GATE Material for Civil Engineering

Grading and Site Preparation

The site you are building on should be flat, with no lumps or hillocks. to level, you could hire a bulldozer and go over the land with the blade close to the ground to remove all lumps or hillocks. If the plot is on a severe incline then lay stilts out so that you can lay out a floor level on top of them. At timesground filling has to be done to raise ground to the required level.

Excavation & Foundation Construction

Foundation Construction is an integral part of home building. It is what the whole house rests on. A firm foundation is a must, so don't use sand. In most construction, concrete is used as foundation. It is laid into a trench up to 3 or 4 feet deep, and provides a very solid foundation for whatever building material used. There are many different types of foundations. The most typical foundation is made up of a footing which is the portion of the steel reinforced concrete that bears on the undisturbed soil. Footings are typically 12" 16" up to 36" inches wide by 10" to 12" deep depending on the soil on which they are bearing. A foundation wall is general applied over the footing to above grade. Foundation walls are typically 8" wide and terminate 8' above the finished grade. like column footing , pile, raft etc. these are used as per condition of soil. Waterproofing, damproofing and drainage are typically applied at the exterior of the foundation and footing. This is particulary important when building in wet soils.

Rough Framing

The Framing begins with the anchoring of the studs to the concrete foundation. Walls are constructed of Wood Studs typical 16" on center - framing studs come in 2 x 4, 2 x 6, 2 x 8, 2 x10 and lengths of 6,8,10,12 feet and longer. Floor/Ceiling joists are typically an engineered lumber product, as are the Roof Trusses, both ordered to size. Sheer Walls are constructed by sheathing the stud walls with plywood onone or both sides - used primarimly in earthquake regions. Exterior walls are sheathed with plywood or oriented strand board (OSB) and usually a Owens Corning Rigid Insulation Product.

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    - NOTE: There are no Federal Building Codes, each State and then County adopts its own building and safety codes. Typically a framing inspection, electrical inspection and plumbing inspection is required prior to walls being insulated and/or drywalled.

Rough Plumbing

Installation of Hot/Cold Supply and Waste Lines, Shower and Tub Valves, Hot Water Heater, Water Purifier/Softner and Fire Supression System

Septic/Sewage Systems

How the septic tank system works - A typical septic system, consists of an underground tank, a distribution line and a soil absorption drainage field. Wastewater leaves the home through an underground pipe connected to the septic tank. Baffles in the septic tank slow the flow of the incoming wastewater and prevent sewage from flowing directly through the tank. Heavier solids settle to the bottom and accumulate as sludge.

The septic tank system is a natural sewage treatment and disposal system. By natural we mean that it relies on bacteria to digest and clean the wastewater. Septic systems are biological systems and must have bacteria to work. The bacteria in the septic tank literally eat & digest the solids in the tank turning them into liquids and gases.

Raw household wastewater will quickly clog all but the most porous gravel formations. A septic tank is a watertight chamber, which conditions the wastewater to reduce clogging so that it may be more readily absorbed into the soil. The septic tank separates the settable and floatable solids, promotes the growth of anaerobic bacteria necessary to decompose the solids, and provides storage for the resulting sludge and scum.

Constructing a septic tank system:

Septic tanks should be located where they will not cause contamination of any well, spring or any other source of water supply.

The septic tank should not be located closer than 10 feet from any building. The tank should not be located in swampy areas, or in areas subject to flooding.

Slopes greater than 25 percent will cause considerable difficulty in construction, and may slide once they become saturated with effluent.

There must be no permanent cover placed over the septic tank or lateral lines.

Masonry

Brick Firebox & Chimney Construction and installation of Prefabricated Firebox Inserts with Flue/Dampers

Brick veneer on wood frame walls is a standard home building practice.

When applied over wood frame walls, a weather-resistive-barrier is critical to protect the wood framing from water damage. Masonry is not waterproof.

Rough Electrical

An important document to read is the IEE Wiring Regulations (or equivilant).

Installation of Transformers, Service Panels, Switches, Outlets, GFI Outlets, Jboxs, Recessed Light Fixtures, Exhaust Fans, Fire/Security Horns and Emergency Lighting

Rough Low Voltage Systems

installation of wiring for Security Camera/Alarm, Telephone/Intercom/Internet Communications, Audio/Video Components, LV Light Fixtures and Home Automation Contolers

Rough HVAC (Heating, Ventilation and Air Conditioning)

Furnace/Air Conditioner Unit Installation, Supply/Return Air Ducts, Exhaust Ducts, Radiant Floor Systems, Air Purifier/Humidity Systems and Vacuum Systems.

Roof

Mansford, Shed, Hip, Gamble, Saltbox and Gable can be covered in Asphalt and Wood Shingle, Terra Cotta Tiles, Slate Tiles, Copper Sheeting and Metal Standing Seam the most common roofing materials. Based on geometry roofs can be classified as sloped, pitched or flat. The "pitch" of the roof is dependant on geographic location - Higher pitched rooves are typically found in cold climates but also in wet climates.

Rain Control

Depending on the archiectual style the Gutter, Downspouts and Collector Box's may be exposed or built with the roofs eaves and walls.

Exterior Openings

Installation of Exterior Windows, Doors, Roof Hatch and Skylights requires weather proofing be at its best. Although older houses may still contain single glazed wooden framed windows and doors; modern house usually have a steel or aluminium reinforced uPVC framed double glazed system. These window system are fixed to the preformed concrete ope by use of usually a galvanised steel fixing bracket which clips to the frame of the window or door and is fix back to reveal.

The window will sit either on the upstand of a precast concrete cill or onto a pvc of aluminium PPC cill. A silicone sealant or expandable foam can be used to weather the window or door once in place.

Doors may have to be sunk a few millimeters in order to achieve the correct height of threshold depending on building regulations governing the use of the door.

Exterior Wall Surfaces

Materials as Stucco, Brick, Stone, Wood Clapboard and Shingle are all cladding products. Modern woodframe construction requires the installation of a weather-resistive-barrier (WRB) under the cladding. The WRB provides the primary funtion of protecting the structure from water.

Exterior Cast Stone

Door/Window Surrounds, Eaves, Balconies, Columns/Pilasters and Steps/Coping

Interior Doors

Installation of Jambs for Pocket/French/BiFold type doors.

Door & Window Hardware

Installation of Door Hinges, Knobs/Lever, Cremone/Lock, Treasholds and Window Locks/Cranks "Taqi"

Interior Wall Surfaces

Drywall, Plaster, Wood Panel and Tile/Stone/Brick are typical selections.

Insulation

A fundamental principle in building. It the means by which the interior environment can be made thermally comfortable. Insulation works by limiting the loss of heat from a relatively warm environment to a relatively cold environment. This is achieved in a number of ways:

Limiting Convection (air) heat loss. Most forms of insulation (such as polystyrene and fiberglass) contain cellular structures that trap air. This trapped air acts as a barrier to the passage of air through the insulating material.

Limiting Radiation heat loss Insulanting materials are generally good at limiting radiated heat by reflecting it in the same way that a mirror reflects light. Shiny metals such as aluminum are good at reflecting heat, and this material is often used as a backing for insulated panels.

