression (negligible bending action) and is directed along *BC*. For a given force in the coupler link, the torque transmitted to the output bar (about point *D*) is maximum when the angle θ between coupler bar *BC* and output bar *CD* is $\pi/2$. Therefore, angle BCD is called **transmission angle**.

(5-3)

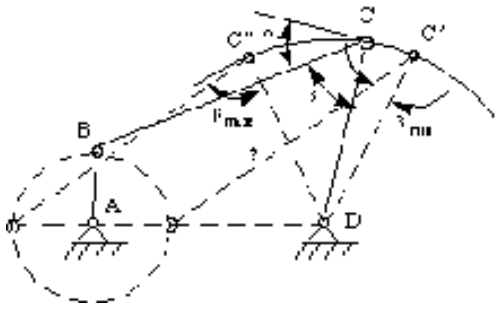


Figure 5-11 Transmission angle

When the *transmission angle* deviates significantly from $\pi/2$, the torque on the output bar decreases and may not be sufficient to overcome the friction in the system. For this reason, the deviation angle $\alpha = |\pi/2 - \theta|$ should not be too great. In practice, there is no definite upper limit for α, because the existence of the inertia forces may eliminate the undesirable force relationships that is present under static conditions. Nevertheless, the following criterion can be followed.

5.2.5 Dead Point

When a side link such as *AB* in Figure 5-10, becomes aligned with the coupler link *BC*, it can only be compressed or extended by the coupler. In this configuration, a torque applied to the link on the other side, *CD*, cannot induce rotation in link *AB*. This link is therefore said to be at a **dead point** (sometimes called a **toggle point**).

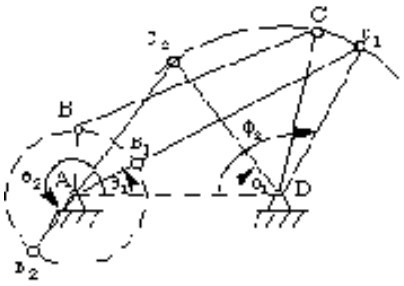


Figure 5-10 Dead point

In Figure 5-11, if *AB* is a crank, it can become aligned with BC in full extension along the line AB_1C_1 or in flexion with AB_2 folded over B_2C_2. We denote the angle *ADC* by $\Phi$and the angle DAB by θ. We use the subscript 1 to denote the extended state and 2 to denote the flexed state of links *AB* and *BC*. In the extended state, link *CD* cannot rotate clockwise without stretching or compressing the theoretically rigid line AC_1. Therefore, link *CD* cannot move into the *forbidden zone* below C_1D, and Φmust be at one of its two extreme positions; in other words, link *CD* is at an extremum. A second extremum of link *CD* occurs with $\Phi = \Phi_1$.

Note that the extreme positions of a side link occur simultaneously with the dead points of the opposite link.

In some cases, the dead point can be useful for tasks such as work fixturing (Figure 5-11).



Figure 5-11 Work fixturing

In other cases, dead point should be and can be overcome with the moment of inertia of links or with the asymmetrical deployment of the mechanism (Figure 5-12).

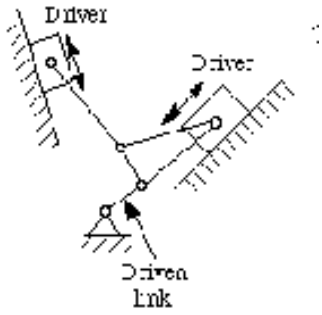


Figure 5-12 Overcoming the dead point by asymmetrical deployment (V engine)

5.2.6 Slider-Crank Mechanism

The slider-crank mechanism, which has a well-known application in engines, is a special case of the crank-rocker mechanism. Notice that if rocker 3 in Figure 5-13a is very long, it can be replaced by a block sliding in a curved slot or guide as shown. If the length of the rocker is infinite, the guide and block are no longer curved. Rather, they are apparently straight, as shown in Figure 5-13b, and the linkage takes the form of the ordinary **slider-crank mechanism**.

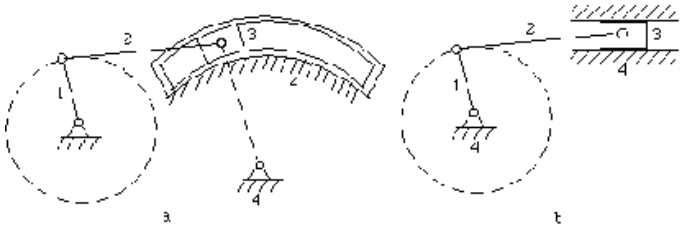


Figure 5-13 Slider-Crank mechanism

5.2.7 Inversion of the Slider-Crank Mechanism

Inversion is a term used in kinematics for a reversal or interchange of form or function as applied to kinematic chains and mechanisms. For example, taking a different link as the fixed link, the slider-crank mechanism shown in Figure 5-14a can be inverted into the mechanisms shown in Figure 5-14b, c, and d. Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in Figure 5-15 is the same as that in Figure 5-14b.

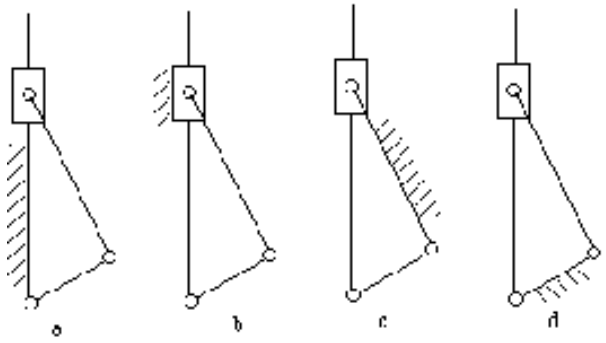


Figure 5-14 Inversions of the crank-slide mechanism

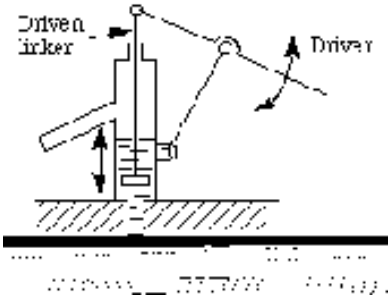


Figure 5-15 A pump device

Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

Cams:

6 Cams

6.1 Introduction

6.1.1 A Simple Experiment: What is a Cam?

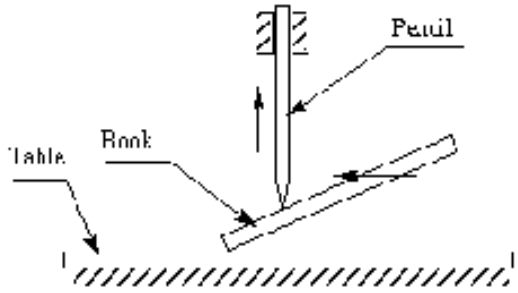


Figure 6-1 Simple Cam experiment

Take a pencil and a book to do an experiment as shown above. Make the book an inclined plane and use the pencil as a slider (use your hand as a guide). When you move the book smoothly upward, what happens to the pencil? It will be pushed up along the guide. By this method, you have transformed one motion into another motion by a very simple device. This is the basic idea of a cam. By rotating the cams in the figure below, the bars will have either translational or oscillatory motion.

6.1.2 Cam Mechanisms

The transformation of one of the simple motions, such as rotation, into any other motions is often conveniently accomplished by means of a **cam mechanism** A cam mechanism usually consists of two moving elements, the cam and the follower, mounted on a fixed frame. Cam devices are

versatile, and almost any arbitrarily-specified motion can be obtained. In some instances, they offer the simplest and most compact way to transform motions.

A ***cam*** may be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element called the ***follower*** . The cam has a very important function in the operation of many classes of machines, especially those of the automatic type, such as printing presses, shoe machinery, textile machinery, gear-cutting machines, and screw machines. In any class of machinery in which automatic control and accurate timing are paramount, the cam is an indispensable part of mechanism. The possible applications of cams are unlimited, and their shapes occur in great variety. Some of the most common forms will be considered in this chapter.

6.2 Classification of Cam Mechanisms

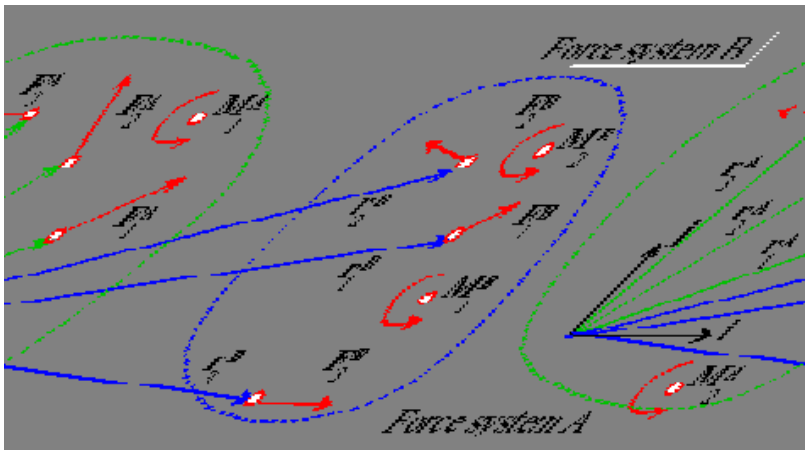
We can classify cam mechanisms by the modes of input/output motion, the configuration and arrangement of the follower, and the shape of the cam. We can also classify cams by the different types of motion events of the follower and by means of a great variety of the motion characteristics of the cam profile. (Chen 82)

Engineering Mechanics:

Statically equivalent force and moment systems:

6.1 Definition of statically equivalent systems of forces and moments

Two force systems that exert the same resultant force and resultant moment are said to be *statically equivalent* (they have the same effect on the motion of a rigid body).



To make this precise, let force system A consist of forces
$\mathbf{F}_1^A, \mathbf{F}_2^A \ldots \mathbf{F}_N^A$
acting at positions
$\mathbf{r}_1^A, \mathbf{r}_2^A \ldots \mathbf{r}_N^A$,
together with a set of moments
$\mathbf{M}_1^A, \mathbf{M}_2^A \ldots \mathbf{M}_n^A$
(we don't need to specify where the moments act).

Let force system B consist of forces
$\mathbf{F}_1^B, \mathbf{F}_2^B \ldots \mathbf{F}_M^B$
acting at positions
$\mathbf{r}_1^B, \mathbf{r}_2^B \ldots \mathbf{r}_M^B$,
together with a set of moments
$\mathbf{M}_1^B, \mathbf{M}_2^B \ldots \mathbf{M}_m^B$
(we don't need to specify where the moments act).

The two force systems are statically equivalent if

$$\mathbf{F}_1^A + \mathbf{F}_2^A + \ldots + \mathbf{F}_N^A = \mathbf{F}_1^B + \mathbf{F}_2^B + \ldots + \mathbf{F}_M^B$$

$$\mathbf{r}_1^A \times \mathbf{F}_1^A + \mathbf{r}_2^A \times \mathbf{F}_2^A + \ldots + \mathbf{r}_N^B \times \mathbf{F}_N^A + \mathbf{M}_1^A + \mathbf{M}_2^A + \ldots + \mathbf{M}_n^A =$$
$$\mathbf{r}_1^B \times \mathbf{F}_1^B + \mathbf{r}_2^B \times \mathbf{F}_2^B + \ldots + \mathbf{r}_M^B \times \mathbf{F}_M^B + \mathbf{M}_1^B + \mathbf{M}_2^B + \ldots + \mathbf{M}_n^B$$

or in a rather more convenient shorthand notation

$$\sum_{i=1}^{N} \mathbf{F}_i^A = \sum_{i=1}^{M} \mathbf{F}_i^B$$

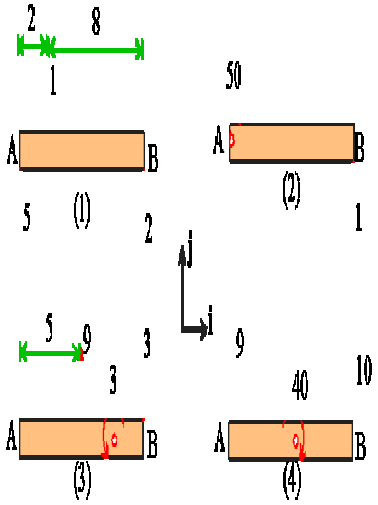
$$\sum_{i=1}^{N} \mathbf{r}_i^A \times \mathbf{F}_i^A + \sum_{i=1}^{n} \mathbf{M}_i^A = \sum_{i=1}^{M} \mathbf{r}_i^B \times \mathbf{F}_i^B + \sum_{i=1}^{m} \mathbf{M}_i^B$$

Note that

1. *Both force and moment sums must be equal*

2. *It doesn't matter what point you take moments about when testing for static equivalence, but you must take moments about the same point for both force systems.*

Here's a simple problems to check that you understand the concept. The picture below shows four different force systems acting on the same beam AB. All forces are specified in N, distances in m, and moments in Nm. Find which force systems are statically equivalent.



All we need to do is to calculate the resultant force and moment for each force system (1)-(4). It's easy to see that

1. Force system 1: resultant force 6j ;
Resultant moment about A 18k

2.Force system 2: resultant force 1j ;
Resultant moment about A 60k

3. Force system 3: resultant force 6j ;
Resultant moment about A 18k

4. Force system 4: resultant force 1j ;
Resultant moment about A 60k

Therefore, force systems (1) and (3) are statically equivalent. Also, force systems (2) and (4) are statically equivalent.

6.2 Replacing complex force and moment systems with simpler statically equivalent systems

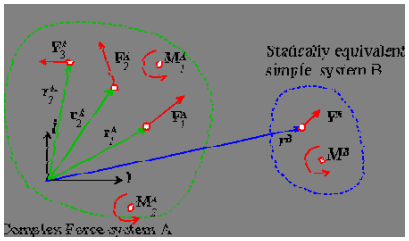
What's the use of this definition? Well, we often find it convenient to replace a complex force system with a simpler, statically equivalent system.

In the preceding section, we just checked to see whether a bunch of force systems were statically equivalent. Usually, however, we are interested in *finding* a simple force system (B, say) that replaces a complex force system (A).

To this end, the following result is helpful

Any complex system of forces and moments is statically equivalent to a single force acting at some specified position, together with a moment.

To use this result, you need to choose where you want the equivalent force to act. Then you can calculate the equivalent force and moment. The moment can act wherever you like, but its magnitude and direction will depend on where you choose to put the force.



In fact, we can find general formulas for the equivalent force and moment. Let the complex force system A consist of forces

$\mathbf{F}_1^A, \mathbf{F}_2^A \ldots \mathbf{F}_N^A$

acting at positions

$\mathbf{r}_1^A, \mathbf{r}_2^A .. \mathbf{r}_N^A,$
together with a set of moments
$\mathbf{M}_1^A, \mathbf{M}_2^A .. \mathbf{M}_n^A.$
Suppose we choose to replace force system A by a single force
$\mathbf{F}^B$
acting at some convenient point
$\mathbf{r}^B,$
together with a moment
$\mathbf{M}^B.$

Clearly the formula for $\mathbf{F}_B$ is simply

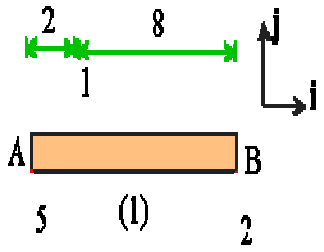
$$\mathbf{F}^B = \sum_{i=1}^{N} \mathbf{F}_i^A$$

whereupon we find the following formula for $\mathbf{M}^B$

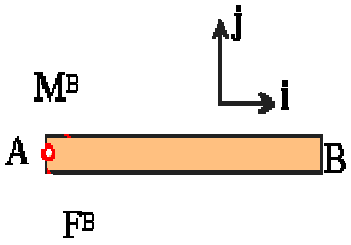
$$\mathbf{M}_i^B = \sum_{i=1}^{N} \mathbf{r}_i^A \times \mathbf{F}_i^A + \sum_{i=1}^{n} \mathbf{M}_i^A - \mathbf{r}^B \times \mathbf{F}^B$$

Let's work through a few examples

Example 1: Replace the force system shown with a statically equivalent force and moment acting (a) at point A; (b) acting at point B



It's quicker to solve this problem by interpreting the meaning of the formulas than by cranking through the vector algebra. The figure shows statically equivalent forces and moments acting at A. We need to determine the unknown force and moment.



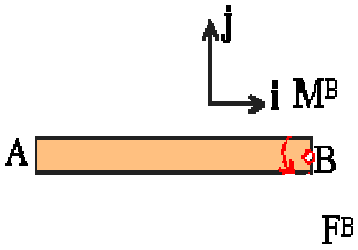
The formulas say:

(1) $\mathbf{F}^B$ is equal to the resultant of the complex force system. Thus, $\mathbf{F}^B = 6\mathbf{j}$

(2) $\mathbf{M}^B$ is equal to the resultant moment of the complex force system, minus the moment of $\mathbf{F}^B$. It doesn't matter what point we take moments about to do this computation. For the sake of variety, let's take moments about B. The moment of the complex force system about B is -42**k.** The moment of $\mathbf{F}^B$ about B is -60**k**. Therefore

$$\mathbf{M}^B = -42\mathbf{k} - (-60\mathbf{k})$$
$$= 18\mathbf{k}$$

Now let's repeat the exercise to get statically equivalent forces at B.



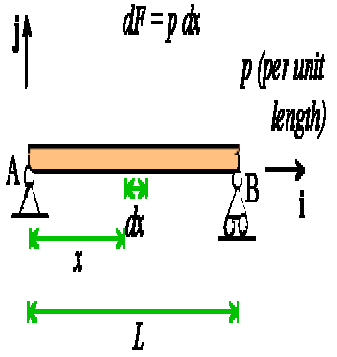
(1) $\mathbf{F}^B$ is equal to the resultant of the complex force system. Thus, $\mathbf{F}^B = 6\mathbf{j}$

(2) $\mathbf{M}^B$ is equal to the resultant moment of the complex force system, minus the moment of $\mathbf{F}^B$. The moment of the complex force system about B is -42**k.** The moment of $\mathbf{F}^B$ about B is zero. Therefore $\mathbf{M}^B = -42\mathbf{k}$.

Notice that the statically equivalent *force* is always the same, regardless of where you put it. The *moment* depends on where you choose to put the force, however.

Example 2: In practice, statically equivalent force systems are used mostly to simplify *distributed* loading. We've already seen that it's a pain to calculate resultant forces and moments of a distributed load. In any problem that involves distributed loading, we usually start by replacing the distributed load by a statically equivalent force and moment (or, as we shall see below, we can sometimes replace the distributed load by a single force acting at a special point).