Wood Flooring

Wood Plank, Strip and Parquet, Decorative Inlays/Boarders and Staircase Risers/Treads

Stone & Tile

Ceramic/Terra Cotta, Terrazo, Concrete, Mosaics and Inlays Floors, Bathroom Walls/Showers/Tub Decks, Fireplace Mantels/Surrounds/Hearths, Countertops/Backsplash, Staircase Risers/Risers, Columns/Pedestals and Exterior Terrace/Walks/Paths/Drives

Cabinetry

Built In Bookcases/Desks, Powder Room Pullmans, Bathroom & Kitchen Cabinetry, Pantry/Linen Shelving and Closetry

Millwork

Wall Paneling, Coffered Cieling, Decorative Beams, BaseBoard, Door/Window Casing, Applied Wall Molding, Crown/Cornice Moldings, Columns/Pilasters Overdoor, Staircase Ballustrades/Hand Rail and Fireplace Mantels

Metalwork

Wrought Iron Railings, Doors/Windows, Pedestrian/Auto Gates, Fences and Grilles. Copper Awnings, Chimney Caps and Hoods

Finish Electrical

Switch/Outlet Covers, Recessed Lighting Trims, Installation of Decorative Light Fixtures (Chandeliers/Sconce/Lanterns)

Finish Plumbing

Installation of Fixtures (Bidet/Toilet), Trims of Faucet/Drain Fittings, Connect Tub/Sink Drains and Volume Tests

Finish HVAC

Decorative Grilles/Diffusers and Controler Trims

Appliances

Kitchen Refrigerator/Freezer, Range, Ovens, Hoods, Dishwasher, Ice makers, Wine Coolers. Laundry Washer and Dryers

Interior Wall/Ceiling Finishes

Primed, Painted, Stained, Wallpaper, Faux Finishes and Venetian Plasters

Glass/Mirror

Decorative Leaded and Etched Glass Window/Doors, Shower and Tub Doors/Enclosures and Wall Mirror used in house.

Landscape Hardscape

Walls, Terraces, Walks, Stairs, Curbs and Driveways

Landscape Watering

Spinkler, Driblers and Misters

Landscape Lighting

Path/Walk, Stair, Plant/Tree, Water Features and Security

Landscape Pool/Spa

Swiming Pool, Spa, Fountains and Reflecting Pools

Landscape Structures

Gazebo, Cabana, Canvas Awnings and BBQ/Cooking Areas

Landscape Plantings

Lawn, Trees, Shrubs, Flowers and Potted Plants

Furnishings

Floor Coverings (Carpet/Area Rugs), Window Coverings (Drapes/Shades and Sun Control: Window Tint)

Punch list

Contractor and Home Owner "To Do" List

Introduction to Materials Science:

Introduction

Different materials have different properties. Think of the difference between the engine of a car and its wheels; the metal in a wire and its insulator. All these objects can only be made out of materials that have properties suited to their application.

Materials science is the study of the properties of materials. It focuses on the factors that make one material different from another. Understandably, there are many such factors, some obvious and some subtle. Examples of these factors might include elemental composition, arrangement, bonding, impurities, surface structure, length scale and so on. The ability to understand the relationships between these factors and the properties of a material has been crucial to most of mankind's technological breakthroughs. Today, materials science is a multidisciplinary subject. It draws upon just about every field of science and engineering, providing insights for other researchers to use in their field.

This book is aimed at those studying materials science at the undergraduate level in university whether as their major field or as a single module of a related engineering course.

Structure of Matter

Atomic Structure and Bonding

Fundamentally, two types of bonding exist- bonds between atoms and bonds between ions. Bonds between atoms of nonmetals are covalent, meaning that they share a pair of electrons in the space between them. These two atoms are bound together and cannot be separated by simple physical means. If these two atoms have similar electronegativity, neither atom has more pull on the electron pair than the other. This type of covalent bond is called **Non Polar**. Examples of non polar covalent compounds are methane, carbon dioxide and graphite. In graphite, all atoms are identical and so no atom has stronger pull than any of the others. In methane, the carbon-hydrogen bonds are very slightly polar, and the polarities are cancelled because the bonds all point to the same locus.

Crystal Structure

Defects

Defects of materials are subject to intense study. However there are some methods to determine the source of defects and, if occurred, the size, shape and position of defects in the materials. There are: **destructive testing methods** and **Non destructive testing methods (NDT)**.

Thermodynamics of Material

Phase Diagrams

Phase diagrams provide a graphical means of presenting the results of experimental studies of complex natural processes, such that at a given temperature and pressure for a specific system at equilibrium the phase or phases present can be determined.

SYSTEM - Any portion of the universe which is of interest and can be studied experimentally.

PHASE- any particular portion of a system, which is physically homogeneous, has a specific composition, and can be mechanically removed or separated from any other phase in the system.

* e.g. A system containing a mixture of ol and pl in equilibrium contains two phases - ol and pl.

In petrology we generally deal with primary phases - any crystalline phase which can coexist with liquid, i.e. it formed/crystallized directly from the liquid.

EQUILIBRIUM - The condition of minimum energy for the system such that the state of a reaction will not change with time provided that pressure and temperature are kept constant.

In experimental petrology there are three practical criteria used to test for equilibrium.

1. Time - with time the system does not change its physical or chemical makeup. 2. Approach equilibrium from two directions, e.g. the melting point of Albite. * begin with a liquid of Ab composition (Na_2O-Al_2O_3-$6SiO_2$) and cool until Ab crystallizes - T=1100°C * begin with the same mixture of solid Albite and heat it up until liquid forms - T=1120°C Melting point of albite = 1110°C + 10°C. 3. Attainment of equilibrium by using different reactants and procedures. To determine the melting temperature of Albite * grind up a sample of pure albite * combine powdered oxides to give pure Ab composition Use both to determine Ab melting point.

One final term to be defined prior to examining phase diagrams.

COMPONENT - the smallest number of independent variable chemical constituents necessary to define any phase in the system.

* components may be oxides, elements or minerals, dependant on the system being examined.

For example, experiments carried out in the H_2O system, show that the phases which appear over a wide temperature and pressure range are ice, liquid water and water vapour. The composition of each phase is H_2O and only one chemical parameter or component is required to describe the composition of each phase. Systems which can be defined by a single component are called Unary Systems. H_2O System In this system pressures from 0 to 15 kbars seven phases, each with the same composition - H_2O have been recognized:

* Ice I * Ice II * Ice III * Ice IV(actually not exist) * Ice V * Water * Steam

SiO_2 System In the one component SiO_2 system in the temperature range from 0 to 2,000°C and a pressure range from 0 to 30 kbars six phases of SiO_2 are recognized. At pressures > 30 kbar a seventh phase, stishovite, exists. The six phases of SiO_2 are:

* coesite * alpha quartz (Trigonal) * beta quartz (hexagonal) * tridymite * cristobalite * anhydrous melt

Materials Processing

In order to produce an article for any application out of a particular material there are several steps that may be required. The first step is usually to obtain the raw materials from our environment. This may involve discovering where these raw materials are located (often achieved with knowledge of geology) and developing processes to extract them from these locations (e.g. mining the ores, drilling for oil etc.). Otherwise, it may be possible to find sources of material suitable for recycling or reprocessing. Once these raw materials have been obtained they may need to undergo some initial processing to get them into a usable form. This may be some form of extractive metallurgy, chemical synthesis or some other chemical process. It may also be necessary to mix different raw materials to achieve a certain composition (e.g. alloying in metals) that is appropriate for or has been optimised the application. The application will usually require that the material be in a particular shape and a suitable shaping process or combination of process must be employed to achieve this. Often, it may be possible to produce a shape out of a

material with any one of the many different shaping processes. However, there is usually one particular process that either results in particular benefits in terms of the properties of the material or the article that is produced or meets some other important criteria - such as low cost - that it is selected over the other options. Finally, it may be necessary or beneficial to process the article further, once it has been formed, in order to optimise the properties of the material.