For example, let's replace the distributed force shown below by a statically equivalent force and moment acting at A.

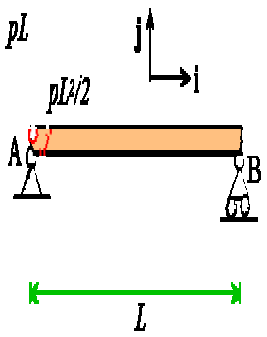


We've already calculated the resultant force and moment of the distributed force. Remember that

(1) The resultant force is $\mathbf{F}^A = -pL\mathbf{j}$

(2) The resultant moment about A is $\mathbf{M}^A = -(pL^2/2)\ \mathbf{k}$

Therefore, the distributed force is statically equivalent to a force $\mathbf{F}^A = -pL\mathbf{j}$ and moment $\mathbf{F}^A = -(pL^2/2)\mathbf{k}$ acting at A, as shown below.



If we needed to calculate reaction forces acting at A and B, for example, we could now work with this simpler equivalent force system and forget about the pressure.

6.3 An Application – Centers of Gravity for a collection of solid objects

Sometimes it's more convenient to try to replace a complex force system by a single force, *which is chosen to act at a special point, such that the moment vanishes.* Actually, this can't always be done – it only works for special force systems. Examples include

1. Any 2D system of forces and moments
2. Any set of parallel forces (eg gravity loading)
3. Any pressure distribution acting normal to a surface.
4. Any set of forces that act through a single point

It's possible to derive a general formula for the force and its position, but the derivation is a bit devious and the results aren't particularly useful. If you have a strong background in vectors and are up for a challenge you might try to derive the result yourself – it's a neat vector math problem. If you give up the full scoop is here.

By far the most common situation where we need to work through this exercise is to find the *center of gravity* of a collection of objects. In this case we are actually replacing the forces of

gravity (which act on each object) with a statically equivalent force, exerting the same resultant force (the total weight), and acting at a special point (the center of gravity) chosen to ensure that the equivalent force also exerts the same resultant moment.

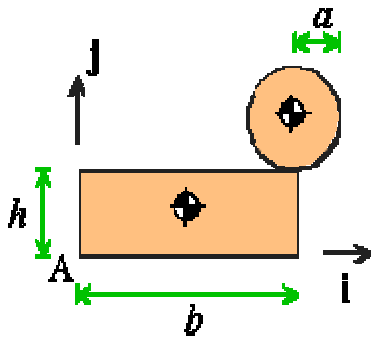
> Most practical engineering applications involve small objects close to the earth's surface. In this case the force of gravity can be taken to act in a fixed direction. Under these conditions the center of gravity is the same as the center of mass.

There's a simple formula for the position of the center of mass. Suppose we have a collection of objects, with masses $m_1, m_2 \ldots m_N$, centers of mass located at $\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_N$. The position of the center of mass is for the collection of objects is

$$\mathbf{r}_C = \frac{1}{m_1 + m_2 + \ldots m_N}\left(m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \ldots m_N\mathbf{r}_N\right)$$

The same formula works with mass replaced by weight.

Example 1: Calculate the position of the center of mass of the object shown below, expressing your answer as coordinates relative to point A in the basis shown. Evaluate the answer for the particular geometry $h=a=b$. All parts have the same mass density.



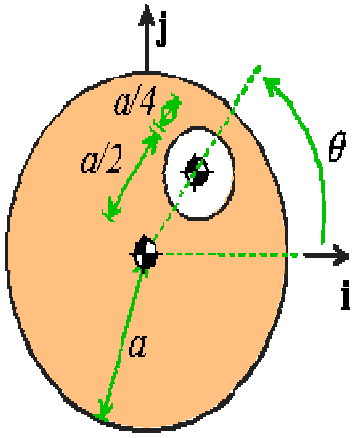
The center of gravity of each of the two parts is shown in the picture. It's convenient to do the calculation in a table. We will denote the mass per unit area of each object by ρ

The CG position follows by dividing the sum of the last column by the sum of the second

$$\mathbf{r}_{CG} = \frac{1}{hb + \pi a^2}\left[b(hb/2 + \pi a^2)\mathbf{i} + (h^2 b/2 + \pi a^2(a+h))\mathbf{j}\right]$$

For the particular case when $a=h=b$ we find

Example 2: Calculate the position of the CG of a disk with a hole in, shown below. Express your answer in terms of a and θ as coordinates relative to the center of the disk.



There's a really neat trick for dealing with a hole – we can regard it as an object with *negative density*!. So if the disk has mass density μ per unit area, we treat the hole as an object with mass density $-\mu$ per unit area, located at the position shown.

So the CG position is

$$-(a/30)\left[\cos\theta\mathbf{i}+\sin\theta\mathbf{j}\right]$$

Example 3: Aircraft weight and balance.

If you fly airplanes, you will regularly need to calculate the position of the center of gravity for your aircraft. The center of gravity moves if you load the plane with passengers, fuel and baggage. It's very important to keep the center of gravity within prescribed limits – if you don't do so, the aircraft may be unable to recover from a stall, or may even become uncontrollable.

FAA regulations require an accurate *weight and balance* record for each aircraft, which is an official record of the planes empty weight, and the position of its center of gravity when empty. The pilots operating handbook specifies the positions of passenger seats, fuel containers and baggage compartment. This information can be used to determine the position of the center of gravity in any loaded configuration. The position can then be compared with allowable limits.

As an example, the picture below gives data for a Cessna Skyhawk C-172N. All positions are reported as distances behind the firewall, in inches. **THIS INFORMATION IS NOT TO BE USED FOR ACTUAL FLIGHT PLANNING PURPOSES. ALWAYS CONSULT THE POH FOR YOUR OWN AIRCRAFT.**

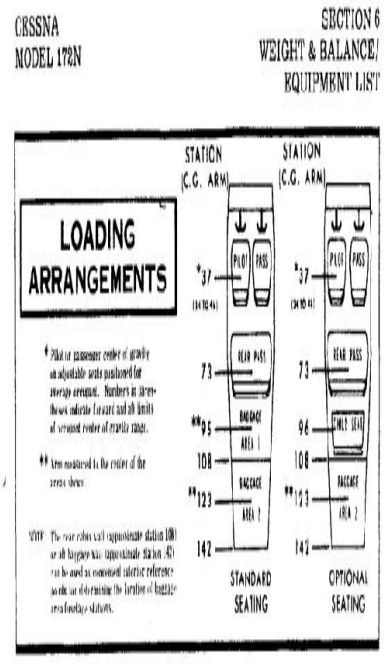


Figure 6-3. Loading Arrangements

The operating manual also specifies admissible ranges for the position of the CG. The data for a C-172N is shown below.

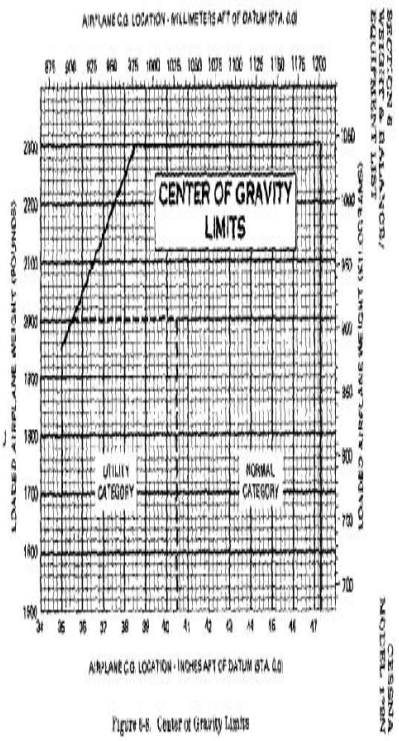


Figure 6-8. Center of Gravity Limits

In addition to the data shown above, each aircraft has to be carefully weighed, with all its equipment on board, but without fuel, oil, passengers and baggage, to find the *basic empty weight (BEW);* and the *basic empty CG arm*.

Let's suppose our airplane has a BEW of 1454lb, and a CG arm of 39.6 inches.

Suppose we plan a family trip with the following loading

1. Pilot – 150lb
2. Front seat passenger – 180lb
3. Rear seat passengers 80lb+60lb

4. Baggage 20lb

Basic empty weight (includes unusable fuel & oil) 1454lb

Fuel capacity – 43 gal, fuel tank CG arm 46 inches behind firewall datum

Fuel weight 6lb/gal

Is the CG within limits with full fuel? Will it remain within limits as fuel is consumed?

We can use our formula to calculate the distance of the CG behind the firewall

For this calculation we only need to work with one component of the position vector – the distance behind the firewall

The CG position with full fuel follows as

$$\mathbf{r}_G = 93776.4\mathbf{i} / 2202 = 42.58\mathbf{i}$$

If all the fuel is used up the CG will move to

$$\mathbf{r}_G = (93776.4\mathbf{i} - 11868\mathbf{i}) / (2202 - 258) = 42.13\mathbf{i}$$

(FAA regulations require that you use at least a 30 min fuel reserve while planning a daytime VFR flight; 45 mins for instrument flight or at night. The laws of physics allow you to use all the fuel, but generally punish any attempt to use more than the total fuel available rather severely)

We can check the position of the CG against the allowable range specified in the operators manual. We are safely within limits both with full fuel and empty. As fuel is used up, the CG moves along a straight line between the two end points, so will remain within limits throughout the flight.

6.4 Simple problems involving replacing distributed loads with statically equivalent forces

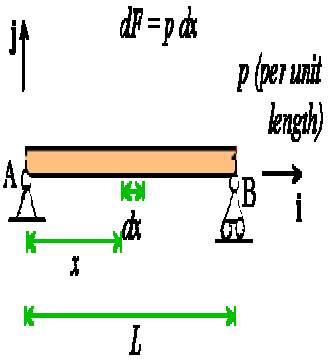
We showed in an earlier example how a distributed load, or pressure, can be replaced by a statically equivalent force and moment. Often, it is preferable to replace the distributed load using a statically equivalent *force* only, which acts at a special point that is chosen so that the moment vanishes.

We've already seen the results of such calculations, in our discussion of gravitational loading (distributed gravity load is replaced by a force acting at the CG); hydrostatic loading (distributed fluid pressure is replaced by a buoyancy force acting at the center of buoyancy) and drag and lift force (distributed loading replaced by equivalent drag and lift forces acting at center of lift).

So far, we don't know how to actually calculate the location of the special point (CG, center of buoyancy, etc).

In this section we show how to do this for simple 1-D distributed loads. Real life is 3D, unfortunately, and you will learn how to solve 3D problems in later courses.

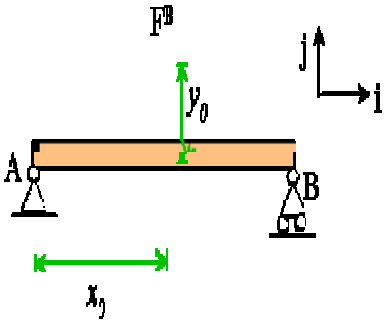
As a simple example, let's find the location of the statically equivalent force for a uniform pressure acting on a beam. The beam has length *L* and is subjected to a distributed load *p* per unit length. We'll work with the coordinate system shown in the picture.



We already know that

(1) The resultant force is $-pL\mathbf{j}$

(2) The resultant moment about A is $-(pL^2/2)\mathbf{k}$



We now need to find a statically equivalent system, which consists of a single force $\mathbf{F}_B$ acting at some special point $\mathbf{r}^B$, chosen so that $\mathbf{F}_B$ has the same resultant force and moment.

For $\mathbf{F}_B$ to have the same resultant as the distributed load, we need to set $\mathbf{F}^B = -pL\mathbf{j}$

To find $\mathbf{r}^B$ we can set $\mathbf{r}^B = x_0\mathbf{i} + y_0\mathbf{j}$ where (x_0, y_0) are the unknown components of the position vector of $\mathbf{F}_B$. The moment of $\mathbf{F}_B$ follows as

$$\mathbf{M} = (x_0\mathbf{i} + y_0\mathbf{j}) \times (-pL\mathbf{j}) = -x_0 pL\mathbf{k}$$

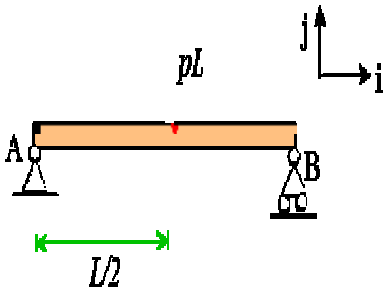
We need to make this moment equal to that of the distributed pressure, so that

$$\mathbf{M} = -x_0 pL\mathbf{k} = -\frac{1}{2} pL^2\mathbf{k}$$

$$\Rightarrow x_0 = \frac{L}{2}$$

How about y_0? We didn't get an equation for it, so *any* value of y_0 will do. The reason for this, of course, is that you can always move the force $\mathbf{F}_B$ parallel to itself without changing its resultant moment. Physically it makes sense to think of the equivalent force as acting on the beam itself, so it makes sense to choose $y_0 = 0$.

The statically equivalent point force and its location are shown in the picture.



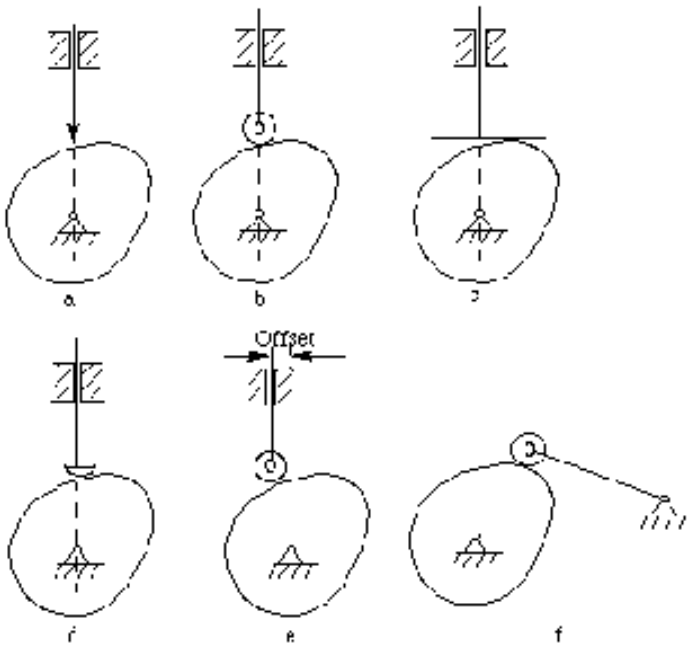


Figure 6-2 Classification of cam mechanisms

4.2.1 Modes of Input/Output Motion

1. Rotating cam-translating follower. (Figure 6-2a,b,c,d,e)
2. Rotating follower (Figure 6-2f):
   The follower arm swings or oscillates in a circular arc with respect to the follower pivot.
3. Translating cam-translating follower (Figure 6-3).
4. Stationary cam-rotating follower:
   The follower system revolves with respect to the center line of the vertical shaft.

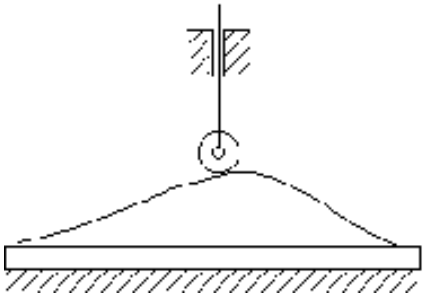


Figure 6-3 Translating cam - translating follower

6.2.1 Follower Configuration

1. Knife-edge follower (Figure 6-2a)
2. Roller follower (Figure 6-2b,e,f)
3. Flat-faced follower (Figure 6-2c)
4. Oblique flat-faced follower
5. Spherical-faced follower (Figure 6-2d)

6.2.2 Follower Arrangement

1. In-line follower:
   The center line of the follower passes through the center line of the camshaft.
2. Offset follower:
   The center line of the follower does not pass through the center line of the cam shaft. The amount of offset is the distance between these two center lines. The offset causes a reduction of the side thrust present in the roller follower.

6.2.3 Cam Shape

1. Plate cam or disk cam:
   The follower moves in a plane perpendicular to the axis of rotation of the camshaft. A translating or a swing arm follower must be constrained to maintain contact with the cam profile.
2. Grooved cam or closed cam (Figure 6-4):
   This is a plate cam with the follower riding in a groove in the face of the cam.

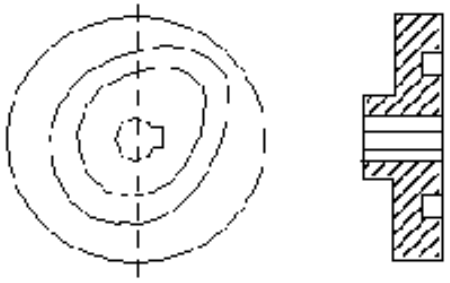


Figure 6-4 Grooved cam

3. Cylindrical cam or barrel cam (Figure 6-5a):
   The roller follower operates in a groove cut on the periphery of a cylinder. The follower may translate or oscillate. If the cylindrical surface is replaced by a conical one, a conical cam results.
4. End cam (Figure 6-5b):
   This cam has a rotating portion of a cylinder. The follower translates or oscillates, whereas the cam usually rotates. The end cam is rarely used because of the cost and the difficulty in cutting its contour.

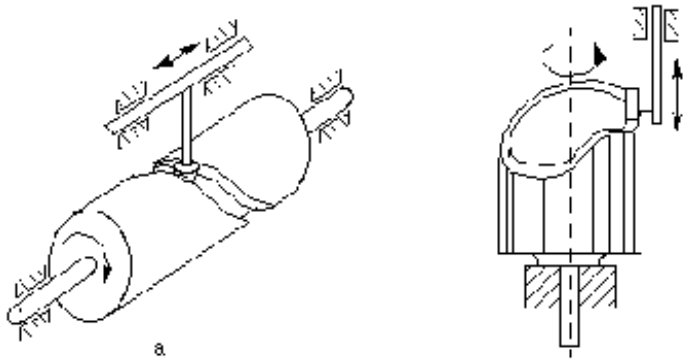


Figure 6-5 Cylindrical cam and end cam

6.2.4 Constraints on the Follower

1. Gravity constraint:
   The weight of the follower system is sufficient to maintain contact.
2. Spring constraint:
   The spring must be properly designed to maintain contact.
3. Positive mechanical constraint:
   A groove maintains positive action. (Figure 6-4 and Figure 6-5a) For the cam in Figure 6-6, the follower has two rollers, separated by a fixed distance, which act as the constraint; the mating cam in such an arrangement is often called a *constant-diameter cam*.

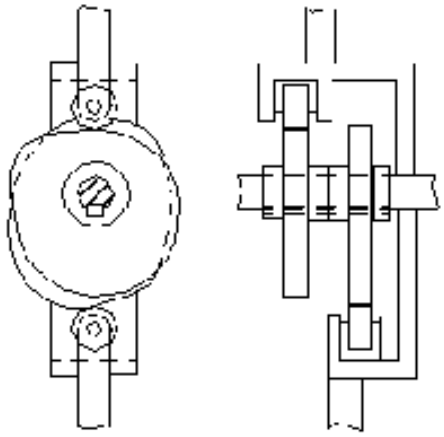


Figure 6-6 Constant diameter cam

A mechanical constraint cam also be introduced by employing a dual or conjugate cam in arrangement similar to what shown in Figure 6-7. Each cam has its own roller, but the rollers are mounted on the same reciprocating or oscillating follower.

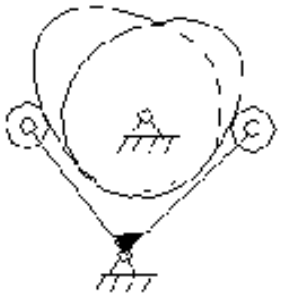


Figure 6-7 Dual cam

6.2.5 Examples in SimDesign

Rotating Cam, Translating Follower

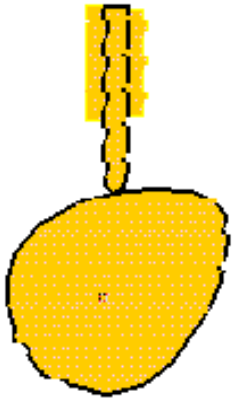


Figure 6-8 SimDesign translating cam

Load the SimDesign file `simdesign/cam.translating.sim`. If you turn the cam, the follower will move. The weight of the follower keeps them in contact. This is called a *gravity constraint* cam.

Rotating Cam/Rotating Follower

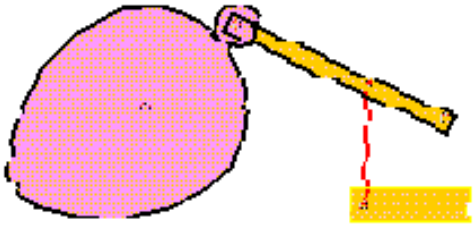


Figure 6-9 SimDesign oscillating cam

The SimDesign file is `simdesign/cam.oscillating.sim`. Notice that a roller is used at the end of the follower. In addition, a spring is used to maintain the contact of the cam and the roller.

If you try to calculate the degrees of freedom (DOF) of the mechanism, you must imagine that the roller is welded onto the follower because turning the roller does not influence the motion of the follower.

6.3 Cam Nomenclature

Figure 6-10 illustrates some cam nomenclature:

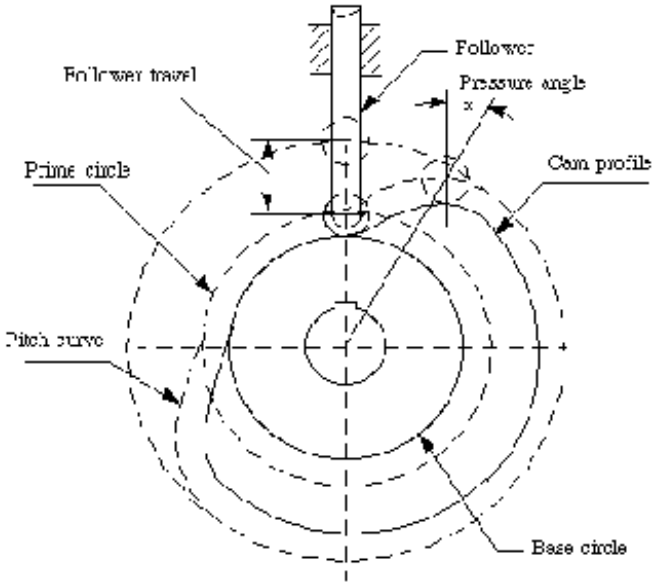


Figure 6-10 Cam nomenclature

- Trace point: A theoretical point on the follower, corresponding to the point of a fictitious *knife-edge follower*. It is used to generate the *pitch curve*. In the case of a *roller follower*, the trace point is at the center of the roller.
- Pitch curve: The path generated by the trace point at the follower is rotated about a stationary cam.
- Working curve: The working surface of a cam in contact with the follower. For the *knife-edge follower* of the plate cam, the *pitch curve* and the *working curves* coincide. In a *close or grooved cam* there is an *inner profile* and an *outer working curve*.
- Pitch circle: A circle from the cam center through the pitch point. The pitch circle radius is used to calculate a cam of minimum size for a given *pressure angle*.
- Prime circle (reference circle): The smallest circle from the cam center through the pitch curve.
- Base circle: The smallest circle from the cam center through the cam profile curve.
- Stroke or throw:The greatest distance or angle through which the follower moves or rotates.

- Follower displacement: The position of the follower from a specific zero or rest position (usually its the position when the *f* ollower contacts with the *base circle* of the cam) in relation to time or the rotary angle of the cam.
- Pressure angle: The angle at any point between the normal to the pitch curve and the instantaneous direction of the follower motion. This angle is important in cam design because it represents the steepness of the cam profile.

6.4 Motion events

When the cam turns through one motion cycle, the follower executes a series of events consisting of rises, dwells and returns. **Rise** is the motion of the follower away from the cam center, **dwell** is the motion during which the follower is at rest; and **return** is the motion of the follower toward the cam center.

There are many follower motions that can be used for the rises and the returns. In this chapter, we describe a number of basic curves.

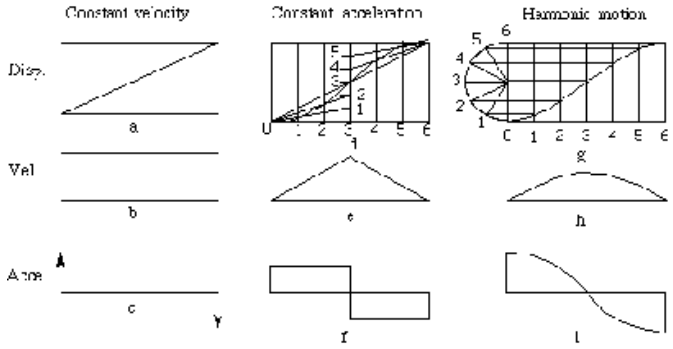


Figure 6-11 Motion events

Notation

Φ: The rotary angle of the cam, measured from the beginning of the motion event;

Φ0: The range of the rotary angle corresponding to the motion event;

h : The stoke of the motion event of the follower;

S : Displacement of the follower;

V : Velocity of the follower;

A : Acceleration of the follower.

6.4.1 Constant Velocity Motion

If the motion of the follower were a straight line, Figure 6-11a,b,c, it would have equal displacements in equal units of time, *i.e.*, uniform velocity from the beginning to the end of the stroke, as shown in b. The acceleration, except at the end of the stroke would be zero, as shown

in c. The diagrams show abrupt changes of velocity, which result in large forces at the beginning and the end of the stroke. These forces are undesirable, especially when the cam rotates at high velocity. The *constant velocity motion* is therefore only of theoretical interest.

$$S(\varphi) = h\frac{\varphi}{\beta}$$
$$V(\varphi) = \frac{h}{\beta}$$
$$A(\varphi) = 0$$

(6-1)

6.4.2 Constant Acceleration Motion

Constant acceleration motion is shown in Figure 6-11d, e, f. As indicated in e, the velocity increases at a uniform rate during the first half of the motion and decreases at a uniform rate during the second half of the motion. The acceleration is constant and positive throughout the first half of the motion, as shown in f, and is constant and negative throughout the second half. This type of motion gives the follower the smallest value of maximum acceleration along the path of motion. In high-speed machinery this is particularly important because of the forces that are required to produce the accelerations.

When

$$0 \le \varphi \le \frac{\beta}{2},$$

$$S(\varphi) = 2h\frac{\varphi^2}{\beta^2}$$
$$V(\varphi) = \frac{4h}{\beta^2}\varphi$$
$$A(\varphi) = \frac{4h}{\beta^2}$$

(6-2)

When

$$\frac{\beta}{2} \le \varphi \le \beta,$$

$$S(\varphi) = h - \frac{2h}{\beta^2}(\beta - \varphi)^2$$
$$V(\varphi) = \frac{4h}{\beta}\left(1 - \frac{\varphi}{\beta}\right)$$
$$A(\varphi) = -\frac{4h}{\beta^2}$$

(6-3)

6.4.3 Harmonic Motion

A cam mechanism with the basic curve like g in Figure 6-7g will impart *simple harmonic motion* to the follower. The velocity diagram at h indicates smooth action. The acceleration, as shown at i, is maximum at the initial position, zero at the mid-position, and negative maximum at the final position.

$$S(\varphi) = \frac{h}{2}\left(1 - \cos\frac{\pi\varphi}{\beta}\right)$$
$$V(\varphi) = \frac{h\pi}{2\beta}\sin\frac{\pi\varphi}{\beta}$$
$$A(\varphi) = \frac{h\pi^2}{2\beta^2}\cos\frac{\pi\varphi}{\beta}$$

(6-4)

6.5 Cam Design

The translational or rotational displacement of the follower is a function of the rotary angle of the cam. A designer can define the function according to the specific requirements in the design. The motion requirements, listed below, are commonly used in cam profile design.

6.5.1 Disk Cam with Knife-Edge Translating Follower

Figure 6-12 is a skeleton diagram of a disk cam with a knife-edge translating follower. We assume that the cam mechanism will be used to realize the displacement relationship between the rotation of the cam and the translation of the follower.

Figure 6-12 A Skeleton Diagram of disk cam with knife-edge translation

Below is a list of the essential parameters for the evaluation of these types of cam mechanisms. However, these parameters are adequate only to define a knife-edge follower and a translating follower cam mechanism.

Parameters:

r_o: The radius of the base circle;

e: The offset of the follower from the rotary center of the cam. Notice: it could be negative.

s: The displacement of the follower which is a function of the rotary angle of the cam -- Φ.

IW: A parameter whose absolute value is 1. It represents the turning direction of the cam. When the cam turns clockwise: *IW=+1*, otherwise: *IW=-1*.

Cam profile design principle:

The method termed inversion is commonly used in cam profile design. For example, in a disk cam with translating follower mechanism, the follower translates when the cam turns. This means that the relative motion between them is a combination of a relative turning motion and a relative translating motion. Without changing this feature of their relative motion, imagine that the cam remains fixed. Now the follower performs both the relative turning and translating motions. We have inverted the mechanism.

Furthermore, imagine that the knife-edge of the follower moves along the fixed cam profile in the inverted mechanism. In other words, the knife edge of the follower draws the profile of the cam. Thus, the problem of designing the cam profile becomes a problem of calculating the trace of the knife edge of the follower whose motion is the combination of the relative turning and the relative translating.

Design equations:

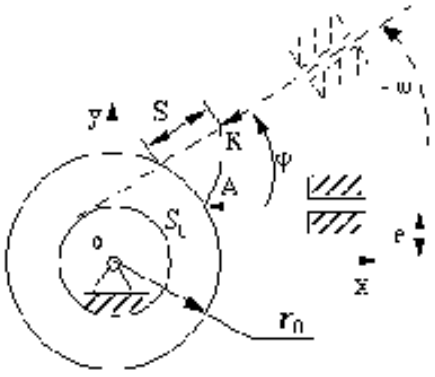


Figure 6-13 Profile design of translating cam follower

In Figure 6-13, only part of the cam profile *AK* is displayed. Assume the cam turns clockwise. At the beginning of motion, the knife edge of the follower contacts the point of intersection *A* of the base circle and the cam profile. The coordinates of *A* are (*So*, *e*), and *So* can be calculated from equation $S_0 = \sqrt{r_0^2 - e^2}$

Suppose the displacement of the follower is *S* when the angular displacement of the cam is ϕ. At this moment, the coordinates of the knife edge of the follower should be (*So* + *S*, *e*).

To get the corresponding position of the knife edge of the follower in the inverted mechanism, turn the follower around the center of the cam in the reverse direction through an angle of ϕ. The knife edge will be inverted to point *K*, which corresponds to the point on the cam profile in the inverted mechanism. Therefore, the coordinates of point *K* can be calculated with the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(IW \cdot \phi) & -\sin(IW \cdot \phi) \\ \sin(IW \cdot \phi) & \cos(IW \cdot \phi) \end{bmatrix} \begin{bmatrix} S_0 + S \\ e \end{bmatrix} \tag{6-5}$$

Note:

- The offset *e* is negative if the follower is located below the *x* axis.
- When the rotational direction of the cam is clockwise: *IW* = *+1*, otherwise: *IW* = *-1*.

6.5.2 Disk Cam with Oscillating Knife-Edge Follower

Suppose the cam mechanism will be used to make the knife edge oscillate. We need to compute the coordinates of the cam profile that results in the required motion of the follower.

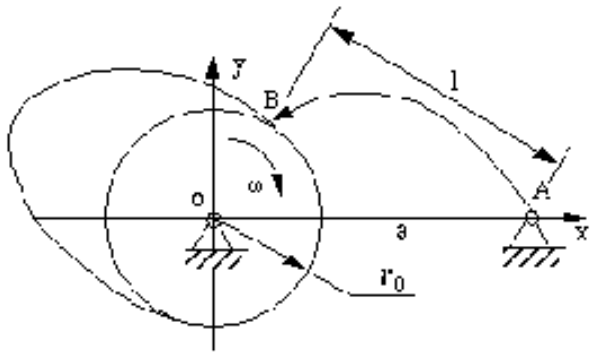


Figure 6-14 Disk cam with knife-edge oscillating follower

The essential parameters in this kind of cam mechanisms are given below.

r_o: The radius of the base circle;

a: The distance between the pivot of the cam and the pivot of the follower.

l: The length of the follower which is a distance from its pivot to its knife edge.

ψ: The angular displacement of the follower which is a function of the rotary angle of the cam -- ϕ.

IP: A parameter whose absolute value is 1. It represents the location of the follower. When the follower is located above the *x* axis: *IP=+1*, otherwise: *IP=-1*.

IW: A parameter whose absolute value is 1. It represents the turning direction of the cam. When the cam turns clockwise: *IW=+1*, otherwise: *IW=-1*.

Cam profile design principle

The fundamental principle in designing the cam profiles is still inversion, similar to that that for designing other cam mechanisms, (*e.g.*, the translating follower cam mechanism). Normally, the follower oscillates when the cam turns. This means that the relative motion between them is a combination of a relative turning motion and a relative oscillating motion. Without changing this feature of their relative motion, let the cam remain fixed and the follower performs both the relative turning motion and oscillating motion. By imagining in this way, we have actually inverted the mechanism.

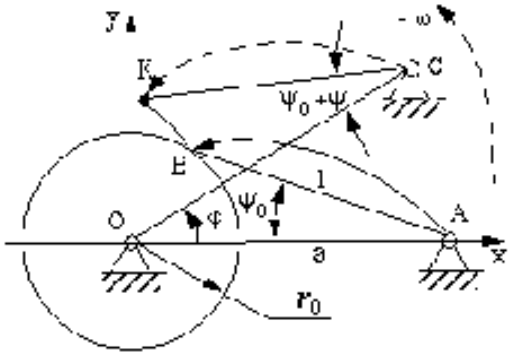


Figure 6-15 Cam profile design for a rotating follower

In Figure 6-15, only part of the cam profile *BK* is shown. We assume that the cam turns clockwise.

At the beginning of motion, the knife edge of the follower contacts the point of intersection (*B*) of the base circle and the cam profile. The initial angle between the follower (*AB*) and the line of two pivots (*AO*) is ψ_0. It can be calculated from the triangle *OAB*.

When the angular displacement of the cam is Φ, the oscillating displacement of the follower is ψ which measures from its own initial position. At this moment, the angle between the follower and the line passes through two pivots should be $\psi + \psi 0$.

The coordinates of the knife edgeat this moment will be

$$(a - l\cos[IP(\psi + \psi_0)], l\sin[IP(\psi + \psi_0)])$$

(6-6)

To get the corresponding knife-edge of the follower in the inverted mechanism, simply turn the follower around the center of the cam in the reverse direction of the cam rotation through an angle of Φ. The knife edge will be inverted to point *K* which corresponds to the point on the cam profile in the inverted mechanism. Therefore, the coordinates of point *K* can be calculated with the following equation:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(IW \cdot \varphi) & -\sin(IW \cdot \varphi) \\ \sin(IW \cdot \varphi) & \cos(IW \cdot \varphi) \end{bmatrix} \begin{bmatrix} a - l\cos[IP(\psi + \psi_0)] \\ l\sin[IP(\psi + \psi_0)] \end{bmatrix}$$

(6-7)

Note:

- When the initial position of the follower is above the *x* axis, *IP* = *+1*, otherwise: *IP* = *-1*.
- When the rotary direction of the cam is clockwise: *IW* = *+1*, otherwise: *IW* = *-1*.

6.5.3 Disk Cam with Roller Follower

Additional parameters:

- *r*: the radius of the roller.
- *IM*: a parameter whose absolute value is 1, indicating which envelope curve will be adopted.
- *RM*: inner or outer envelope curve. When it is an inner envelope curve: *RM*=*+1*, otherwise: *RM*=*-1*.

Design principle:

The basic principle of designing a cam profile with the inversion method is still used. However, the curve is not directly generated by inversion. This procedure has two steps:

1. Imagine the center of the roller as a knife edge. This concept is important in cam profile design and is called the trace point) of follower. Calculate the pitch curve *aa*, that is, the trace of the pitch point in the inverted mechanism.
2. The cam profile *bb* is a product of the enveloping motion of a series of rollers.

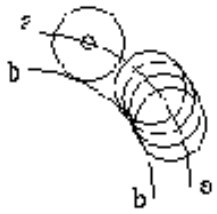


Figure 6-16 The trace point of the follower on a disk cam

Design equations:

The problem of calculating the coordinates of the cam profile is the problem of calculating the tangent points of a sequence of rollers in the inverted mechanism. At the moment shown Figure 6-17, the tangent point is *P* on the cam profile.