Firstly, this chapter will present the various chemical processes that may be necessary to produce suitable materials from the raw materials in our environment. The different methods for shaping these materials will then be presented. Finally, the processes used to optimise the properties of the materials will be discussed.

Chemical Processing
Extraction of Raw Materials
Chemical Synthesis of Materials
Shaping Processes
Melt Processing
Casting
Physical Processing
Forging
Rolling
Extrusion

Powder Process

Powder processes are used in the production of metallic and ceramic parts. The use of metal powders is commonly referred to as Powder Metallurgy (P/M).

There are 4 main stages to producing products with, they are: Powder Mixing, Compaction, Sintering and final finishing.

A metal or ceramic powder is prepared, then compacted into a desired shape. This part is then heated in a furnace causing the powders to weld together forming a solid part. The part is then final processed by final shaping, minor smoothing, or drilling.

Using Powders to produce parts is viable when you require a high volume of simple parts that need to be cost efficient. All though casting can also do this, P/M offers near net shape products. This means that the part that comes out of the process needs little or no finishing done to it.

Ceramics lend themselves well to powder processing as they are very hard and brittle, thus a near net shape is highly beneficial.

Mixing

Mixing is mainly done to add waxes for the compaction, binders to temporarily strengthen the compacts and sometimes to get the right chemistry.

As most suppliers recommend lubricant for idea compaction, mixing is a very important process, so a homogenous mixture is required. Optimum mixing occurs with turbulent mixing and at low centrifugal forces.

Along with ensuring a homogenous mix, the mixing process also provides some milling of the powders. As we all know you can put more tennis balls in an area than beach balls, thus increasing the surface area of the balls. The same is true with powders, more surface area, the better the final product is.

Compaction

Compaction is the process of squishing the powders into the desired shape with enough force so as to hold its shape. This is called a green body, as it still has moisture in it and needs to be Sintered. Same basic concept as pottery, the plate or cup is considered "green" until it is fired

There are 2 categories of pressing: Isostatic and Axial.

Sintering

Sintering is simply the furnace heating of a compacted powder object, also known as a green body to form a solid part. The powders can be either metallic or ceramic. They can be in elemental form, as an alloy, or mixture of both. Most sintering processes are done in a protective atmosphere, such as nitrogen or hydrogen mixed gas, to avoid degradation of the green bodies, and at a temperature lower than the melting point, approximately 60~90% of the main elements meting point. The specific atmosphere and temperature is dependant upon the material being processed.

If the material being sintered is an alloy, it is possible that one or more of the constitutes has a melting point lower than the sintering temperature, thus causing a small amount of liquid to form. This is called Liquid Phase Sintering. Caution needs to be taken when choosing a temperature as too much liquid will result in the deformation of the part. This is referred to as slumping.

The mechanism of sintering is the diffusion of the atoms across the particle boundaries of compacted powders. As the atoms diffuse, all voids are filled and the material forms one solid part. As the voids between particles are no longer present, the part increases in density, and experiences shrinkage. However, due to the nature of this process, only 93%-98% theoretical density can be achieved, thus further mechanical processing is needed to obtain 100% dense material.

The resultant microscopic structure resembles the starting green compact. The starting particle boundaries eventually turn into the final grain boundaries.

As the voids between the powder particles are filled during the sintering process, the gases need to be expelled from the compact. These gases are; air trapped between powders and gasses from additives added during the mixing and/or compaction process. These gases are expelled through

capillaries formed by the particle boundaries. If the compacts are hated to fast, these capillaries can be “pinched” off and if these gasses are not expelled, the part will have defects such as warpage, porosity, or even holes.

A typical industrial sintering process is done on a traveling grate furnace with a 2 stages of heating. The green bodies are placed on a conveyor which travels into the furnace which has a positive pressure protective atmosphere blown onto the conveyor belt. The parts travel into the first temperature zone to vaporize and wax and degas. The second temperature zone is to do the actual sintering of the material. After the appropriate sinter time, the parts travel through a cooling zone to allow the parts to be handled, or to lock properties for continued processing. Degas and sinter times vary based on material.

Finishing Machining Welding

Materials Optimisation

Heat Treatment:It is defined as combination of heating and cooling cycles given to a particular material of interest to achieve desired properties.

Surface Engineering

Materials Characterisation

An important aspect of materials science is the characterisation of the materials that we use or study in order to learn more about them. Today, there is a vast array of scientific techniques available to the materials scientist that enables this characterisation. These techniques will be introduced and explained in this section.

Macroscopic Observation

The first step in any characterisation of a material or an object made of a material is often a macroscopic observation. This is simply looking at the material with the naked eye. This simple process can yield a large amount of information about the material such as the colour of the material, its lustre (does it display a metallic lustre), its shape (whether it displays a regular, crystalline form), its composition (is it made up of different phases), its structural features (does it contain porosity) etc. Often, this investigation yields clues as to what other tests could be performed to fully identify the material or to solve a problem that has been experienced in use.

Microscopic Observation

Microscopy is a technique that, combined with other scientific techniques and chemical processes, allows the determination of both the composition and the structure of a material. It is essentially the process of viewing the structure on a much finer scale than is possible with the naked eye and is necessary because many of the properties of materials are dependent on extremely fine features and defects that are only possible to observe using one of the following techniques in this field.

Optical Microscopy

Optical microscopes are formed of lenses that magnify and focus light. This light may have been transmitted through a material or reflected from a material's surface and can be used to ascertain a great deal of information about that material under evaluation. This can include whether the material is dense or contains porosity, what colour the material is, whether the material is composed of a single phase or contains multiple phases etc.

A common practice performed in conjunction with optical microscopy is that of targeted and controlled chemical attack of the material using one of many chemical reagents available. For metallic materials, this technique combined with optical microscopy is know as optical metallography. The basis of this combined technique is that regions of different composition within a material as well as entirely different materials are affected differently when exposed to certain chemicals. These chemical effects are catalogued in various works (for example the ASM Metals Handbook or Metallographic Etching by G. Petzow) and through an understanding of these effects and a systematic experimental process they can be used to determine material composition and structure.

There are several limitations to the usefulness of optical microscopy. The first is that the maximum resolving power is limited by diffraction effects to approximately 0.2 micrometres at a magnification of around 1500X (see reference). Many of the defects and structural features important in determining material properties, and therefore of interest to materials scientists, are of atomic scale. (for reference, the diameter of a helium atom is approximately 100 picometers) The second major limitation in optical microscopy is limited depth of field. This limitation means that surfaces with features at different heights - such as the rough surfaces of a fractured specimen for example - cannot be seen in sharp focus at the same time. This means that flat or polished surfaces are preferred for this technique. Furthermore, the chemical techniques required for identifying different phases within a structure are destructive. Thus, if a only a small amount of a certain portion of the sample is present then this may be destroyed by the process by the etching technique.

Electron Microscopy
Scanning Electron Microscopy
Transmission Electron Microscopy
Chemical Analysis in Electron Microscopy
Diffraction Techniques
Principles of Diffraction
X-Ray Diffraction
Neutron Diffraction
Electron Diffraction
Spectroscopic Techniques
Energy Dispersive X-Ray Spectroscopy
Wavelength Dispersive X-Ray Spectroscopy
Electron Energy Loss Spectroscopy
X-Ray Photoelectron Spectroscopy
Auger Electron Spectroscopy
Infra-red and Raman Spectroscopy
Ultra-violet and Visible Spectroscopy

Electrical and Magnetic Techniques
Electrical Resistance
Impedance Spectroscopy
Thermal Techniques
Thermogravimetric Analysis (TGA)
Differential Scanning Calorimetry (DSC)
Mechanical Testing

Strength

Hardness

Hardness is defined as the resistance of a material to penetration by an indentor. The Mohs scale of hardness has ten level and diamond is the material with the highest level of hardness ever known. There are several **methods** used to determine material's hardness, such as: **Brinell**, **Rockwell**, **Vickers** and **Poldy**.

Hardness Brinell (HB)

Is the method used for raw metallic materials. It uses a spherical ball indentor in order to stamp a print in the material. An external force transmitted through the indentor over the surface of the material determines the material's penetration.

Hardness Rockwell (HRB/HRC)

Is the method used for heat treated metallic materials. It has two variants regarding the indenter shape (ball or cone).

Hardness Vickers (HV)

Is a method used for the determination of hardness of special metallic materials, such as high alloyed materials, characterized by a very high degree of hardness.

Non destructive testing (NDT)

Some of the NDT methods available are: **ultrasonic method**, **radiation penetration method**.

Metals

Metals are materials made of elements on the left hand side of the periodic tables 'stair step' border starting on the left of Boron and going down and right and finishing at polonium. These elements can be mixed and combined with other elements (metals or non-metals) to create materials called alloys. Alloys are just a mix of elements and materials to create a new material with favorable properties.

Metals can be generally identified by a set of few physical properties (these a very general and there are plenty of exceptions). The main definition of a metal is an element that readily loses

electrons and forms positive ions. The general bulk properties that are used to simply identify metals is that they tend to be lustrous (shiny when not oxidised), they are malleable (so can be beaten into a shape and not break), they are ductile (they can be drawn out into a wire) and that they conduct electricity; this rises from the fact that they readily lose electrons so there is a free electron 'gas' where the electrons can move around and this means that a charge can flow when an electric field is placed across the metal.

The metal that has changed the way the whole world functions and takes up a huge majority of the industry even now after over a century of its discovery and use (in terms of its modern production and composition). This metal is steel and is an alloy of mainly iron (Fe) and carbon (C) with many other constituent elements added depending on the type of steel wanted and the properties required.

Steel can be produced in a number of ways. Traditional methods utilise integrated steel processes which use energy intensive blast furnaces (to produce iron) sand basic oxygen steelmaking (to convert iron to steel). More modern methods use electric arc furnaces in which scrap steel is melted using electric currents and then formed into slabs or ingots for further processing. When the steel slab or ingot has cooled, a variety of forming operations such as rolling or extrusion are used to form the metal into flat sheet for use in cars, fridges, filing cabinets or radiators, or into beams, and heavy plate for use in construction and ship building

Ceramics

inorganic and non-metallic materials bimevox

Polymers

Polymer is a group of substances that has large molecules consisting of at least five repeated chemical units bonded together with a same type of linkage, like beads on a string.Polymer usually contains more than five repeated units and some polymers contain hundreds or thousands of monomers in each of their polymer chains. Polymer materials can be natural or synthetic. Polymer material is a large group of materials whereby they can be further classified specifically into plastics, elastomers and composites!

Materials Science:

Materials Processing

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some form of extractive metallurgy, chemical synthesis or some other chemical process. It may also be necessary to mix different raw materials to achieve a certain composition (e.g. alloying in metals) that is appropriate for or has been optimised the application. The application will usually require that the material be in a particular shape and a suitable shaping process or combination of process must be employed to achieve this. Often, it may be possible to produce a shape out of a material with any one of the many different shaping processes. However, there is usually one particular process that either results in particular benefits in terms of the properties of the material or the article that is produced or meets some other important criteria - such as low cost - that it is selected over the other options. Finally, it may be necessary or beneficial to process the article further, once it has been formed, in order to optimise the properties of the material.

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Single Crystal Methods:

Heat treatment is a process in which the material is heated and cooled to change the properties of the metal.

Materials Characterisation:

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Introduction

Materials science is an interdisciplinary field involving the properties of matter and its applications to various areas of science and engineering. It includes elements of applied physics and chemistry, as well as chemical, mechanical, civil and electrical engineering. With significant media attention to nanoscience and nanotechnology in the recent years, materials science has been propelled to the forefront at many universities, sometimes controversially.

The choice material of a given era is often its defining point: the stone age, Bronze Age, and steel age are examples. Materials science is one of the oldest forms of engineering and applied science. Modern materials science evolved directly from metallurgy, which itself evolved from mining. A major breakthrough in the understanding of materials occurred in the late 19th century, when Willard Gibbs demonstrated that thermodynamic properties relating to atomic structure in various phases are related to the physical properties of the material. Important elements of modern materials science are a product of the space race: the understanding and engineering of the metallic alloys and other materials that went into the construction of space vehicles was one of the enablers of space exploration. Materials science has driven, and been driven by, the development of revolutionary technologies such as plastics, semiconductors, and biomaterials.

Before the 1960s (and in some cases decades after), many materials science departments were named metallurgy departments, from a 19th and early 20th century emphasis on metals. The field has since broadened to include every class of materials, including: ceramics, polymers, semiconductors, magnetic materials, medical implant materials and biological materials.

In materials science, rather than haphazardly looking for and discovering materials and exploiting their properties, one instead aims to understand materials fundamentally so that new materials with the desired properties can be created.

The basis of all materials science involves relating the desired properties and relative performance of a material in a certain application to the structure of the atoms and phases in that material through characterization. The major determinants of the structure of a material and thus of its properties are its constituent chemical elements and the way in which it has been processed into its final form. These, taken together and related through the laws of thermodynamics, govern the material’s microstructure, and thus its properties.

An old adage in materials science says: "materials are like people; it is the defects that make them interesting". The manufacture of a perfect crystal of a material is physically impossible. Instead materials scientists manipulate the defects in crystalline materials such as precipitates, grain boundaries (Hall-Petch relationship), interstitial atoms, vacancies or substitutional atoms, creating a material with the desired properties.