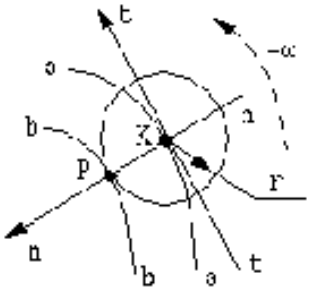


Figure 6-17 The tangent point, P, of a roller to the disk camm

The calculation of the coordinates of the point *P* has two steps:

1. Calculate the slope of the tangent *tt* of point *K* on pitch curve, *aa*.
2. Calculate the slope of the normal *nn* of the curve *aa* at point *K*.

Since we have already have the coordinates of point *K:* (*x*, *y*), we can express the coordinates of point *P*as

$$\begin{cases} x_P = x - IW \cdot RM \cdot r \cdot \dfrac{dy/d\varphi}{\sqrt{(dx/d\varphi)^2 + (dy/d\varphi)^2}} \\ y_P = y - IW \cdot RM \cdot r \cdot \dfrac{dx/d\varphi}{\sqrt{(dx/d\varphi)^2 + (dy/d\varphi)^2}} \end{cases} \tag{6-8}$$

Note:

- When the rotary direction of the cam is clockwise: *IW = +1*, otherwise: *IW = -1*.
- when the envelope curve (cam profile) lies inside the pitch curve: *RM = +1*, otherwise: *RM = -1*.

Gears:

7 Gears

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers.

7.1 Gear Classification

Gears may be classified according to the relative position of the axes of revolution. The axes may be

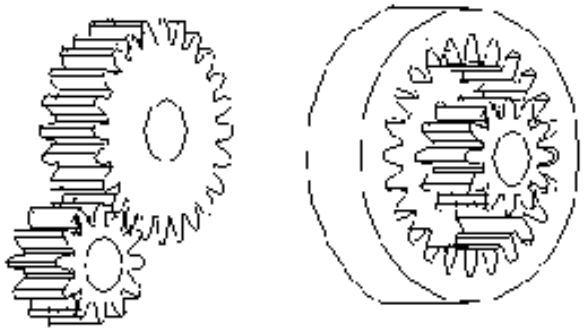
1. parallel,
2. intersecting,
3. neither parallel nor intersecting.

Here is a brief list of the common forms. We will discuss each in more detail later.

- Gears for connecting parallel shafts
- Gears for connecting intersecting shafts
- Neither parallel nor intersecting shafts

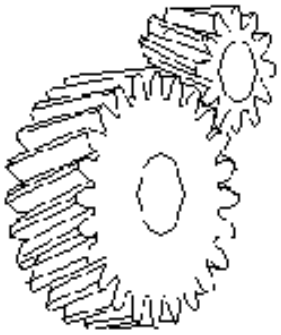
Gears for connecting parallel shafts

1. *Spur gears*

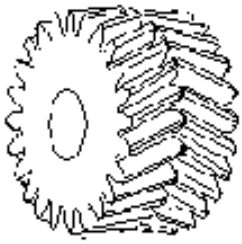


The left pair of gears makes **external contact**, and the right pair of gears makes **internal contact**

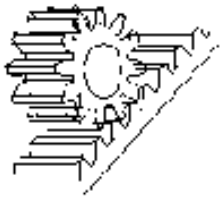
2. *Parallel helical gears*



3. *Herringbone gears* (or double-helical gears)

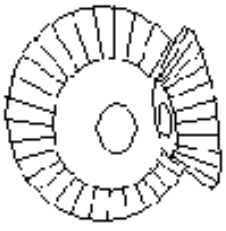


4. *Rack* and *pinion* (The rack is like a gear whose axis is at infinity.)



Gears for connecting intersecting shafts

1. *Straight bevel gears*



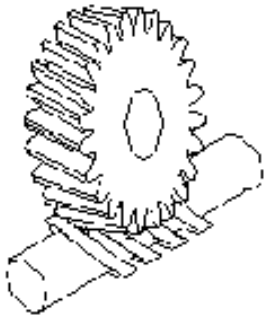
2. *Spiral bevel gears*

Neither parallel nor intersecting shafts

1. *Crossed-helical gears*



2. *Hypoid gears*
3. *Worm and wormgear*



7.2 Gear-Tooth Action

7.2.1 Fundamental Law of Gear-Tooth Action

Figure 7-2 shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point *K*.
- N_1N_2 is the common normal of the two profiles.
- N_1 is the foot of the perpendicular from O_1 to N_1N_2
- N_2 is the foot of the perpendicular from O_2 to N_1N_2.

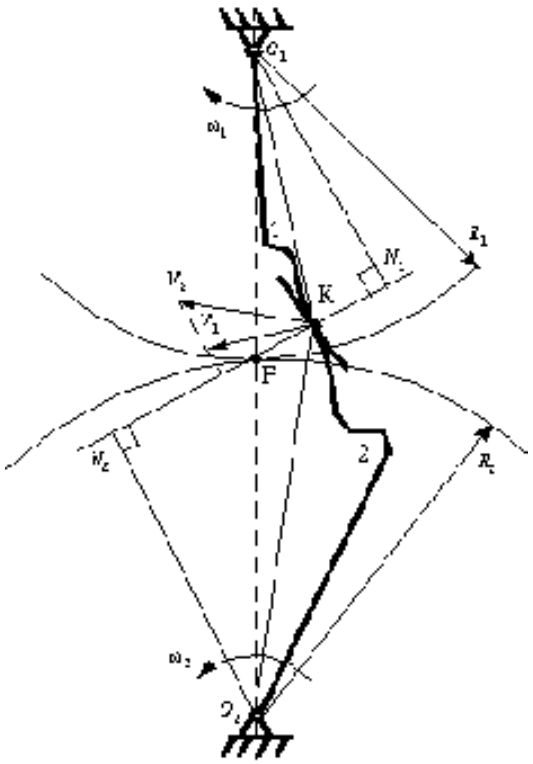


Figure 7-2 Two gearing tooth profiles

Although the two profiles have different velocities $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ at point K, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

$$O_1N_1 \cdot \omega_1 = O_2N_2 \cdot \omega_2$$

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or

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We notice that the intersection of the tangency N_1N_2 and the line of center O_1O_2 is point P, and

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Thus, the relationship between the angular velocities of the driving gear to the driven gear, or **velocity ratio**, of a pair of mating teeth is

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Point *P* is very important to the velocity ratio, and it is called the **pitch point**. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

7.2.2 Constant Velocity Ratio

For a constant velocity ratio, the position of *P* should remain unchanged. In this case, the motion transmission between two gears is equivalent to the motion transmission between two imagined slipless cylinders with radius R_1 and R_2 or diameter D_1 and D_2. We can get two circles whose centers are at O_1 and O_2, and through pitch point *P*. These two circle are termed **pitch circles**. The velocity ratio is equal to the inverse ratio of the diameters of pitch circles. This is the fundamental law of gear-tooth action.

The **fundamental law of gear-tooth action** may now also be stated as follow (for gears with fixed center distance) (Ham 58):

The common normal to the tooth profiles at the point of contact must always pass through a fixed point (the pitch point) on the line of centers (to get a constant velocity ration).

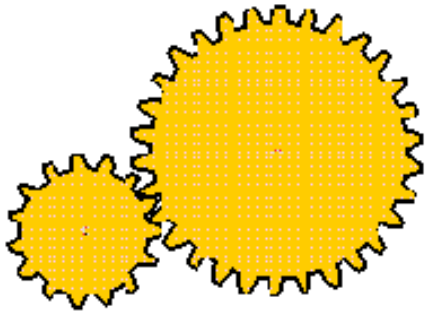
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Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the *cycloidal* and *involute* profiles. The involute has important advantages -- it is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required when using the involute profile. The most commonly used *conjugate* tooth curve is the *involute curve* (Erdman & Sandor 84).

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The following examples are involute spur gears. We use the word *involute* because the contour of gear teeth curves inward. Gears have many terminologies, parameters and principles. One of the important concepts is the *velocity ratio*, which is the ratio of the rotary velocity of the driver gear to that of the driven gears.



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7.3.1 Generation of the Involute Curve

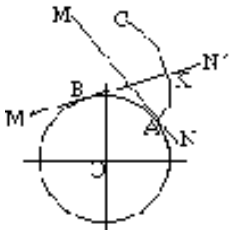


Figure 7-3 Involute curve

The curve most commonly used for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line rolls without slipping on the

circumference of a circle. It may also be defined as a path traced by the end of a string which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle**.

In Figure 7-3, let line *MN* roll in the counterclockwise direction on the circumference of a circle without slipping. When the line has reached the position *M'N'*, its original point of tangent *A* has reached the position *K*, having traced the involute curve *AK* during the motion. As the motion continues, the point *A* will trace the involute curve *AKC*.

7.3.2 Properties of Involute Curves

1. The distance *BK* is equal to the arc *AB*, because link *MN* rolls without slipping on the circle.
2. For any instant, the *instantaneous center* of the motion of the line is its point of tangent with the circle.
   Note: We have not defined the term *instantaneous center* previously. The instantaneous center or instant center is defined in two ways (Bradford & Guillet 43):
   1. When two bodies have planar relative motion, the instant center is a point on one body about which the other rotates at the instant considered.
   2. When two bodies have planar relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.
3. The normal at any point of an involute is tangent to the base circle. Because of the property (2) of the involute curve, the motion of the point that is tracing the involute is perpendicular to the line at any instant, and hence the curve traced will also be perpendicular to the line at any instant.
4. There is no involute curve within the base circle.

7.4 Terminology for Spur Gears

Figure 7-4 shows some of the terms for gears.

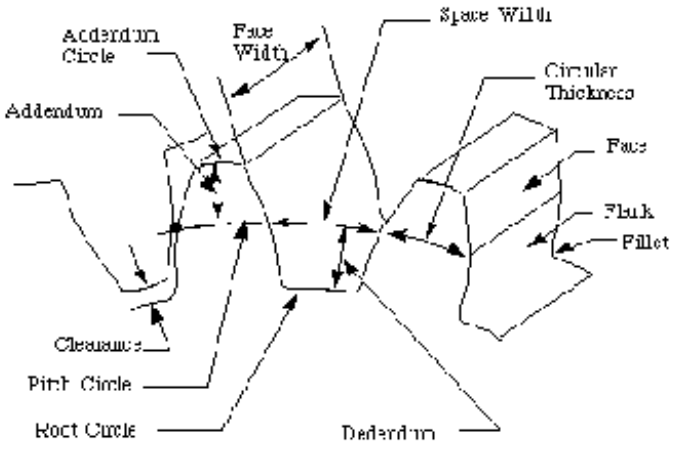


Figure 7-4 Spur Gear

In the following section, we define many of the terms used in the analysis of spur gears. Some of the terminology has been defined previously but we include them here for completeness. (See (Ham 58) for more details.)

- Pitch surface : The surface of the imaginary rolling cylinder (cone, etc.) that the toothed gear may be considered to replace.
- Pitch circle: A right section of the pitch surface.
- Addendum circle: A circle bounding the ends of the teeth, in a right section of the gear.
- Root (or dedendum) circle: The circle bounding the spaces between the teeth, in a right section of the gear.
- Addendum: The radial distance between the pitch circle and the addendum circle.
- Dedendum: The radial distance between the pitch circle and the root circle.
- Clearance: The difference between the dedendum of one gear and the addendum of the mating gear.
- Face of a tooth: That part of the tooth surface lying outside the pitch surface.
- Flank of a tooth: The part of the tooth surface lying inside the pitch surface.
- Circular thickness (also called the tooth thickness) : The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.
- Tooth space: The distance between adjacent teeth measured on the pitch circle.
- Backlash: The difference between the circle thickness of one gear and the tooth space of the mating gear.
- Circular pitch p: The width of a tooth and a space, measured on the pitch circle.
- Diametral pitch P: The number of teeth of a gear per inch of its pitch diameter. A toothed gear must have an integral number of teeth. The *circular pitch*, therefore, equals the pitch circumference divided by the number of teeth. The *diametral pitch* is, by definition, the number of teeth divided by the *pitch diameter*. That is,

$$p = \frac{\pi D}{N}$$

(7-5)

and

$$P = \frac{N}{D}$$

(7-6)

Hence

$$pP = \pi$$

(7-7)

where

p = circular pitch

P = diametral pitch

N = number of teeth

D = pitch diameter

That is, the product of the diametral pitch and the circular pitch equals π.

- Module m: Pitch diameter divided by number of teeth. The pitch diameter is usually specified in inches or millimeters; in the former case the module is the inverse of diametral pitch.
- Fillet : The small radius that connects the profile of a tooth to the root circle.
-
- Pinion: The smaller of any pair of mating gears. The larger of the pair is called simply the gear.
- Velocity ratio: The ratio of the number of revolutions of the driving (or input) gear to the number of revolutions of the driven (or output) gear, in a unit of time.
- Pitch point: The point of tangency of the pitch circles of a pair of mating gears.
- Common tangent: The line tangent to the pitch circle at the pitch point.
- Line of action: A line normal to a pair of mating tooth profiles at their point of contact.
- Path of contact: The path traced by the contact point of a pair of tooth profiles.
- Pressure angle α: The angle between the common normal at the point of tooth contact and the common tangent to the pitch circles. It is also the angle between the line of action and the common tangent.
- Base circle :An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.

7 Gears

Gears are machine elements that transmit motion by means of successively engaging teeth. The gear teeth act like small levers.

7.1 Gear Classification

Gears may be classified according to the relative position of the axes of revolution. The axes may be

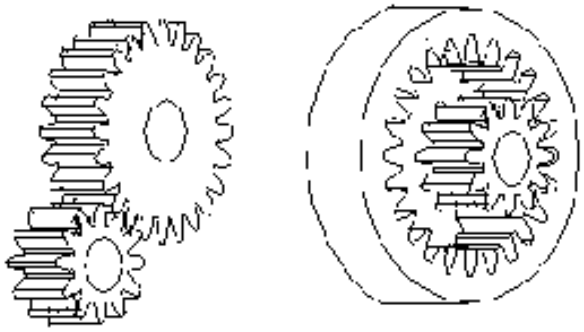
1. parallel,
2. intersecting,
3. neither parallel nor intersecting.

Here is a brief list of the common forms. We will discuss each in more detail later.

- Gears for connecting parallel shafts
- Gears for connecting intersecting shafts
- Neither parallel nor intersecting shafts

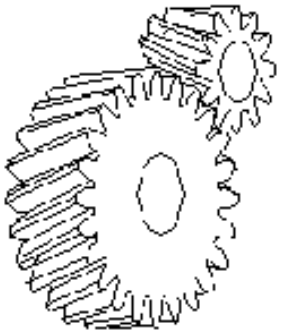
Gears for connecting parallel shafts

1. *Spur gears*

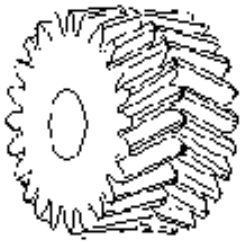


The left pair of gears makes **external contact**, and the right pair of gears makes **internal contact**

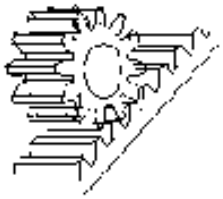
2. *Parallel helical gears*



3. *Herringbone gears* (or double-helical gears)

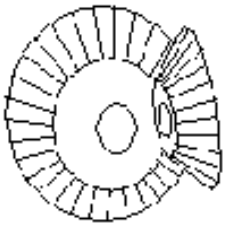


4. *Rack* and *pinion* (The rack is like a gear whose axis is at infinity.)



Gears for connecting intersecting shafts

1. *Straight bevel gears*



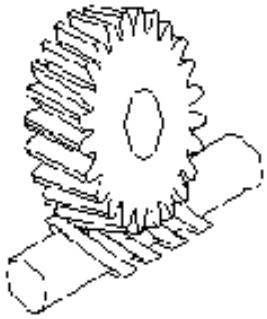
2. *Spiral bevel gears*

Neither parallel nor intersecting shafts

1. *Crossed-helical gears*



2. *Hypoid gears*
3. *Worm and wormgear*



7.2 Gear-Tooth Action

7.2.1 Fundamental Law of Gear-Tooth Action

Figure 7-2 shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point *K*.
- N_1N_2 is the common normal of the two profiles.
- N_1 is the foot of the perpendicular from O_1 to N_1N_2
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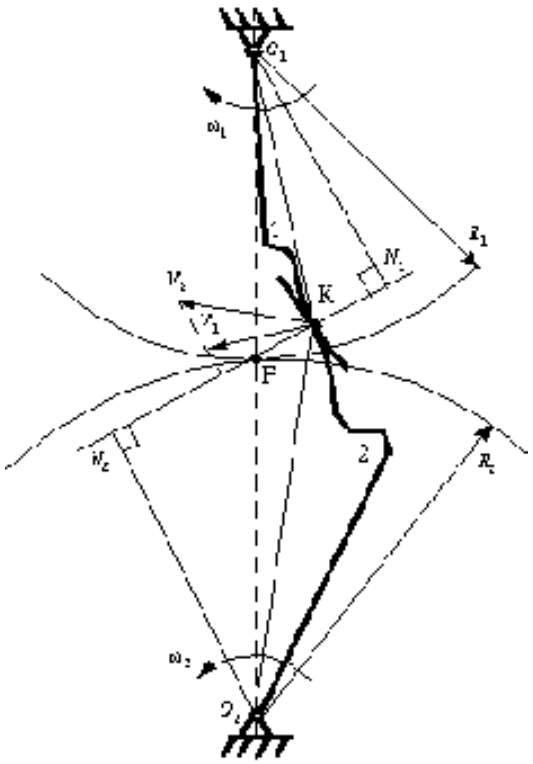


Figure 7-2 Two gearing tooth profiles

Although the two profiles have different velocities $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ at point K, their velocities along N_1N_2 are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, we have

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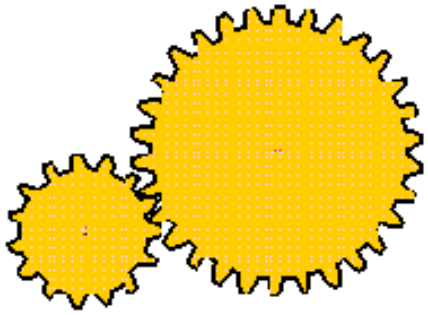
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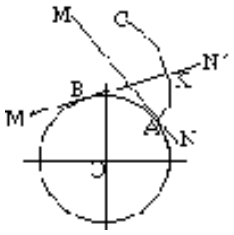


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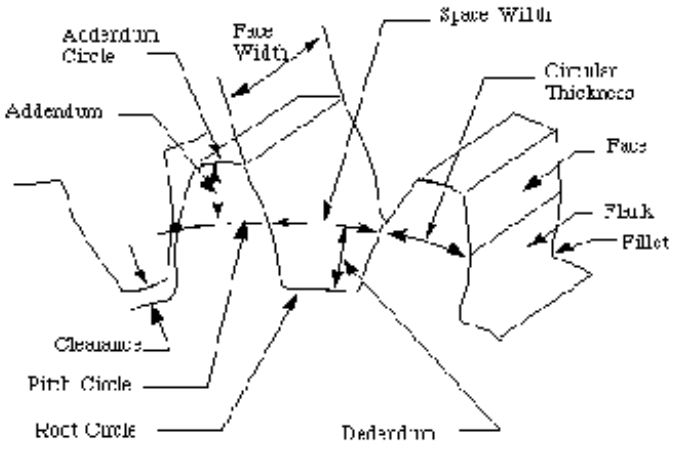


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Other Mechanisms:

Chapter 8. Other Mechanisms

8.1 Ratchet Mechanisms

A wheel provided with suitably shaped teeth, receiving an intermittent circular motion from an oscillating or reciprocating member, is called a **ratchet wheel**. A simple form of ratchet mechanism is shown in Figure 8-1.