Not all materials have a regular crystal structure. Polymers display varying degrees of crystallinity. Glasses, some ceramics, and many natural materials are amorphous, not possessing any long-range order in their atomic arrangements. These materials are much harder to engineer than crystalline materials. Polymers are a mixed case, and their study commonly combines

elements of chemical and statistical thermodynamics to give thermodynamical, rather than mechanical descriptions of physical properties.

In addition to industrial interest, materials science has gradually developed into a field which provides tests for condensed matter or solid state theories. New physics emerges because of the diverse new material properties needed to be explained.

Radical materials advances can drive the creation of new products or even new industries, but stable industries also employ materials scientists to make incremental improvements and troubleshoot issues with currently used materials. Industrial applications of materials science include materials design, cost-benefit tradeoffs in industrial production of materials, processing techniques (casting, rolling, welding, ion implantation, crystal growth, thin-film deposition, sintering, glassblowing, etc.), and analytical techniques (characterization techniques such as electron microscopy, x-ray diffraction, calorimetry, nuclear microscopy (HEFIB), Rutherford backscattering, neutron diffraction, etc.).

Besides material characterisation, the material scientist/engineer also deals with the extraction of materials and their conversion into useful forms. Thus ingot casting, foundry techniques, blast furnace extraction, electrolytic extraction all are part of the required knowledge of a materials scientist/engineer. Often the presence, absence or variation of minute quantities of secondary elements and compounds in a bulk material will have a great impact on the final properties of the materials produced, for instance, steels are classified based on 1/10th and 1/100 weight percentages of the carbon and other alloying elements they contain. Thus, the extraction and purification techniques employed in the extraction of iron in the blast furnace will have an impact of the quality of steel that may be produced.

The overlap between physics and materials science has led to the offshoot field of materials physics, which is concerned with the physical properties of materials. The approach is generally more macroscopic and applied than in condensed matter physics. See the important publications in materials physics for more details on this field of study.

Alloys of metals is an important and significant part of materials science. Of all the metallic alloys in use today, the alloys of iron (steel, stainless steel, cast iron, tool steel, alloy steels) make up the largest proportion both by quantity and commercial value. Iron alloyed with various weight percentages of carbon gives low, mid and high carbon steels. For the steels, the hardness and tensile strength of the steel is directly related to the amount of carbon present, while increasing carbon levels lead to lower ductility and toughness. The addition of silicon and graphitization will produce cast irons (although some cast irons are made precisely with no graphitization). The addition of chromium, nickel and molybdenum to carbon steels (more than 10%) gives us stainless steels.

Other significant metallic alloys are those of aluminium, titanium, copper and magnesium. Copper alloys have been know for a long time (during the Bronze Age), while the alloys of the other three metals have been relatively recently developed, due to the chemical reactivity of these metals and the resultant difficulty in their extraction which wasn't accomplished (electrolytically) until recently. The alloys of aluminium, titanium and magnesium are also

known and valued for their high strength to weight ratios and, in the case of magnesium, their ability to provide electromagnetic shielding. These materials find special applications where high strength-weight ratios are desired (aero-space industry).

Other than metals, polymers and ceramics are also an important part of material science. Polymers are the raw materials (the resins) used to make what we commonly call plastics. Plastics are actually the final product after many polymers and additives have been processed and shaped into a final shape and form. Polymers that have been around and are in current widespread use include polyethylene, polypropylene, polyvinyl-chloride, polystyrene, nylons, polyesters, acrylics, polyurethane, polycarbonates. Plastics are generally classified as "commodity", "specialty" and "engineering" plastics.

PVC is a commodity plastic, it is widely used, low cost and annual quantities are huge. It lends itself to an incredible array of applications, from faux leather to electrical insulation to cabling to packaging and vessels. Its fabrication and processing are simple and well-established. The versatility of PVC is due to the wide range of additives that it accepts. Additives in polymer science refers to the chemicals and compounds added to the polymer base to modify its physical and material properties.

Polycarbon would be normally considered an engineering plastic (other examples include PEEK, ABS). Engineering plastics are valued for their superior strengths and other special material properties. They are usually not used for disposable applications, unlike commodity plastics.

Specialty plastics are really the materials with unique characteristics, such as ultrahigh strength, electrical conductivity, electro-florescence, high thermal stability, etc.

It should be noted here that the dividing line between the various types of plastics is not based on material but rather their properties and applications. For instance, polypropylene (PP) is a cheap, slippery polymer commonly used to make disposable shopping bags and trash bags. It is commodity. But a variety of PP called Ultra-high Molecular Weight Polypropylene (UHMWPE) is an engineering plastic which is used extensively as the glide rails for industrial equipment.

Another application of material science in industry is the making of composite materials. Composite materials are structured materials composed of at least two different macroscopic phases. An example would be steel-reinforced concrete. Also, take a look at the plastic casing of your telly set, cell-phone: these plastic casings are usually a composite made up of a thermoplastic matrix such as acrylonitrile-butadiene-styrene (ABS)in which calcium carbonate chalk, talc, glass fibres or carbon fibres have been added (dispersants) for added strength, bulk, or electro-static dispersion.

Soil mechanics:

Geology is the foundation of soil mechanics. Without a proper understanding of the underlying geologic processes that have formed the particular soil you have to deal with, a soils or geotechnical engineer will be playing with fire.

This is where a well trained engineering geologist is so vitally important. They can help other people involved in the design and construction of a facility or project understand the geologic antecedents of the materials they have to use.

Geotechnical engineers classify soils, or more properly earth materials, for their properties relative to foundation support or use as building material. These systems are designed to predict some of the engineering properties and behavior of a soil based on a few simple laboratory or field tests, though some earlier systems were adaptations of soil-science classification systems. The most common is the Unified Soil Classification System, with three major groups:

(1) coarse-grained, sands and gravels, (2) fine-grained, silts and clays, and (3) highly organic soils (referred to as peat even when the soil is not truly a peat). The first two groups are then subdivided as follows:

Coarse grain: gravels, sands, based on the grain-size of the coarse-grained fraction. **Fine grain:** silts, clays, organics silts & clays, based on plasticity and organic content.

Fine-grained soils are then subdivided according to their plasticity, while coarse-grained soils are subdivided by the presence and properties of the fines or the grain-size distribution of the soil.

A full geotechnical engineering soil description will include other properties of the soil, including color, in-situ moisture content, in-situ strength, and somewhat more detail about the material properties of the soil than is provided by the USCS code.

The objective of a geotechnical investigation is to characterize the geologic environment. This may include;

1) The lateral distribution and thickness of the soil and rock strata.

2) Groundwater Conditions and quality.

3) Ground response to changing conditions such as surface loading from structures and unloading by excavations or extraction.

4) Physical properties of soils and rock formations.

5) Engineering properties of soils and rock formations.

6) Hazardous conditions such as unstable slopes, faults, flood plains, ground subsidence and heave potential.

The magnitude of the site investigation depends upon the geological variability of the site as well as the size (spatially and financially) of the planned project.