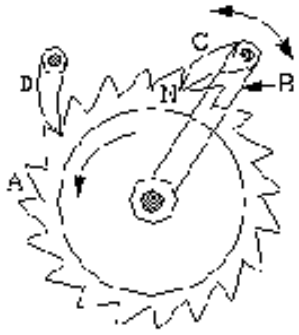


Figure 8-1 Ratchet

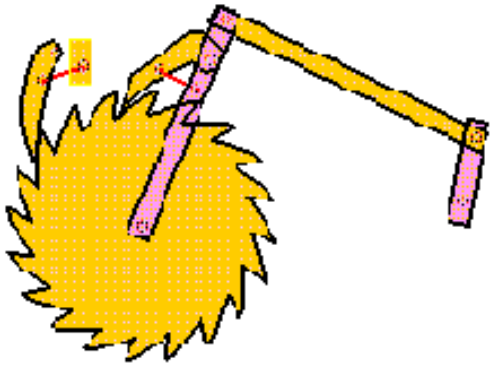
A is the **ratchet wheel**, and *B* is an oscillating lever carrying the **driving pawl**, *C*. A **supplementary pawl** at *D* prevents backward motion of the wheel.

When arm *B* moves counterclockwise, pawl *C* will force the wheel through a fractional part of a revolution dependent upon the motion of *B*. When the arm moves back (clockwise), pawl *C* will slide over the points of the teeth while the wheel remains at rest because of fixed pawl *D*, and will be ready to push the wheel on its forward (counterclockwise) motion as before.

The amount of backward motion possible varies with the pitch of the teeth. This motion could be reduced by using small teeth, and the expedient is sometimes used by placing several pawls side by side on the same axis, the pawls being of different lengths.

The contact surfaces of wheel and pawl should be inclined so that they will not tend to disengage under pressure. This means that the common normal at *N* should pass between the pawl and the ratchet-wheel centers. If this common normal should pass outside these limits, the pawl would be forced out of contact under load unless held by friction. In many ratchet mechanisms the pawl is held against the wheel during motion by the action of a spring.

The usual form of the teeth of a ratchet wheel is that shown in the above Figure, but in feed mechanisms such as used on many machine tools it is necessary to modify the tooth shape for a reversible pawl so that the drive can be in either direction. The following SimDesign example of a ratchet also includes a four bar linkage.



If you try this mechanism, you may turn the crank of the link mechanism. The rocker will drive the driving pawl to drive the ratchet wheel. The corresponding SimDesign data file is:

`/afs/andrew.cmu.edu/cit/ce/rapidproto/simdesign/ratchet.sim`

8.2 Overrunning Clutch

A special form of a ratchet is the **overrunning clutch**. Have you ever thought about what kind of mechanism drives the rear axle of bicycle? It is a free-wheel mechanism which is an overrunning clutch. Figure 8-2 illustrates a simplified model. As the driver delivers torque to the driven member, the rollers or balls are wedged into the tapered recesses. This is what gives the positive drive. Should the driven member attempt to drive the driver in the directions shown, the rollers or balls become free and no torque is transmitted.

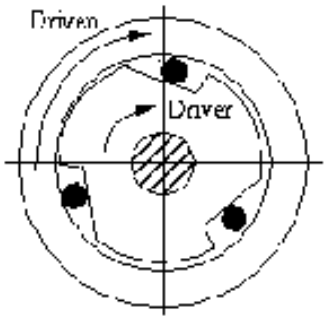


Figure 8-2 Overrunning clutch

8.3 Intermittent Gearing

A pair of rotating members may be designed so that, for continuous rotation of the driver, the follower will alternately roll with the driver and remain stationary. This type of arrangement is know by the general term **intermittent gearing**. This type of gearing occurs in some counting mechanisms, motion-picture machines, feed mechanisms, as well as others.

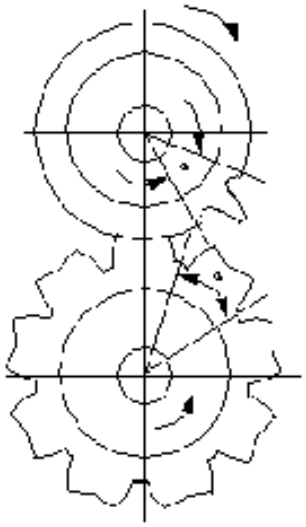


Figure 8-3 Intermittent gearing

The simplest form of intermittent gearing, as illustrated in Figure 8-3 has the same kind of teeth as ordinary gears designed for continuous rotation. This example is a pair of 18-tooth gears modified to meet the requirement that the follower advance one-ninth of a turn for each turn of the driver. The interval of action is the two-pitch angle (indicated on both gears). The single tooth on the driver engages with each space on the follower to produce the required motion of a one-ninth turn of the follower. During the remainder of a driver turn, the follower is locked against rotation in the manner shown in the figure.

To vary the relative movements of the driver and follower, the meshing teeth can be arranged in various ways to suit requirements. For example, the driver may have more than one tooth, and the periods of rest of the follower may be uniform or may vary considerably. Counting mechanisms are often equipped with gearing of this type.

8.4 The Geneva Wheel

An interesting example of intermittent gearing is the **Geneva Wheel** shown in Figure 8-4. In this case the **driven wheel**, *B*, makes one fourth of a turn for one turn of the **driver**, *A*, the **pin**, *a*,

working in the **slots**, *b*, causing the motion of *B*. The circular portion of the driver, coming in contact with the corresponding hollow circular parts of the driven wheel, retains it in position when the pin or tooth *a* is out of action. The wheel *A* is cut away near the pin *a* as shown, to provide clearance for wheel *B* in its motion.

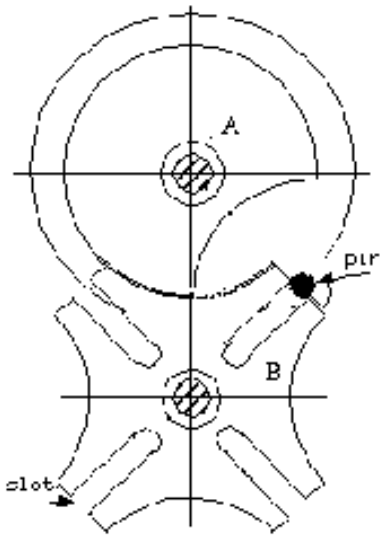
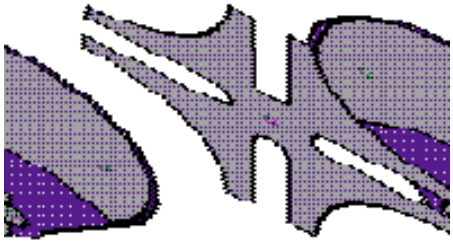


Figure 8-4 Geneva wheel

If one of the slots is closed, *A* can only move through part of the revolution in either direction before pin *a* strikes the closed slot and thus stops the motion. The device in this modified form was used in watches, music boxes, *etc*., to prevent overwinding. From this application it received the name **Geneva stop**. Arranged as a stop, wheel *A* is secured to the spring shaft, and *B* turns on the axis of the spring barrel. The number of slots or interval units in *B* depends upon the desired number of turns for the spring shaft.

An example of this mechanism has been made in SimDesign, as in the following picture.



The corresponding SimDesign data file is:

```
/afs/andrew.cmu.edu/cit/ce/rapidproto/simdesign/geneva.sim
```

8.5 The Universal Joint

The engine of an automobile is usually located in front part. How does it connect to the rear axle of the automobile? In this case, universal joints are used to transmit the motion.

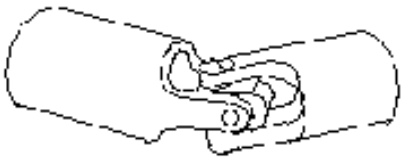


Figure 8-5 Universal joint

The **universal joint** as shown in Figure 8-5 is also known in the older literature as **Hooke's coupling**. Regardless of how it is constructed or proportioned, for practical use it has essentially the form shown in Figure 8-6, consisting of two semicircular forks 2 and 4, pin-jointed to a right -angle cross 3.

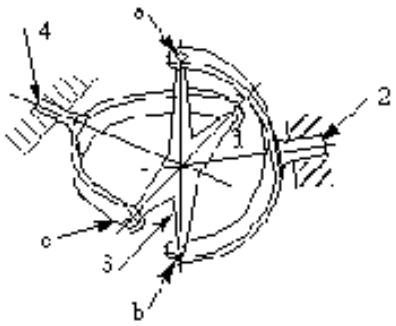


Figure 8-6 General form for a universal joint

The driver 2 and the follower 4 make the complete revolution at the same time, but the velocity ratio is not constant throughout the revolution. The following analysis will show how complete information as to the relative motions of driver and follower may be obtained for any phase of the motion.

8.5.1 Analysis of a Universal Joint

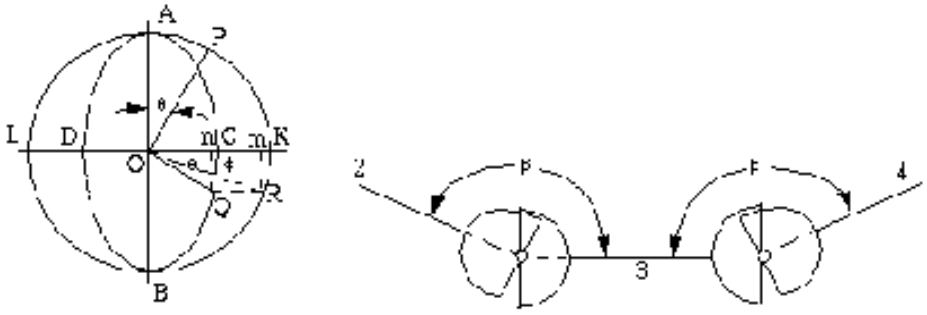


Figure 8-7 Analysis of a universal joint

If the plane of projection is taken perpendicular to the axis of 2, the path of *a* and *b* will be a circle *AKBL* as shown in Figure 8-7.

If the angle between the shafts is θ, the path of *c* and *d* will be a circle that is projected as the ellipse *ACBD*, in which

$$OC = OD = OK\cos\theta = OA\cos\theta$$

(8-1)

If one of the arms of the driver is at *A*, an arm of the follower will be at *C*. If the driver arm moves through the angle θ to *P*, the follower arm will move to *Q*. *OQ* will be perpendicular to *OP*; hence: angle *COQ* = θ. But angle *COQ* is the projection of the real angle describes by the follower. *Qn* is the real component of the motion of the follower in a direction parallel to *AB*, and line *A*B is the intersection of the planes of the driver's and the follower's planes. The true angle ϕ described by the follower, while the driver describes the angle θ, can be found by revolving *OQ* about AB as an axis into the plane of the circle *AKBL*. Then *OR* = the true length of *OQ*, and *ROK* = ϕ= the true angle that is projected as angle *COQ* = θ.

Now

$$\tan\phi = Rm/Om$$

and

$$\tan\theta = Qn/On$$

But

$$Qn = Rm$$

Hence

$$\frac{\tan\theta}{\tan\phi} = \frac{Om}{On} = \frac{OK}{OC} = \frac{\cdot}{\cos\beta}$$

Therefore

$$\tan\phi = \cos\beta\tan\theta$$

The ratio of the angular motion of the follower to that of the driver is found as follower, by differentiating above equation, remembering that β is constant

$$\frac{\omega_4}{\omega_2} = \frac{d\psi}{d\theta} = \frac{\cos\beta\sec^2\theta}{\sec^2\phi} = \frac{\cos\beta\sec^2\theta}{1+\tan^2\phi}$$

Eliminating ϕ:

$$\frac{\omega_4}{\omega_2} = \frac{\cos\beta\sec^2\theta}{1-\cos^2\beta\tan^2\theta} = \frac{\cos\beta}{1-\sin^2\beta\sin^2\theta}$$

Similarly, θ can be eliminated:

$$\frac{\omega_4}{\omega_2} = \frac{1-\cos^2\phi\sin^2\beta}{\cos\beta}$$

According to the above equations, when the driver has a uniform angular velocity, the ratio of angular velocities varies between extremes of *cos* β and *1/cos* β. These variations in velocity give rise to inertia forces, torques, noise, and vibration which would not be present if the velocity ratio were constant.

8.5.2 Double Universal Joint

By using a double joint shown on the right in Figure 8-7, the variation of angular motion between driver and follower can be entirely avoided. This compensating arrangement is to place an intermediate shaft 3 between the driver and follower shafts. The two forks of this intermediate shaft must lie in the same plane, and the angle between the first shaft and the intermediate shaft must exactly be the same with that between the intermediate shaft and the last shaft. If the first shaft rotates uniformly, the angular motion of the intermediate shaft will vary according to the result deduced above. This variation is exactly the same as if the last shaft rotated uniformly, driving the intermediate shaft. Therefore, the variable motion transmitted to the intermediate shaft by the uniform rotation of the first shaft is exactly

compensated for by the motion transmitted from the intermediate to the last shaft, the uniform motion of either of these shafts will impart, through the intermediate shaft, uniform motion to the other.

Universal joints, particularly in pairs, are used in many machines. One common application is in the drive shaft which connects the engine of an automobiles to the axle.

Engineering Thermodynamics:

Engineering Thermodynamics

Introduction to Classical Thermodynamics

Thermodynamics is the study of energies. More specifically, introductory thermodynamics is the study of energy transfer in systems. Classical thermodynamics consists of methods and constructs that are used to "account" for macroscopic energy transfer. In fact, energy accounting is an appropriate synonym for classical thermodynamics. In much the same way that accountants balance money in and money out of a bank account, rocket scientists simply balance the energy in and out of a rocket engine. Of course just as a bank account's balance is obfuscated by arcane devices such as interest rates and currency exchange, so too is thermodynamics clouded with seemingly difficult concepts such as irreversibility and enthalpy. But, also just like accounting, a careful review of the rules suggests a coherent strategy for maintaining tabs on a particular account.

If a statement about the simplicity of thermodynamics failed to convert would-be students, they may be captured with a few words on the importance of understanding energy transfer in our society. Up until about 150 years ago or so, the earth's economy was primarily fueled by carbohydrates. That is to say, humans got stuff done by converting food, through a biological process, to fuel we could spend to do work (e.g. raise barns). This was a hindrance to getting things accomplished because, as it turned out, most individuals had to use the brunt of that energy to grow and cultivate more carbohydrates (e.g. crops and livestock). We won't even talk about how much food the horses ate!

Today, we have the luxury, primarily through an understanding of energy, to concentrate our energy production into efficient low maintenance operations. Massive power plants transfer energy to power tools for raising barns. Extremely efficient rocket engines tame and direct massive amounts of energy to blast TV satellites into orbit. This improvement in energy mastery frees humanity's time to engage in more worthwhile activities such as watching cable TV. Although most are content to blissfully ignore the intricacies that command their way of life, I challenge you to embrace the contrary.

By no means is the energy battle over. Understanding energy transfer and energy systems is the second step to destroying the limits to what humanity can next accomplish. The first step is commanding an interest in doing so from an inclined portion of the population. Given the reader

(and editor) has read this far through this aggrandizing rhetoric, I welcome your interest and hope to see it continue until the end.

The Main Macroscopic Forms of Energy

It will be in the best interest of the reader to have defined energy before it is discussed further. There are three primary forms of energy that are discussed in macroscopic thermodynamics. Several other forms of energy exist, but they generally exist on a microscopic level and should be deferred to more advanced study.

Kinetic Energy

The first form (probably most easily understood idea of energy) is defined by the motion of an object. Kinetic energy is the energy of a moving mass. For instance, a moving car will have more kinetic energy than a stationary car. The same car traveling at 60 km/h has more kinetic energy than it does traveling at 30 km/h.

Kinetic Energy = (1/2) x (mass) x $(\text{velocity})^2$

Ratio of $v_{60}^2 / v_{30}^2 = ((60)(1{,}000 \text{ meters/sec})(3600 \text{ sec/hour}))^2 / ((30)(1{,}000 \text{ meters/sec})(3{,}600 \text{ sec/hour}))^2$

So, once the algebra is completed properly we find the vehicle traveling at 60 km/h has four times the kinetic energy as when it is traveling at 30 km/h, while the vehicle has zero kinetic energy when it is stationary because the velocity = zero results in $1/2mv^2$ being zero.

Potential Energy

The second type of energy is called potential energy. Gravitational potential energy describes the energy due to elevation. A car at a height of 50 m has more potential energy than a car at a height of 25 m. This may be understood more easily if the car is allowed to drop from its height. On impact with the earth at 0 m, the car that initially rested at 50 m will have more kinetic energy because it was moving faster (allowed more time to accelerate). The idea that potential energy can convert to kinetic energy is the first idea of energy transfer. Transfers between kinetic and potential energy represent one type of account balance rocket scientists need be aware of.

Internal Energy of Matter

The third and most important concept of energy is reflected by temperature. The internal energy of matter is measured by its temperature. Hot water has more internal energy than the same amount of cold water. Internal energy is a measure of kinetic energy of the molecules and atoms that make up the substance. Since each atom or molecule is acting on its own accord, this internal energy is different from the bulk kinetic energy associated with the movement of the entire solid. The internal energy of matter is exhibited by molecular motion. The molecules of a gas at high temperature zip around their container constantly colliding with walls and other

molecules. The molecules of a high temperature solid also move around a lot; however, since they are stuck together with other molecules, the most they can do is vibrate in place.