Structural Engineering:

Structural Engineering Activities

Structural engineering is mainly involved with two activities

1. Structural Analysis
2. Structural Design

Structural Analysis

It deals with analysing a particular structural system. A structural system may vary from simple systems (like beams, columns, slabs, etc) to more complex systems (like frames, bridges, piers, foundations, retaining walls, etc). The objective behind analysis is to estimate or find resultant stresses (or forces) so that these elements can be designed to withstand the load that comes over it.

Analysis of Statically Determinate Structures

Idealized Structures

In reality the exact analysis of a structure can never be carried out. Idealizing a structure is a method of conservatively simplifying the components of the structural system, while keeping its behavior under loading the same. This is done in order to simplify calculations. Without an idealized structure, design could take a massively longer amount of time... and sometimes becomes impossible. It is important for a structual engineer to develop methods to idealize a structure in order to carry out analysis.

Supports for Coplanar Structures

Structural Members are joined together by supports depending on the intent of the engineer. Three types of joints most often used are pin connections, roller supports, and fixed joints. Pin connections provide vertical and lateral support, but cannot provide a moment reaction. A roller support can only supply a reaction of a force in one direction. A fixed joint provides vertical, lateral as well as a moment reaction.

Analysis of Statically Determinate Trusses

Typically done using method of joints, or method of sections.

Internal Loadings Developed in Structural Members

Internal loadings are found by "cutting" the member and applying the equations of equilibrium.

Cables and Arches

What are cables and arches?

Influence Lines for Statically Determinate Structures

Influence lines are a diagram that represents variation of a loading function at one location on a structure. For example, a truck drives across a bridge:

When the truck is at the left side on top of the bridge, very little loading is felt at the right support. The function represented by influence lines changes as the truck moves towards the right end of the bridge. The loading at the right support grows as the truck nears it, until the truck reaches the far end, where the end realizes the maximum loading. Influence lines for statically determinate structures consist of a straight line.

Deflections

Deflections occur naturally in structures from various sources. Loads and temperature changes are sources of deflections that structural engineers have to design for because they are unavoidable. Designs must be made in order to avoid cracking of the materials used. Fabrication or Design Errors can lead to failure of structures and should be avoided through careful planning and analysis.

Structures in most cases are built with materials that can withstand the designed loading and only have a linear elastic response. Under these conditions loads may cause deflections, but when the load is removed the structure will return to it original shape and strength. Overloading beyond the linear elastic response may cause damage and failure of the structure. This is referred to as "plastic deformation".

Elastic-Beam Theory

If a material is homogeneous and behaves in a linear elastic manner we can derive a nonlinear second order differential equation that when solved through the double integration method can give a solution deflection as a function of x. We must assume $dv / dx = 0$ relative to the length of the beam in the x-axial direction.

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

M = the internal moment in the beam E = the material's modulus of elasticity I = the beam's moment of inertia computed about the neutral axis v = deflection

The Double Integration Method

When Moment can be expressed as a function of position x, then performing double integration will yield the beam's slope as a function of x, and the equation of the elastic curve.

The two integrations will yield two constants of intergration. Using the boundary conditions the constants can be solved for.

Castigliano's theorems

First theorem

> *If the strain energy of an elastic structure can be expressed as a function of generalised displacement qi; then the partial derivative of the strain energy with respect to generalised displacement gives the generalised force Qi.*

Using this theorem the generalised forces acting on the structure can be calculated.

Second theorem

> *If the strain energy of a linearly elastic structure can be expressed as a function of generalised force Qi; then the partial derivative of the strain energy with respect to generalised force gives the generalised displacement qi in the direction of Qi.*

Using this theorem the generalised displacement of the structure can be calculated. **Castigliano's method** usually refers to the application of his second theorem.

Structural Design

Structural design is the process of selecting members of required dimensions such that they provide adequate stability under service loads. There are two conditions that a structural designer must keep in mind. One is "stability" and the other is "serviceability". Stability of a structure means that it can resist the loads acting on it satisfactorily and that the structure will not collapse immediately (that is, it provides enough time to escape to safety). Serviceability refers to certain conditions that are required so that the structure remains serviceable. For example, consider a bridge that can resist service loads without collapse. This is a "stable" structure. Now assume that this bridge shows abnormal deflections. The deflections could be such that the bridge feels bouncy and could lead to steering problems for vehicles crossing it at high speeds. As such this may not cause the structure to collapse. So we can say that the structure is stable, but according to serviceability criterion it is not serviceable because people could feel afraid of using the bridge.

Vibrations of Structures:

Introduction

Many a times, we use the terms vibration and oscillation without knowing the difference between them.

The term oscillation refers strictly to the repeating motion of a point mass or that of a rigid body while the term vibration refers to the repeating motion or deformations of an elastic structure. Thus any oscillation is a term used only in cases like the motion of a pendulum or that of a ship as a rigid body moving on the wavy seas while vibration is a term used for phenomenon exhibited by structures such as rotating fans or motors etc. Vibration involves deformation by definition while an oscillating structure does not deform.

Dynamic Load and D'Alembert's Principle

We have studied how we find the response of a structure to static load. When we come to study vibrations, what we have is not static load but dynamic load. The very nature of the load means that the response of the structure i.e the displacement, stress, reactions etc. also varies with time.

The fundamental difference between static and dynamic loads is that acceleration of the body must be taken into account. From Newton's Law, we know that any force produces acceleration, however in statics we assumed that the load had been applied slowly and whatever effects were to take place are over and the body is now in equilibrium. This cannot be assumed under dynamic load.

The solution to this problem was proposed by D'Alembert who said that the acceleration can be treated as another force acting opposite to the acceleration and of magnitude the mass times the acceleration. This force is called the inertial force. The forces that act on the body can now be treated as a set of forces in equilibrium.

Forces Involved

In any vibrating system, there are a total of four types of forces that need to be taken into account. These are listed here.

- Inertial Force
- Spring Force
- Damping Force
- Total External Force

The inertial force has been described already. It occurs because acceleration is present.

The Spring force arises due to the elasticity of the material. It follows the Hooke's Law and is proportional to the displacement.

While these forces are enough to set up harmonic vibrations in a system, most systems also have intrinsic damping even if an external damper is not used. The damping force tends to oppose motion and acts against the velocity in most cases (the exception is negative damping). It generally varies with some power of the velocity though the most useful is viscous damping which varies linearly with velocity.

The total External force is the remaining force that acts on the system. It is the force that causes the excitation in the first place and may or may not be present while the system vibrates.

Fluids Mechanics and Fluid Properties

What is fluid mechanics? As its name suggests it is the branch of applied mechanics concerned with the statics and dynamics of fluids - both liquids and gases. The analysis of the behaviour of fluids is based on the fundamental laws of mechanics which relate continuity of mass and energy with force and momentum together with the familiar solid mechanics properties.

1. Objectives of this section

- Define the nature of a fluid.
- Show where fluid mechanics concepts are common with those of solid mechanics and indicate some fundamental areas of difference.
- Introduce viscosity and show what are Newtonian and non-Newtonian fluids
- Define the appropriate physical properties and show how these allow differentiation between solids and fluids as well as between liquids and gases.

2. Fluids

There are two aspects of fluid mechanics which make it different to solid mechanics:

1. The nature of a fluid is much different to that of a solid
2. In fluids we usually deal with *continuous* streams of fluid without a beginning or end. In solids we only consider individual elements.

We normally recognise three states of matter: solid; liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it *flows* under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely.

The deformation is caused by *shearing* forces which act tangentially to a surface. Referring to the figure below, we see the force F acting tangentially on a rectangular (solid lined) element ABDC. This is a shearing force and produces the (dashed lined) rhombus element A'B'DC.

$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at4^\circ c)}}$$

Shearing force, F, acting on a fluid element.

We can then say:

A Fluid is a substance which deforms continuously,
or flows, when subjected to shearing forces.

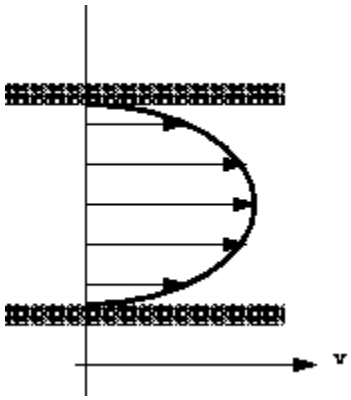
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and conversely this definition implies the very important point that:

If a fluid is at rest there are no shearing forces acting.
All forces must be perpendicular to the planes which the are acting.

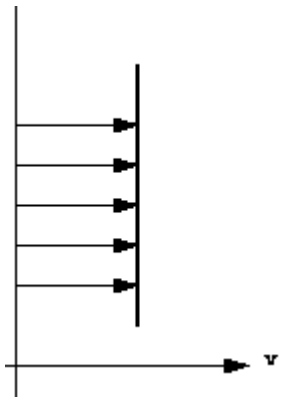
When a fluid is in motion shear stresses are developed if the particles of the fluid move relative to one another. When this happens adjacent particles have different velocities. If fluid velocity is the same at every point then there is no shear stress produced: the particles have zero *relative* velocity.

Consider the flow in a pipe in which water is flowing. At the pipe wall the velocity of the water will be zero. The velocity will increase as we move toward the centre of the pipe. This change in velocity across the direction of flow is known as velocity profile and shown graphically in the figure below:



Velocity profile in a pipe.

Because particles of fluid next to each other are moving with different velocities there **are** shear forces in the moving fluid i.e. shear forces are **normally** present in a moving fluid. On the other hand, if a fluid is a long way from the boundary and all the particles are travelling with the same velocity, the velocity profile would look something like this:

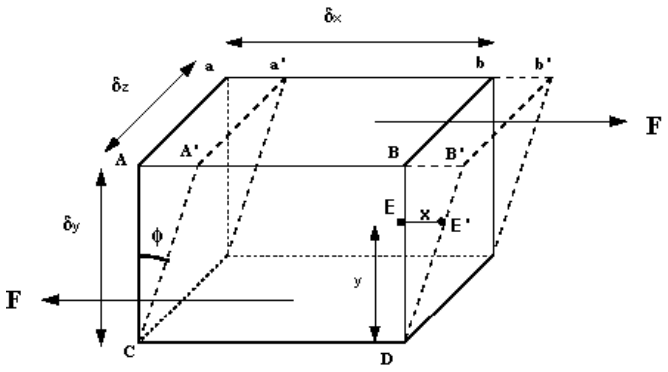


Velocity profile in uniform flow

and there will be no shear forces present as all particles have zero relative velocity. In practice we are concerned with flow past solid boundaries; aeroplanes, cars, pipe walls, river channels etc. and shear forces will be present.

3. Newton's Law of Viscosity

How can we make use of these observations? We can start by considering a 3d rectangular element of fluid, like that in the figure below.



Fluid element under a shear force

The shearing force F acts on the area on the top of the element. This area is given by $A = \delta z \times \delta x$. We can thus calculate the *shear stress* which is equal to force per unit area i.e.

$$\text{shear stress, } \tau = \frac{F}{A}$$

The deformation which this shear stress causes is measured by the size of the angle □ □□□□

It has been found experimentally that the *rate of shear stress* (shear stress per unit time,
□
˜ □□□□

$$\tau = \mu \frac{du}{dy}$$

This is known as **Newton's law of viscosity**.

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4. *Fluids vs. Solids*

In the above we have discussed the differences between the behaviour of solids and fluids under an applied force. Summarising, we have;

---

1. For a **solid** the strain is a function of the applied stress (providing that the elastic limit has not been reached). For a **fluid**, the rate of strain is proportional to the applied stress.

   ---

2. The strain in a **solid** is independent of the time over which the force is applied and (if the elastic limit is not reached) the deformation disappears when the force is removed. A **fluid** continues to flow for as long as the force is applied and will not recover its original form when the force is removed.

It is usually quite simple to classify substances as either solid or liquid. Some substances, however, (e.g. pitch or glass) appear solid under their own weight. Pitch will, although appearing solid at room temperature, deform and spread out over days - rather than the fraction of a second it would take water.

As you will have seen when looking at properties of solids, when the elastic limit is reached they seem to flow. They become plastic. They still do **not** meet the definition of true fluids as they will only flow after a certain minimum shear stress is attained.

5. *Newtonian / Non-Newtonian Fluids*

Even among fluids which are accepted as fluids there can be wide differences in behaviour under stress. Fluids obeying Newton's law where the value of □
□□□

where A, B and n are constants. For Newtonian fluids A = 0, B = □
□□□̇□̇□̃1
□

Below are brief description of the physical properties of the several categories:

- *Plastic:* Shear stress must reach a certain minimum before flow commences.
- *Bingham plastic:* As with the plastic above a minimum shear stress must be achieved. With this classification n = 1. An example is sewage sludge.
- *Pseudo-plastic:* No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. colloidial substances like clay, milk and cement.
- *Dilatant substances;* Viscosity increases with rate of shear e.g. quicksand.
- *Thixotropic substances:* Viscosity decreases with length of time shear force is applied e.g. thixotropic jelly paints.
- *Rheopectic substances:* Viscosity increases with length of time shear force is applied
- *Viscoelastic materials:* Similar to Newtonian but if there is a sudden large change in shear they behave like plastic.

There is also one more - which is not real, it does not exist - known as the **ideal fluid**. This is a fluid which is assumed to have no viscosity. This is a useful concept when theoretical solutions are being considered - it does help achieve some practically useful solutions.

Liquids vs. Gasses

Although liquids and gasses behave in much the same way and share many similar characteristics, they also possess distinct characteristics of their own. Specifically

- A liquid is difficult to compress and often regarded as being incompressible.
  A gas is easily to compress and usually treated as such - it changes volume with pressure.
- A given mass of liquid occupies a given volume and will occupy the container it is in and form a free surface (if the container is of a larger volume).
  A gas has no fixed volume, it changes volume to expand to fill the containing vessel. It will completely fill the vessel so no free surface is formed.