In a nutshell, the above forms of energy are studied in classical thermodynamics. Those forms of energy are allowed to transfer among each other as well as in to or out of a system. Thermodynamics essentially provides some definitions for interpreting thermodynamic systems. It then goes on to define an important rule about fairly balancing energy and one rule about the quality of energy. (some energy is more valuable) Understanding the framework and the few rules that govern macroscopic thermodynamics proves to be an incredibly powerful set of tools for analyzing a myriad of not only engineering problems, but issues of practical concern.
CONTRIBUTION BY CHANGES

Engineering Thermodynamics

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Fluid Mechanics:

Introduction

A fluid is a substance that deforms continuously when subjected to a tangential or shear stress, however small the shear stress may be. Such a continuous deformation under the stress constitutes a flow. Fluid mechanics is therefore the study of mechanics of such matter. As such, this pertains mostly to the study of liquids and gases, however the general theories may be applied to the study of amorphous solids, colloidal suspensions and gelatinous materials.

Fluid mechanics is a subdivision of continuum mechanics. Consequentially, fluids are considered continuous media for analysis, and their discrete nature is of no consequence for most applications. This assumption is valid mostly on length scales much larger than intramolecular distances. The departure from continuum is characterised by a dimensionless parameter, the Knudsen Number, defined by $K_n = \lambda / L$, where L is a characteristic length scale of the flow. The continuum hypothesis holds good if Kn < 0.01. However, recent applications in nanotechnology and biotechnology are demonstrating that the governing equations are still relevant on smaller scales, specifically when they are modified to include the effects of electrostatic, magnetic, colloidal and surface-tension driven forces.

Some fluid mechanics problems can be solved by applying conservation laws (mass, momentum, energy) of mechanics to a finite control volume. However, in general, it is necessary to apply those laws to an infintesimal control volume, then use the resulting differential equations. Additionally, boundary values, initial conditions and thermodynamic state equations are generally necessary to obtain numeric or analytic solutions.

Fluid Properties

In addition to the properties like mass, velocity, and pressure usually considered in physical problems, the following are the basic properties of a fluid:

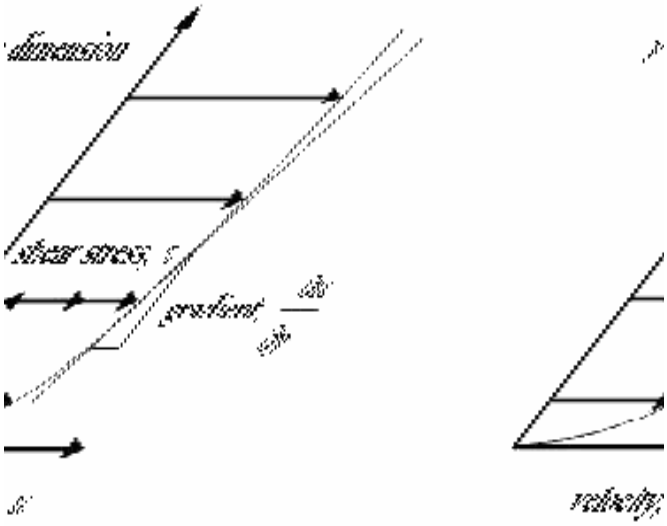
Density

The density of a fluid is defined as the mass per unit volume of the fluid over an infinitesimal volume.

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

Viscosity

Viscosity (represented by μ) is a material property, unique to fluids, that measures the fluid's resistance to flow. Though this is a property of the fluid, its effect is understood only when the fluid is in motion. When different elements move with different velocities, then the each element tries to drag its neighbouring elements along with it. Thus shear stress can be identified between fluid elements of different velocities.



Velocity gradient in laminar shear flow

The relationship between the shear stress and the velocity field was studied by Isaac Newton and he proposed that the shear stresses are directly proportional to the velocity gradient.

$$\tau = \mu \frac{\partial u}{\partial y}$$

The constant of proportionality is called the coefficient of dynamic viscosity.

Another coefficient, known as the kinematic viscosity is defined as the ratio of dynamic viscosity and density. $\nu = \mu / \rho$

Reynolds Number

There are several dimensionless parameters that are important in fluid dynamics. *Reynolds number* (after Osborne Reynolds, 1842-1912) is an important parameter in the study of fluid flows. Physically it is the ratio between inertial and viscous forces. The value of Reynolds number determines the kind of flow of the fluid.

$$Re - \frac{\rho V L}{\mu} - \frac{V L}{\nu}$$

where ρ(rho) is the density, μ(mu) is the viscosity, V is the velocity of the flow, and L is the dimension representing length for the flow. Additionally, we define a parameter ν(nu) as the *kinematic viscosity.*

Low *Re* indicates creeping flow, medium *Re* is *laminar* flow, and high *Re* indicates *turbulent* flow.

Reynolds number can also be transformed to take account of different flow conditions. For example the reynolds number for flow within a pipe is expressed as

$$Re = \frac{\rho u d}{\mu}$$

where u is the average fluid velocity within the pipe and d is the inside diameter of the pipe.

Application of dynamic forces (and the Reynolds number) to the real world: sky-diving, where friction forces equal the falling body's weight. (jjam)

Pathlines and Streamlines

The path which a fluid element traces out in space is called a *pathline*. For steady non fluctuating flows where a pathline is followed continuously by a number of fluid elements , the pathline is called *streamline*. A *streamline* is the imaginary line whose tangent gives the velocity of flow at all times if the flow is steady, however in an unsteady flow, the streamline is constantly changing and thus the tangent gives the velocity of an element at an instant of time. A common practice in analysis is taking some of the walls of a control volume to be along streamlines. Since there is no flow perpendicular to streamlines, only the flow across the other boundaries need be considered.

Hydrostatics

The pressure distribution in a fluid under gravity is given by the relation $dp/dz = -\rho g$ where dz is the change in the direction of the gravitational field (usually in the vertical direction). Note that it

is quite straightforward to get the relations for arbitrary fields too, for instance, the pseudo field due to rotation.

The pressure in a fluid acts equally in all directions. When it comes in contact with a surface, the force due to pressure acts normal to the surface. The force on a small area *dA* is given by *p dA* where the force is in the direction normal to *dA*. The total force on the area *A* is given by the vector sum of all these infinitesimal forces.

Control Volume Analysis

A fluid dynamic system can be analysed using a *control volume*, which is an imaginary surface enclosing a volume of interest. The control volume can be fixed or moving, and it can be rigid or deformable. Thus, we will have to write the most general case of the laws of mechanics to deal with control volumes.

The first equation we can write is the conservation of mass over time. Consider a system where mass flow is given by *dm/dt*, where *m* is the mass of the system. We have,

$$\dot{m} = \int_{CS} \rho (\mathbf{V} \cdot \mathbf{n}) \, dA$$

For steady flow, we have

$$\int_{CS} \rho (\mathbf{V} \cdot \mathbf{n}) \, dA = 0$$

And for incompressible flow, we have

$$\int_{CS} (\mathbf{V} \cdot \mathbf{n}) \, dA = 0$$

If we consider flow through a tube, we have, for steady flow,

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

and for incompressible steady flow, $A_1 V_1 = A_2 V_2$.

Law of conservation of momentum as applied to a control volume states that

$$\sum F = \frac{d}{dt} \left(\int_{CV} \mathbf{V} \rho \right) + \int_{CS} \mathbf{V} \rho (\mathbf{V} \cdot \mathbf{n}) \, dA$$

where *V* is the velocity vector and *n* is the unit vector normal to the control surface at that point.

Law of Conservation of Energy (First Law of Thermodynamics)

$$\frac{d\mathbf{Q}}{dt} + \frac{d\mathbf{W}}{dt} = \frac{d}{dt}\left(\int_{CV} e\rho\right) + \int_{CS} e\rho\,(\mathbf{V}\cdot\mathbf{n})\,dA$$

where *e* is the energy per unit mass.

Bernoulli's Equation

Bernoulli's equation considers frictionless flow along a streamline.

For steady, incompressible flow along a streamline, we have

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

We see that Bernoulli's equation is just the law of conservation of energy without the heat transfer and work.

It may seem that Bernoulli's equation can only be applied in a very limited set of situations, as it requires ideal conditions. However, since the equation applies to streamlines, we can consider a streamline near the area of interest where it is satisfied, and it might still give good results, *i.e.*, you don't need a control volume for the actual analysis (although one is used in the derivation of the equation).

Energy in terms of Head

Bernoulli's equation can be recast as

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = constant$$

This constant can be called head of the water, and is a representation of the amount of work that can be extracted from it. For example, for water in a dam, at the inlet of the penstock, the pressure is high, but the velocity is low, while at the outlet, the pressure is low (atmospheric) while the velocity is high. The value of head calculated above remains constant (ignoring frictional losses).

Heat Transfer:

Contents

Introduction The basics

Heat Balances What goes in must come out

Conduction A mechanism of heat transfer

Convection A mechanism of heat transfer

Radiation A mechanism of heat transfer

Heat Transfer with Phase Change Different rules apply when this happens

Heat Exchangers Industrial devices for heating, cooling and saving energy

Appendices There may be something in here later

Bibliography Recommended for more in-depth study

Pro Engineer:

Pro/E is the software product of PTC (Parametric Technology corporation) its intial versions were Pro/E 1, 2...upto 17. Later Pro/E launched sequentially versions as Pro/E 2000; 2000i, 2000i^2;2001i, wildfire1.0; wildfire2.0, wildfire3.0; wildfire4.0 till date. This book will explore the various philosophies and methodologies employed by professionals who use the Computer Aided Design (CAD) package Pro/Engineer. Additional topics include how to acquire the software and optimal support.

Notes to Contributors:

Please write Pro/Engineer and Pro/Intralink fully not pro/e, pro/engineer or any other variation.
Conventions:

```
*RMB  - Right Mouse Button
*LMB  - Left Mouse Button
*MMB  - Middle Mouse Button
```

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*BMB  - Both Mouse Buttons
*ICON - Link to image
```

All items assumed to be from top menu unless left menu is mentioned Proposed contents

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Robotics:

An Introduction to Robotics

Defining Robotics

Robotics can be described as the current pinnacle of technical development. Robotics is a confluence science using the continuing advancements of mechanical engineering, material science, sensor fabrication, manufacturing techniques, and advanced algorithms. The study and practice of robotics will expose a dabbler or professional to hundreds of different avenues of study. For some, the romanticism of robotics brings forth an almost magical curiosity of the world leading to creation of amazing machines. A journey of a lifetime awaits in robotics.

Robotics can be defined **as the science or study of the technology primarily associated with the design, fabrication, theory, and application of robots**. While other fields contribute the mathematics, the techniques, and the components, robotics creates the magical

end product. The practical applications of robots drive development of robotics and drive advancements in other sciences in turn. Crafters and researchers in robotics study more than just robotics.

The promise of robotics is easy to describe but hard for the mind to grasp. Robots hold the promise of moving and transforming materials with the same elan and ease as a computer program transforms data. Today, robots mine minerals, assemble semi-processed materials into automobile components, and assemble those components into automobiles. On the immediate horizon are self-driving cars, robotics to handle household chores, and assemble specialized machines on demand. It is not unreasonable to imagine robots that are given some task, such as reclaim desert into photovoltaic cells and arable land, and left to make their own way. Then the promise of robotics exceeds the minds grasp.

In summary, robotics is the field related to science and technology primarily related to robotics. It stands tall by standing the accomplishments of many other fields of study.

Defining Robots

Robot used in English describes any construct that automates some behavior. For example, a garage door opener automates the behavior of opening a door. A garage door opener has a sensor to detect the signal from the remote control, actuators to open the door, and a control system to stop turn off the motors and lights when the garage is fully closed. In practice, this type of a machine is better described as a Mechatronic device, and is a subset of the more interesting robots that include **autonomy** or **resourcefulness**. This book will consider mechatronic devices to be degenerate robots.

A **Mechatronic Device** is a degenerate robot with these components:

1. Sensors, which detect the state of the environment
2. Actuators, which modify the state of the environment
3. A Control System, which controls the actuators based on the environment as depicted by the sensors

A **Robot** is a mechatronic device which also includes **resourcefulness** or **autonomy**. A device with **autonomy** does its thing "on its own" without a human directly guiding it moment-by-moment. Some authors would contend that all mechatronic devices are robots, and that this book's restriction on **robot** entails only specialized software.

Various types of robots are usually classified by their capabilities. Two examples will be used to capture most of what we see as a "robot".

1. Machine Pet: A machine, capable of moving in some way, that can sense its surroundings and can act on what it senses autonomously. Most of these robots have no real useful purpose, other than to entertain and challenge. These are also commonly used for experimenting with sensors, artificial intelligence, actuators and more. Most of this book

covers this type of robot.
2. Autonomous Machine: A machine with sensors and actuators that can do some sort of work "on its own". This includes things like robotic lawnmowers and vacuum cleaners, and also self-operating construction machines such as CNC cutters. Most industrial and commercial robots fall in this category.

What isn't considered a "robot" in this book? Pretty much everything you see on RobotWars; those are remote-controlled vehicles without any form of autonomy, no sensors, and a just enough of a control system to drive the actuators. These devices use many of the same mechanical technologies described in this book, but not the advanced controls.

In short: If it has autonomy it's a robot (in this book). If it's remote controlled, it isn't.

Student Questions

1. Which of these studies would be considered robotics by this definition?
    1. Studying the strength and flexibility of a titanium alloy used to make a robotic arm?
    2. Integrating sensor data from sonar, laser, and CCD cameras and to build an accurate map of surroundings?
    3. The real-time software needed to drive two motors to make a robot go in a straight line?

1. Classify each of these as a robot, a mechatronic device, a machine, or something else?
    1. A spam email filter.
    2. A garage door opener.
    3. A remote controlled boat.
    4. A 1970's automobile.
    5. A current model automobile which includes lane-following.
    6. An Apple IPod.
    7. An actor in a silver suit.

Design Basics

Note to potential contributors: this section could be used to discuss the basics of robot design/construction.

1. What you should know
2. Physical Design
3. Design software
4. Tools and Equipment
5. Electronic Components
6. Mechanical Components
7. Building materials
8. Basic Programming

Physical Construction

1. The Platform
2. Construction Techniques
3. Resourcefulness

Components

1. Power Sources
2. Actuation Devices
    1. Motors
    2. Shape Memory Alloys
    3. Air muscle
    4. Linear Electromagnetic
    5. Piezoelectric Actuators
    6. Pneumatics/Hydraulics
    7. Miniature internal combustion engines
3. Grippers
4. Audio
5. Video

Computer Control

1. Control Architectures
    1. Reactive Systems
    2. Sense-Plan-Act
    3. Brooks' Subsumption Architecture (w:Subsumption architecture)
    4. Hybrid Systems
2. The Interface
    1. Personal Computers
    2. Single Board Computers and multichip modules
    3. Microcontrollers
    4. Remote Control
    5. Networks

Sensors

Sensors that a robot uses generally fall into three different categories:

1. Environment sensors tell the robot what is happening around it
    1. Thermal Sensors
    2. Pressure Sensors
2. Feedback sensors tell the robot what it is actually doing, and
3. Communication sensors allow a human or computer to provide a robot other information.

Sensors aren't perfect. When you use a sensor on your robot there will be a lot of times where

the sensors acts funny. It could miss an obstacle, or see one where none is. Key to successfuly using sensors is knowing how they function and what they *really* measure.

- Real World Sensors

Navigation

1. Navigation
    1. Localization
    2. Collision Avoidance
    3. Exploration
    4. Mapping
    5. Trajectory Planning

Robotics

1. Design Basics
    1. Physical Design
    2. Tools and Components
    3. Electronic Components
    4. Construction Techniques
    5. Basic Programming
2. Physical Construction
    1. The platform
    2. Resourcefulness
    3. The LEGO World
        1. Modded Lego Robots
        2. The RCX
        3. Programming the RCX
3. Components
    1. Batteries
    2. Power Supplies
    3. Actuation Devices
        1. Motors
        2. Muscles
        3. Electromagnetic
    4. Audio
    5. Visual
4. Robot Designs
    1. The Rover
    2. The Legged Robot
    3. Advanced Locomotion

    4. Robotic Arms
        1. Revolute-coord Arm
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    5. Environment Interfaces
5. Computer Control
    1. Laws of Robotics
    2. The Interface
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        1. Computers
        2. Microcontrollers
        3. Remote Control
6. Environment Sensors
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2. BEAM
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4. Contest Robot
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6. Modular and fractal Robots
7. The LEGO World
    1. LEGO Robots
    2. Introduction to the RCX
    3. Programming the RCX

An Introduction to Robotics

Defining Robotics

Robotics can be described as the current pinnacle of technical development. Robotics is a confluence science using the continuing advancements of mechanical engineering, material science, sensor fabrication, manufacturing techniques, and advanced algorithms. The study and practice of robotics will expose a dabbler or professional to hundreds of different avenues of study. For some, the romanticism of robotics brings forth an almost magical curiosity of

the world leading to creation of amazing machines. A journey of a lifetime awaits in robotics.

Robotics can be defined **as the science or study of the technology primarily associated with the design, fabrication, theory, and application of robots**. While other fields contribute the mathematics, the techniques, and the components, robotics creates the magical end product. The practical applications of robots drive development of robotics and drive advancements in other sciences in turn. Crafters and researchers in robotics study more than just robotics.

The promise of robotics is easy to describe but hard for the mind to grasp. Robots hold the promise of moving and transforming materials with the same elan and ease as a computer program transforms data. Today, robots mine minerals, assemble semi-processed materials into automobile components, and assemble those components into automobiles. On the immediate horizon are self-driving cars, robotics to handle household chores, and assemble specialized machines on demand. It is not unreasonable to imagine robots that are given some task, such as reclaim desert into photovoltaic cells and arable land, and left to make their own way. Then the promise of robotics exceeds the minds grasp.

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Machines Kinematics:

Basic Kinematic concepts: Links, Kinematic pairs, Kinematic chains, Mechanism and

Inversions, single and double slider crank chains, straight line motion mechanisms, Velocity and acceleration. in mechanism, Relative velocity methods, Instantaneous center of rotation, centroids, Acceleration diagram, Acceleration center, Friction Devices: Introduction to friction, Belt, chain arnd rope drive, Transmission of Power through friction clutch, Fundamental law of gearing: Classification of gears and basic terminology, Geometric and Kinematic characteristics of involute and cycloidal tooth profiles, under cutting and interference, Gear trains: Simple, compound and planetary, tooth load and torque, Balancing: Balancing of revolving in the same Irlane by a single revolving mass, Balancing of several revolving masses in different planes by two revolving masses in suitable planes

Machine Kinematics Dynamics - Notes, Tutorials

- Force, Curvature, and Uncertainty - with lots of equations
- Reflections on Relativity - Book in HTML format
- Kinematics – the study of constrained motion - Common single loop, closed chain mechanisms
- Kinematics Dictionary - From Free Directory
- Acceleration, velocity, and Position - The connections between position, velocity, and acceleration ...
- Relative Velocity - Java Applet
- Relative Velocity and Riverboat Problems - Vectors - Fundamentals and Operations
- Acceleration, Forces and Free Body Diagrams - Newton's Second Law and Elevators

- Introduction to Friction - Java Applet

- Gears and Gear Systems - A good example of a ‘gear train’

- What is Torque? - A measure of how much a force acting on an object causes that object to rotat

Strength of Materials:

Strength of Materials (SOM) - Notes, Tutorials

Introduction/Review of Statics, Stress, Stress, Mechanical Properties of Materials, Transformation of Stress, Axial Load, Transformation of Strain, Torsion, Beam Bending, Bending of 'Composite ' Beams, Transverse Shear, Combined Loadings, Deflections of Beams and Shafts

Strength of Materials (SOM)

- Material Properties: Strength of Material - Simple Compression Studies, Plasticity and Adiabatic Shear Bands, Metastable Phases, Phase-Change Measurements, Rear-Surface Studies etc.

- Material Selection Guide - Abbreviations and Acronyms, Metals,Plastics, Coefficient of Friction Testing, Compressive Strength Testing, Deflection Temperature Testing, Metal Matrix Composites, Plastic Recycling Codes etc

- Torsion Definitions - Dictionary meaning of word Torsion

- Cast Iron - Cast iron usually refers to grey cast iron, but can mean any of a group of iron-based alloys containing more than 2% carbon ... read more

- The Basics of Torsion Mechanics - The general principle of inertia as a generalization of Newton's mechanics, Torsion Interactions, Torsion mechanics as a generalization of Einstein's mechanics

- SOM Syllabus - Strength of Materials Syllabus

- A Study on Controlled Low Strength Material - The aim of this research is to address CLSM's engineering properties and chemical resistance. Various tests were done to quantify the workability, strength and chemical resistance.

- Steel Terms and Definitions - Brinell Hardness, Yield Strength, Tensile Strength, Hi-Tensile Steel, Corten Steel, Domex Steel, T-1 Steel, AR 400 Steel

- Flexural Strength and Modulus of Polymers - Flexural Strength Testing of Plastics, Typical Flexural Strength and Flexural Modulus of Polymers

- Allowable Strength - Safety Factors, Shear Stresses, Thermal Stresses, Material Properties, Buckling Stress, Oscillating Stresses, Impact Strength

- Structural Joints - Fastener Quality Standards, Fastener Assembly, Welded Joints, Shear Loads (bolted joints), Screw Joints, Fastener Locking etc

- Analysis of Stress - State of Stress at a Point, Principal Stresses and Directions, Maximum Shear Stresses and Directions, Mohr's Circles, The Maximum Normal Stress Theory, The Maximum Shear Stress Theory etc

- Strength Of Materials - A to Z - Alphabetical listing of terminologies used in SOM

- Shear Strength - Shear strength Laboratory tests

- Plane Shear Flow of Cohesive Granular Materials - Article

- Principal Stress Calculator - Calculator Introduction, Equations behind the Calculator. The same page has links to Plane Strain Transform Calculator, Elastic Constant Converter and Plane Stress Transform Calculator

- Materials for young's modulus - Young's modulus formula for most of the materials

- Strength of Materials Video - Learning strengths has never been easier than with streaming video.

- Strength of Materials Basic - Modules of elasticity, Yield Strength, Bending stress etc

- Calculation of young's modulus - Online calculator to calculate young's modulus

- Materials and Formulae Page - Formulae, tables, converters specific to materials

- Poisson's Ratio - Poisson's ratio is a measure of the simultaneous change in elongation and in cross-sectional area within the elastic range during a tensile or compressive test.

- How Force, Power, Torque and Energy Work - How Force, Power, Torque and Energy Work

- Torque - Units, Moment arm formula, Static equilibrium, Force at an angle, Relationship between torque and power

- Eformulae - Strength of Materials - Stress, Strain, Hooke's Law, Piosson's Ratio, Unit Volume Change, Elongation due to its weight, Thin Rings, Strain Energy, Mohr's Circle for Biaxial Stress, Torsion formula for Thin walled tubes, Torsion formula for Circular Shafts, Flexure Formula, Shear Stress In Bending, Thin-Walled Hollow Members (Tubes), Stress Concentration, Curved Beam in Pure Bending

- Angle of Twist - Torsion: Deformation - Angle of Twist

Machine Elements:

Design and Manufacturing of Machine Elements - Notes, Tutorials

Introduction to Design Considerations, Stresses in Machine Elements, Deflection, Stiffness, and Stability, Critical Material Properties for Machine Parts, Static Stress Failure Theories, Variable Loading: Fatigue Failure, Introduction to Fracture Mechanics, Springs

Design and Manufacturing of Machine Elements Books

- Fundamentals of Machine Elements - by Bernard J. Hamrock, Bo O. Jacobson, Steven R. Schmid
  -- Provides undergraduates and praticing engineers with a clear understanding of the theory and applications behind the fundamental concepts of machine elements.

- Schaum's Outline of Machine Design - by Hall, Holowenko, Laughlin, Holowenko, Laughlin
  -- Master machine design with Schaum's--the high-performance study guide. It will help you cut study time, hone problem-solving skills, and achieve your personal best on exams!

- Machine Design : An Integrated Approach - by Robert L. Norton
  -- Machine Design presents the subject matter in an up-to-date and thorough manner with a strong design emphasis. This textbook emphasizes both failure theory and analysis as well as emphasizing the synthesis and design aspects of machine elements.

- Standard Handbook of Machine Design - by Joseph Shigley, Charles Mischke, Thomas H. Brown
  -- The definitive machine design handbook for mechanical engineers, product designers, project engineers, design engineers, and manufacturing engineers covers every aspect of machine construction and operation.

Machine Elements

- Design of Machine Elements I Syllabus

- Design of Machine Elements II Syllabus

- Machine Learning

- Tolerances and Fits

- 3-D Stress State in Mechanical Design

- Curved Steel - Practical Applications

- Contact Stresses, Castigliano's Theorem

- Machine Elements : Fundamentals Presentation
- Fatigue Design Criteria
- Preface - Fundamentals of Machine Elements
- MD Notes
- Design and Analysis of Machine Elements
- Design of Mechanical Springs
- Design Of Shafts for Deflections
- Shear and bending moment diagrams
- Mohr's circle
- 3-D Mohr's circle
- Stresses due to tension, bending, torsion

- Octahedral Stresses
- Solid Materials
- Combined Stresses and Strains - Crank
- Von Mises Stresses
- Deflection with singularity functions
- Three-support beam deflection problem
- Superposition
- Design for Deflection
- Castigliano's energy method
- How to Predict Failure
- Stress concentration

- Welded Joints
- Gears
- Gear Failure and Design
- Preliminary design of gearbox
- Tolerance
- Beam Cantil
- Bolts
- Bearings
- Shaft 1
- Shaft 2
- Rivets

- Simply Supported Beam

- Overhung Beams

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Machine Design (MD)- Notes, Tutorials

Stress-strain analysis: Stress: Definition and notation for stress; transformation of stress; principal stresses, Strain: Definition and notation for strain; transformation of strain; principal strains,Stress-strain relations for linear elastic materials: Generalized Hooke's Law, Design criteria, Maximum principal stress theory, Maximum shear stress theory, Maximum distortion energy theory, Truss finite element, 1-D, 2-D, 3-d truss element, Beam finite element, Total potential energy, Finite element approximation, Principle of minimum total potential energy, Calculation of element moments, shear forces and stresses, Finite element analysis using I-DEAS, Two-dimensional elasticity, Constant strain triangular element, Four node rectangular element, Four node iso-parametric quadrilateral element, Design optimization, Optimality condition, Convex problem, Numerical method

Machine Design Resources

Machine Design Free Subscription

Design Analysis of Machine Elements

Mechanics of Materials Software (MD Solids)

Mechanisms and Mechanical Devices Sourcebook

Section modulus calculator- beam analysis

Case Studies in Engineering Design

Aspect Oriented Programming with Spring

Engineering Statics

Building Confidence in the Design

An Ontology of Mechanical Devices

Integrating safety during the machine design stage

References On Machine Design

Machine Design Problem

Textbook for Machine Design

Introduction of Precision Machine

Principle of Measurement

Principle of Precision Design

Error Formulation and Error Budget(1)

Error Formulation and Error Budget(2)

Design of Thermomechanical System

Vibration of Precision Machine

Complinance and Residual Vibration

Friction and Accuracy

Positioning Control

Flexure Spring, Traction Drive

Optical measuring system

Nano-measuring systems

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Positioning Control

Flexure Spring, Traction Drive

Optical measuring system

Nano-measuring systems

Nano-positioning systems

Finite Elements:

Finite Element Analysis FEA, Finite Element Methods FEM Notes Tutorials

Finite Element Analysis FEA, Finite Element Methods FEM Books

- Matlab Guide to Finite Elements: An Interactive Approach - by Peter Issa Kattan
  -- This book explores the numerical implementation of Finite Element Analysis using the computer program MATLAB, which is very popular today in engineering and engineering education. The book contains a short tutorial on MATLAB as well as a systematic strategy for the treatment of finite element methods.

- Schaum's Outline of Finite Element Analysis - by George R. Buchanan
  -- If you want top grades and thorough understanding of finite element analysis, this powerful study tool is the best tutor you can have! It takes you step-by-step through the subject and gives you accompanying related problems with fully worked solutions.

- Finite Elements for Electrical Engineers - by Peter P. Silvester, Ronald L. Ferrari
  -- This third edition of the principal text on the finite element method for electrical engineers and electronics specialists presents the method in a mathematically undemanding style, accessible to undergraduates who may be encountering it for the first time. Like the earlier editions, it begins by deriving finite elements for the simplest familiar potential fields, and then formulates finite elements for a wide range of applied electromagnetics problems.

- The Finite Element Method in Electromagnetics - by Jianming Jin
  -- A comprehensive collection of the methods for the discretization of electromagnetic problems. A systematic treatment of the finite element method

- Matrix Algebra -- Review of Matrix Algebra for FEM, Eigenvalues and eigenvectors, Inverse Matrices

- Nonlinear FEA of piezoelectric transducers -- Finite Element Analysis of Piezoelectric Materials, The limitations of FEA packages with respect to piezoelectric materials, General FEA questions

- Introduction to Finite Element Methods FEM -- FEM in different fields of applications, History of FEM, FEM and other numerical methods

- FEM Accuracy -- Finite Element Method - Accuracy Improvement by Using Both E and H Formulations with Applications to Waveguide Discontinuity Problems

- The open Finite Element Project -- Finite element stress analysis (FEM) is a standard tool for engineers to check mechanical properties of parts they construct. The theory behind (linear) FEM is known since many years, and these algorithms need only a modest amount of processing power

- A New Finite Element Formulation for Electromechanics -- A new finite element formulation for the solution of electromechanical boundary value problems is presented. As opposed to the standard formulation that utilizes a scalar electric potential as nodal variables, this new formulation implements a vector potential from which components of electric displacement are derived.

- Applying FEA to Perform Heat Transfer Calculations -- Applying FEA to Perform Heat Transfer Calculations to Increase the Utility of IR Thermography

- FEA Applications -- Heat Transfer Analysis by FEA, Thermal-Stress Analysis by FEA

Syllabus : Introduction to FEM, Review of Matrix Algebra, Stiffness Matrix for Spring Element; Finite Elements Equations, Assembly of Stiffness Matrices, Bar and Beam Elements. Linear Static Analysis, Linear Static Analysis; Bar Element, Distributed Load; Transformation of Coordinate Systems; Element Stress, Beam Elements, Distributed Load, Frame Analysis, Two-Dimensional Problems, Review of the Basic Theory in 2-D Elasticity, Stiffness Matrices for 2-D Problems; Distributed Loads; Stress Calculation; FE Modeling and Solution Techniques, Plate and Shell Elements, Plate Theory, Shell Theory and Shell Elements, 3-D Elasticity; FE Formulation, Element Formulation; Solids of Revolution; Axisymmetric Elements; Structural Vibration and Dynamics, Dynamic, Equations, Free Vibration Analysis, Damping; Modal Equations; Frequency Response Analysis, Transient Response Analysis

Mathematics:

Matrix Algebra: Introduction

Matrix Algebra: Introduction

Matrices and Determinants were discovered and developed in the eighteenth and nineteenth centuries. Initially, their development dealt with transformation of geometric objects and solution of systems of linear equations. Historically, the early emphasis was on the determinant, not the matrix. In modern treatments of linear algebra, matrices are considered first. We will not speculate much on this issue. Matrices provide a theoretically and practically useful way of approaching many types of problems including:

- Solution of Systems of Linear Equations,
- Equilibrium of Rigid Bodies (in physics),
- Graph Theory,
- Theory of Games,
- Leontief Economics Model,
- Forest Management,
- Computer Graphics, and Computed Tomography,
- Genetics,
-
- Cryptography,
- Electrical Networks,
- Fractals.

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Introduction and Basic Operations:

Matrices, though they may appear weird objects at first, are a very important tool in

expressing and discussing problems which arise from real life cases.

Our first example deals with economics. Indeed, consider two families A and B (though we may easily take more than two). Every month, the two families have expenses such as: utilities, health, entertainment, food, etc... Let us restrict ourselves to: food, utilities, and health. How would one represent the data collected? Many ways are available but one of them has an advantage of combining the data so that it is easy to manipulate them. Indeed, we will write the data as follows:

$$\text{Month} = \begin{pmatrix} \textbf{Family} & \textbf{Food} & \textbf{Utilities} & \textbf{Health} \\ A & a & b & c \\ B & d & e & f \end{pmatrix}$$

If we have no problem confusing the names and what the expenses are, then we may write

$$\text{Month} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

This is what we call a **Matrix**. The size of the matrix, as a block, is defined by the number of **Rows** and the number of **Columns**. In this case, the above matrix has 2 rows and 3 columns. You may easily come up with a matrix which has m rows and n columns. In this case, we say that the matrix is a **(mxn) matrix** (pronounce m-by-n matrix). Keep in mind that the first entry (meaning m) is the number of rows while the second entry (n) is the number of columns. Our above matrix is a (2x3) matrix.

When the numbers of rows and columns are equal, we call the matrix a **square matrix**. A square matrix of **order n**, is a (nxn) matrix.

Back to our example, let us assume, for example, that the matrices for the months of January, February, and March are

$$J=\begin{pmatrix} 600 & 250 & 350 \\ 550 & 180 & 400 \end{pmatrix}, F=\begin{pmatrix} 650 & 330 & 250 \\ 600 & 270 & 400 \end{pmatrix}, \text{and } M=\begin{pmatrix} 580 & 270 & 350 \\ 625 & 350 & 410 \end{pmatrix}, \text{ respectively.}$$

To make sure that the reader knows what these numbers mean, you should be able to give the Health-expenses for family A and Food-expenses for family B during the month of February. The answers are 250 and 600. The next question may sound easy to answer, but requires a new concept in the matrix context. Indeed, what is the matrix-expense for the two families for the first quarter? The idea is to add the three matrices above. It is easy to determine the total expenses for each family and each item, then the answer is

$$\textbf{First Quarter} = \begin{pmatrix} 1830 & 850 & 950 \\ 1775 & 800 & 1210 \end{pmatrix}$$

So how do we add matrices? An approach is given by the above example. The answer is to add entries **one by one**. For example, we have

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} \alpha & \beta & \gamma \\ \theta & \nu & \mu \end{pmatrix} = \begin{pmatrix} a+\alpha & b+\beta & c+\gamma \\ d+\theta & e+\nu & f+\mu \end{pmatrix}.$$

Clearly, if you want to double a matrix, it is enough to add the matrix to itself. So we have

we get

$$\text{double of} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

which implies

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} a+a & b+b & c+c \\ d+d & e+e & f+f \end{pmatrix} = \begin{pmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \end{pmatrix}.$$

This suggests the following rule

$$2 \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \end{pmatrix}$$

and for any number λ, we will have

$$\lambda \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} \lambda a & \lambda b & \lambda c \\ \lambda d & \lambda e & \lambda f \end{pmatrix}$$

Let us summarize these two rules about matrices.

Addition of Matrices: In order to add two matrices, we add the entries one by one. Note: Matrices involved in the addition operation must have the same size.

Multiplication of a Matrix by a Number: In order to multiply a matrix by a number, you multiply every entry by the given number.

Keep in mind that we always write numbers to the left and matrices to the right (in the case of multiplication).

What about subtracting two matrices? It is easy, since subtraction is a combination of the two above rules. Indeed, if *M* and *N* are two matrices, then we will write

$$M\text{-}N = M + (\text{-}1)N$$

So first, you multiply the matrix *N* by -1, and then add the result to the matrix *M*.

Example. Consider the three matrices *J*, *F*, and *M* from above. Evaluate

$$J + 2F \ , \ J - M \ , \ \text{and} \ J - F + 2M.$$

Answer. We have

$$J + 2F = \begin{pmatrix} 600 & 250 & 350 \\ 550 & 180 & 400 \end{pmatrix} + 2 \begin{pmatrix} 650 & 330 & 250 \\ 600 & 270 & 400 \end{pmatrix}$$

and since

$$2 \begin{pmatrix} 650 & 330 & 250 \\ 600 & 270 & 400 \end{pmatrix} = \begin{pmatrix} 1300 & 660 & 500 \\ 1200 & 540 & 800 \end{pmatrix}$$

we get

$$J + 2F = \begin{pmatrix} 1900 & 910 & 850 \\ 1750 & 720 & 1200 \end{pmatrix}.$$

To compute *J-M*, we note first that

$$(-1)M = \begin{pmatrix} -580 & -270 & -350 \\ -625 & -350 & -410 \end{pmatrix}.$$

Since *J-M* = *J* + (-1)*M*, we get

$$J-M = \begin{pmatrix} 600 & 250 & 350 \\ 550 & 180 & 400 \end{pmatrix} + \begin{pmatrix} -580 & -270 & -350 \\ -625 & -350 & -410 \end{pmatrix} = \begin{pmatrix} 20 & -20 & 0 \\ -75 & -170 & -10 \end{pmatrix}$$

And finally, for *J-F*+2*M*, we have a choice. Here we would like to emphasize the fact that addition of matrices may involve more than one matrix. In this case, you may perform the calculations in any order. This is called **associativity of the operations**. So first we will take care of -*F* and 2*M* to get

$$-F = (-1)F = \begin{pmatrix} -650 & -330 & -250 \\ -600 & -270 & -400 \end{pmatrix} \text{ and } 2M = \begin{pmatrix} 1160 & 540 & 700 \\ 1250 & 700 & 820 \end{pmatrix}.$$

Since J-F+2M = J + (-1)F + 2M, we get

$$J-F+2M = \begin{pmatrix} 600 & 250 & 350 \\ 550 & 180 & 400 \end{pmatrix} + \begin{pmatrix} -650 & -330 & -250 \\ -600 & -270 & -400 \end{pmatrix} + \begin{pmatrix} 1160 & 540 & 700 \\ 1250 & 700 & 820 \end{pmatrix}.$$

So first we will evaluate J-F to get

$$J - F = \begin{pmatrix} -50 & -80 & 100 \\ -50 & -90 & 0 \end{pmatrix}$$

to which we add 2M, to finally obtain

$$J-F+2M = \begin{pmatrix} -50 & -80 & 100 \\ -50 & -90 & 0 \end{pmatrix} + \begin{pmatrix} 1160 & 540 & 700 \\ 1250 & 700 & 820 \end{pmatrix} = \begin{pmatrix} 1110 & 460 & 800 \\ 1200 & 610 & 820 \end{pmatrix}.$$

For the addition of matrices, one special matrix plays a role similar to the number zero. Indeed, if we consider the matrix with all its entries equal to 0, then it is easy to check that this matrix has behavior similar to the number zero. For example, we have

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

and

$$\lambda \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

What about multiplying two matrices? Such operation exists but the calculations involved are complicated.

Application of Invertible Matrices: Coding

There are many ways to encrypt a message. And the use of coding has become particularly significant in recent years (due to the explosion of the internet for example). One way to encrypt or code a message uses matrices and their inverse. Indeed, consider a fixed invertible matrix A. Convert the message into a matrix B such that AB is possible to perform. Send the message generated by AB. At the other end, they will need to know A^{-1} in order to decrypt or decode the message sent. Indeed, we have



which is the original message. Keep in mind that whenever an undesired intruder finds A, we must be able to change it. So we should have a mechanical way of generating simple matrices A which are invertible and have simple inverse matrices. Note that, in general, the inverse of a matrix involves fractions which are not easy to send in an electronic form. The best is to have

both A and its inverse with integers as their entries. In fact, we can use our previous knowledge to generate such class of matrices.

Indeed, if A is a matrix such that its determinant is ± 1 and all its entries are integers, then A^{-1} will have entries which are integers. So how do we generate such class of matrices? One practical way is to start with an upper triangular matrix with ± 1 on the diagonal and integer-entries. Then we use the elementary row operations to change the matrix while keeping the determinant unchanged. Do not multiply rows with non-integers while doing elementary row operations. Let us illustrate this on an example.

Example. Consider the matrix

$$\begin{pmatrix} -1 & 5 & -1 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{pmatrix}.$$

First we keep the first row and add it to the second as well as to the third rows. We obtain

$$\begin{pmatrix} -1 & 5 & -1 \\ -1 & 6 & 7 \\ -1 & 5 & 0 \end{pmatrix}.$$

Next we keep the first row again, we add the second to the third, and finally add the last one to the first multiplied by -2. We obtain

$$\begin{pmatrix} -1 & 5 & -1 \\ -2 & 11 & 7 \\ 1 & -5 & 2 \end{pmatrix}.$$

This is our matrix A. Easy calculations will give $det(A) = -1$, which we knew since the above elementary operations did not change the determinant from the original triangular matrix

which obviously has -1 as its determinant. We leave the details of the calculations to the reader. The inverse of *A* is

$$\begin{pmatrix} 57 & -5 & 46 \\ 11 & -1 & 9 \\ -1 & 0 & -1 \end{pmatrix}.$$

Back to our original problem. Consider the message

I LOVE MONICA

To every letter we will associate a number. The easiest way to do that is to associate 0 to a blank or space, 1 to A, 2 to B, etc... Another way is to associate 0 to a blank or space, 1 to A, -1 to B, 2 to C, -2 to D, etc... Let us use the second choice. So our message is given by the string

I		*L*	*O*	*V*	*E*		*M*	*O*	*N*	*I*	*C*	*A*
5	0	−6	8	−11	3	0	7	8	−7	5	2	1

Now we rearrange these numbers into a matrix *B*. For example, we have

$$B = \begin{pmatrix} 5 & 8 & 0 & -7 & 1 \\ 0 & -11 & 7 & 5 & 0 \\ -6 & 3 & 8 & 2 & 0 \end{pmatrix}.$$

Then we perform the product *AB*, where *A* is the matrix found above. We get

$$AB = \begin{pmatrix} -1 & 5 & -1 \\ -2 & 11 & 7 \\ 1 & -5 & 2 \end{pmatrix} \begin{pmatrix} 5 & 8 & 0 & -7 & 1 \\ 0 & -11 & 7 & 5 & 0 \\ -6 & 3 & 8 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -66 & 27 & 30 & -1 \\ -52 & -116 & 133 & 83 & -2 \\ -7 & 69 & -19 & -28 & 1 \end{pmatrix}.$$

The encrypted message to be sent is

$$1, \ -52, \ -7, \ -66, \ \ldots, \ -1, \ -2, \ 1$$

System of Equations: An Introduction

Many books on linear algebra will introduce matrices via systems of linear equations. We tried a different approach. We hope this way you will appreciate matrices as a powerful tool useful not only to solve linear systems of equations. Basically, the problem of finding some unknowns linked to each others via equations is called a system of equations.

For example,

$$\begin{cases} 2x - 3y &= 1 \\ x + 3y &= -2 \end{cases}$$

and

$$\begin{cases} x^2 + y^2 &= 1 \\ x + 3y &= -2 \end{cases}$$

are systems of two equations with two unknowns (x and y), while

$$\begin{cases} 2x - 3y^2 &= -1 \\ x + y + z &= 1 \end{cases}$$

Introduction to Determinants

For any square matrix of order 2, we have found a necessary and sufficient condition for invertibility. Indeed, consider the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

The matrix A is invertible if and only if $ad - bc \neq 0$. We called this number the **determinant** of A. It is clear from this, that we would like to have a similar result for bigger matrices (meaning higher orders). So is there a similar notion of determinant for any square matrix, which determines whether a square matrix is invertible or not?

In order to generalize such notion to higher orders, we will need to study the determinant and see what kind of properties it satisfies. First let us use the following notation for the determinant

$$\text{determinant of} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Properties of the Determinant :

1. Any matrix A and its transpose have the same determinant, meaning

$$\det A = \det A^T$$

This is interesting since it implies that whenever we use rows, a similar behavior will result if

we use columns. In particular we will see how row elementary operations are helpful in finding the determinant. Therefore, we have similar conclusions for elementary column operations.

2. The determinant of a triangular matrix is the product of the entries on the diagonal, that is

$$\begin{vmatrix} a & b \\ 0 & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ b & d \end{vmatrix} = ad.$$

3. If we interchange two rows, the determinant of the new matrix is the opposite of the old one, that is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = - \begin{vmatrix} c & d \\ a & b \end{vmatrix}$$

4. If we multiply one row with a constant, the determinant of the new matrix is the determinant of the old one multiplied by the constant, that is

$$\begin{vmatrix} \lambda a & \lambda b \\ c & d \end{vmatrix} = \lambda \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ \lambda c & \lambda d \end{vmatrix}.$$

In particular, if all the entries in one row are zero, then the determinant is zero.

5. If we add one row to another one multiplied by a constant, the determinant of the new matrix is the same as the old one, that is

$$\begin{vmatrix} a+\lambda c & b+\lambda d \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c+\lambda a & d+\lambda b \end{vmatrix}.$$

Note that whenever you want to replace a row by something (through elementary operations), do not multiply the row itself by a constant. Otherwise, you will easily make errors (due to Property 4).

6. We have

$$\det(AB) = \det(A)\det(B).$$

In particular, if A is invertible (which happens if and only if $\det(A) \neq 0$), then

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

If A and B are similar, then $\det(A) = \det(B)$.

Let us look at an example, to see how these properties work.

Example. Evaluate

$$\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix}.$$

Let us transform this matrix into a triangular one through elementary operations. We will keep the first row and add to the second one the first multiplied by $\frac{1}{2}$. We get

$$\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 0 & \frac{7}{2} \end{vmatrix}.$$

Using the Property 2, we get

$$\begin{vmatrix} 2 & 1 \\ 0 & \frac{7}{2} \end{vmatrix} = 2 \cdot \frac{7}{2} = 7.$$

Therefore, we have

$$\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} = 7$$

which one may check easily.

is a system of two equations with three unknowns (x, y, and z).

These systems of equations occur naturally in many real life problems. For example, consider

a nutritious drink which consists of whole egg, milk, and orange juice. The food energy and protein for each of the ingredients are given by the table:

	Food Energy (Calories)	Protein (Grams)
1 egg	80	6
1 cup milk	160	9
1 cup orange juice	110	2

A natural question to ask is how much of each ingredient do we need to produce a drink of 540 calories and 25 grams of protein. In order to answer that, let x be the number of eggs, y the amount of milk (in cups), and z the amount of orange of juice (in cups). Then we need to have

$$\begin{cases} 80x + 160y + 110z &= 540 \\ 6x + 9y + 2z &= 25 \end{cases}$$

The task of **Solving** a system consists of finding the unknowns, here: x, y and z. A solution is a set of numbers once substituted for the unknowns will satisfy the equations of the system. For example, (2,1,2) and (0.325, 2.25, 1.4) are solutions to the system above.

The fundamental problem associated to any system is to find all the solutions. One way is to study the structure of its set of solutions which, in some cases, may help finding the solutions. Indeed, for example, in order to find the solutions to a linear system, it is enough to find just a few of them. This is possible because of the rich structure of the set of solutions.

Determinant and Inverse of Matrices:

Finding the inverse of a matrix is very important in many areas of science. For example, decrypting a coded message uses the inverse of a matrix. Determinant may be used to answer this problem. Indeed, let A be a square matrix. We know that A is invertible if and only if

$\det(A) \neq 0$. Also if A has order n, then the cofactor $A_{i,j}$ is defined as the determinant of the square matrix of order (n-1) obtained from A by removing the row number i and the column number j multiplied by $(-1)^{i+j}$. Recall

$$\det(A) = \sum_{j=1}^{j=n} a_{ij} A_{ij}$$

for any fixed i, and

$$\det(A) = \sum_{i=1}^{i=n} a_{ij} A_{ij}$$

for any fixed j. Define the **adjoint** of A, denoted $adj(A)$, to be the transpose of the matrix whose ij^{th} entry is A_{ij}.

Example. Let

$$A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 1 & -1 \end{pmatrix}.$$

We have

$$adj(A) = \begin{pmatrix} -2 & 5 & -1 \\ 5 & -7 & 8 \\ 6 & -4 & 3 \end{pmatrix}^T = \begin{pmatrix} -2 & 5 & 6 \\ 5 & -7 & -4 \\ -1 & 8 & 3 \end{pmatrix}.$$

Let us evaluate $A \cdot adj(A)$. We have

$$A \cdot adj(A) = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 2 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} -2 & 5 & 6 \\ 5 & -7 & -4 \\ -1 & 8 & 3 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}.$$

Note that $\det(A) = 11$. Therefore, we have

$$A \cdot adj(A) = \det(A) I_3.$$

Is this formula only true for this matrix, or does a similar formula exist for any square matrix? In fact, we do have a similar formula.

Theorem. For any square matrix *A* of order n, we have

$$A \cdot adj(A) = \det(A) I_n.$$

In particular, if $\det(A) \neq 0$, then

$$A^{-1} = \frac{1}{\det(A)} adj(A).$$

For a square matrix of order 2, we have

$$adj\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}^T = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

which gives

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Strength of Materials:

Engineering and Applications Factor of Safety Review:

Factor of safety for structural applications is the ratio of the allowable working unit stress, allowable stress or working stress. The term was originated for determining allowable stress. The ultimate strength of a given material divided by an arbitrary factor of safety, dependant on material and the use to which it is to be put, gives the allowable stress.

$$s_w = s_m / f_s$$

Where:

S_m = Allowable working unit stress
s_w = Working stress (Allowable stress)
f_s = Factor of Safety

In present design and engineering practice, it is customary to use allowable stress as specified by recognized industry standards or authorities as applicable rather than to use an arbitrary factor of safety. One reason for this is that the factor of safety is misleading, in that it implies a greater degree of safety than may actually exists. For example, a factor of safety of 4 does not mean that a component or assembly application can carry a load four times as great as that for which it was designed. It should also be clearly understood that, even though each part of a machine may be designed with the same factor of safety, the machine as a whole does not have that factor of safety. In the event that one part is stressed beyond the proportional limit, or particularly the yield point, the load or stress distribution may be completely changed throughout the entire machine or structure, and its ability to function at rated load may be

changed, even though no part has failed or ruptured.

The following should be considered when designing and analyzing a structural or machine component or assembly:

- Effects of associated assemblies or components.
- Thermal cycling and operating or extreme temperature effects.
- Intensity of stress concentrations.
- Effects of wear
- Likely ness of structural or machine abuse. Controls may not be in place to prevent an overstress condition. Classic example is the general public exceeding automotive (truck) rated towing capacity. Such events particularly when repeated, tend accelerate wear and failure of critical mechanical drive train components and assemblies.
- Quality or likeness of scheduled maintenance.
- Dimensional control and effects on quality of assembly. Excessive internal stresses can never be properly estimated. Material stresses induced by non-perfect geometry are difficult to model. Stresses can also be introduced by assembly misalignment between components.
- Influence of fatigue loading over the life cycle of the machine or structure.

Although no definite or universal rules can be given , if a factor of safety needs to be determined or established, the following circumstances should be taken into account in its selection:

1. When the ultimate strength of the material is known within narrow limits, as for structural steel for which tests of samples have been made, when the load is entirely a steady one of a known value a factor of safety should be adopted is 3.
2. When circumstances of (1) are modified by a portion of the load being variable, as in, gear boxes, floors or warehouse operations, the factor of safety should not be less than 4.
3. When the whole load , or nearly the whole, is likely to be alternately put on and taken off, as in suspension rods as used with suspension floors or bridges, the factor should be 5 or 6.
4. When the stresses are reversed in direction from tension to compression, as in some structural load bearing diagonals and parts of machines, the factor should be not less than 6.
5. When the components are subjected to repeated shock loading the factor should not be less than 10.
6. When the structure or component is subjected to deterioration from corrosion the components or structure factor of safety should be sufficiently increased to allow for a definite amount of material reduction before the system is weakened by the process.
7. When the strength of the material or the amount of the load or both are uncertain the factor of safety should be increased by an allowance sufficient to cover the amount of the uncertainty.
8. When the stress and strains are complex and of uncertain amount, such as those in the

crankshaft of a reversing engine, a very high factor is necessary, possibly even as high as 40 or more.

9. If property loss caused by failure of the part or system may be large or if loss of life may result, the factor of safety should be large and the structure or machine performance should be verified by functional static or fatigue testing.

Von Mises Criterion:

This criterion is based on the determination of the distortion energy in a given material, i.e., of the energy associated with changes in the shape in that material (as opposed to the energy associated with the changes in volume in the same material).

According to this criterion, named after German-American applied mathematician Richard von Mises (1883-1953), a given structural material is safe as long as the maximum value of the distortion energy per unit volume in that material remains smaller than the distortion energy per unit volume required to cause yield in a tensile-test specified of the same material.

Cold and Hot Working:

Plastic deformation which is carried out in a temperature region and over a time interval such that the strain hardening is not relieved is called *cold work*. Considerable knowledge on the structure of the cold-worked state has been obtained. In the early stages of plastic deformation, slip is essentially on primary glide planes and the dislocations form coplanar arrays. As deformation proceeds, cross slip takes place. The cold-worked structure forms high dislocation density regions that soon develop into networks. The grain size decreases with strain at low deformation but soon reaches a fixed size. Cold working will decrease ductility.

Hot working refers to the process where metals are deformed above their recrystallization temperature and strain hardening does not occur. Hot working is usually performed at elevated temperatures. Lead, however, is hot-worked at room temperature because of its low melting temperature. At the other extreme, molybdenum is cold-worked when deformed even at red heat because of its high recrystallization temperature.

The resistance of metals to plastic deformation generally falls with temperature. For this reason, larger massive sections are always worked hot by forging, rolling, or extrusion. Metals display distinctly viscous characteristics at sufficiently high temperatures, and their resistance to

flow increases at high forming rates. This occurs not only because it is a

characteristic of viscous substances, but because the rate of recrystallization may not be fast enough.

Heat Treatment:

During manufacture, by varying the rate of cooling (*quenching*) of the metal, grain size and grain patterns are controlled. Grain characteristics are controlled to produce different levels of hardness and tensile strength. Generally, the faster a metal is cooled, the smaller the grain sizes will be. This will make the metal harder. As hardness and tensile strength increase in heat-treated steel, toughness and ductility decrease.

The cooling rate used in quenching depends on the method of cooling and the size of the metal. Uniform cooling is important to prevent distortion. Typically, steel components are quenched in oil or water.

Welding can induce internal stresses that will remain in the material after the welding is completed. In stainless steels, such as type 304, the crystal lattice is face-centered cubic (austenite). During high temperature welding, some surrounding metal may be elevated to between 500°F and 1000°F. In this temperature region, the austenite is transformed into a bodycentered cubic lattice structure (bainite). When the metal has cooled, regions surrounding the weld contain some original austenite and some newly formed bainite. A problem arises because the "packing factor" (PF = volume of atoms/volume of unit cell) is not the same for FCC crystals as for BCC crysta

The bainite that has been formed occupies more space than the original austenite lattice. This elongation of the material causes residual compressive and tensile stresses in the material. Welding stresses can be minimized by using heat sink welding, which results in lower metal temperatures, and by annealing.

Annealing is another common heat treating process for carbon steel components. During annealing, the component is heated slowly to an elevated temperature and held there for a long period of time, then cooled. The annealing process is done to obtain the following effects.

a. to soften the steel and improve ductility

b. to relieve internal stresses caused by previous processes such as heat treatment, welding, or machining

c. to refine the grain structure