Causes of Viscosity in Fluids

0. Viscosity in Gasses

The molecules of gasses are only weakly kept in position by molecular cohesion (as they are so far apart). As adjacent layers move by each other there is a continuous exchange of molecules. Molecules of a slower layer move to faster layers causing a drag, while molecules moving the other way exert an acceleration force. Mathematical considerations of this momentum exchange can lead to Newton law of viscosity.

If temperature of a gas increases the momentum exchange between layers will increase thus increasing viscosity.

Viscosity will also change with pressure - but under normal conditions this change is negligible in gasses.

1. *Viscosity in Liquids*

There is some molecular interchange between adjacent layers in liquids - but as the molecules are so much closer than in gasses the cohesive forces hold the molecules in place much more rigidly. This cohesion plays an important roll in the viscosity of liquids.

Increasing the temperature of a fluid reduces the cohesive forces and increases the molecular interchange. Reducing cohesive forces reduces shear stress, while increasing molecular interchange increases shear stress. Because of this complex interrelation the effect of temperature on viscosity has something of the form:

$$\mu_T = \mu_0(1 + AT + BT)$$

where μ_T is the viscosity at temperature TC, and μ_0 is the viscosity at temperature 0C. A and B are constants for a particular fluid.

High pressure can also change the viscosity of a liquid. As pressure increases the relative movement of molecules requires more energy hence viscosity increases.

Properties of Fluids

The properties outlines below are general properties of fluids which are of interest in engineering. The symbol usually used to represent the property is specified together with some typical values in SI units for common fluids. Values under specific conditions (temperature, pressure etc.) can be readily found in many reference books. The dimensions of each unit is also give in the MLT system (see later in the section on dimensional analysis for more details about dimensions.)

1. *Density*

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

1. *Mass Density*

Mass Density, ρ, is defined as the mass of substance per unit volume.

Units: Kilograms per cubic metre, kg/m^3 (or kgm^{-3})

Dimensions: ML^{-3}

Typical values:

Water = 1000 kgm^{-3}, Mercury = 13546 kgm^{-3} Air = 1.23 kgm^{-3}, Paraffin Oil = 800 kgm^{-3}.

(at pressure =1.013 $\times 10^{-5}$ Nm^{-2} and Temperature = 288.15 K.)

2. *Specific Weight*

Specific Weight ω, (sometimes , and sometimes known as *specific gravity*) is defined as the weight per unit volume.

or

The force exerted by gravity, g, upon a unit volume of the substance.

The Relationship between g and ω can be determined by Newton's 2nd Law, since

weight per unit volume = mass per unit volume g

$$\omega = \rho g$$

Units: Newton's per cubic metre, N/m^3 (or Nm^{-3})

Dimensions: $ML^{-2}T^{-2}$.

Typical values:

Water =9814 Nm^{-3}, Mercury = 132943 Nm^{-3}, Air =12.07 Nm^{-3}, Paraffin Oil =7851 Nm^{-3}

3. *Relative Density*

Relative Density, σ, is defined as the ratio of mass density of a substance to some standard mass density.

For solids and liquids this standard mass density is the maximum mass density for water (which occurs at $4^{\circ}c$) at atmospheric pressure.

$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at4^{\circ}c)}}$$

Units: None, since a ratio is a pure number.

Dimensions: 1.

Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

2. *Viscosity*

Viscosity, □, is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water.

All fluids are viscous, "Newtonian Fluids" obey the linear relationship

given by Newton's law of viscosity. $\tau = \mu \frac{du}{dy}$, which we saw earlier.

where τ is the shear stress,

Units Nm^{-2}; $kg\,m^{-1}s^{-2}$

Dimensions $ML^{-1}T^{-2}$.

$\frac{du}{dy}$ is the velocity gradient or rate of shear strain, and has

Units: $radianss^{-1}$,

Dimensions t^{-1}

μ is the "coefficient of dynamic viscosity" - see below.

1. *Coefficient of Dynamic Viscosity*

The Coefficient of Dynamic Viscosity, μ, is defined as the shear force, per unit area, (or shear stress τ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \tau \Big/ \frac{du}{dy} = \frac{\text{Force}}{\text{Area}} \Big/ \frac{\text{Velocity}}{\text{Distance}} = \frac{\text{Force} \times \text{Time}}{\text{Area}} = \frac{\text{Mass}}{\text{Length} \times \text{Area}}$$

Units: Newton seconds per square metre, Nsm^{-2} or Kilograms per meter per second, $kgm^{-1}s^{-1}$.

(Although note that μ is often expressed in Poise, P, where 10 P = 1 $kgm^{-1}s^{-1}$.)

Typical values:

Water =1.14 $\times 10^{-3}$ $kgm^{-1}s^{-1}$, Air =1.78 $\times 10^{-5}$ $kgm^{-1}s^{-1}$, Mercury =1.552 $kgm^{-1}s^{-1}$,

Paraffin Oil =1.9 $kgm^{-1}s^{-1}$.

2. *Kinematic Viscosity*

Kinematic Viscosity, ν, is defined as the ratio of dynamic viscosity to mass density.

$$\nu = \frac{\mu}{\rho}$$

Units: square metres per second, m^2s^{-1}

(Although note that □ is often expressed in Stokes, St, where 10^4 St = 1 m^2s^{-1}.)

Dimensions: L^2T^{-1}.

Typical values:

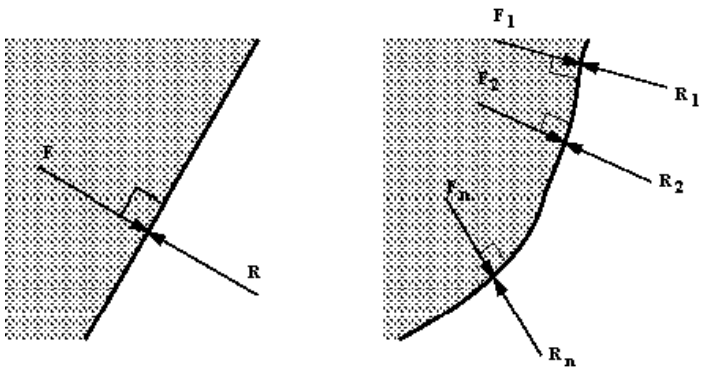
Water =1.14 $\times 10^{-6}\ m^2 s^{-1}$, Air =1.46 $\times 10^{-5}\ m^2 s^{-1}$, Mercury =1.145 $\times 10^{-4}\ m^2 s^{-1}$,
Paraffin Oil =2.375 $\times 10^{-3}\ m^2 s^{-1}$.

Hydrostatic Forces:

1. Fluids statics

The general rules of statics (as applied in solid mechanics) apply to fluids at rest. From earlier we know that:

- a static fluid can have **no shearing force** acting on it, and that
- any force between the fluid and the boundary must be acting at right angles to the boundary.



Pressure force normal to the boundary

Note that this statement is also true for curved surfaces, in this case the force acting at any point is normal to the surface at that point. The statement is also true for any imaginary plane in a static fluid. We use this fact in our analysis by considering elements of fluid bounded by imaginary planes.

We also know that:

- For an element of fluid at rest, the element will be in equilibrium - the sum of the components of forces in any direction will be zero.
- The sum of the moments of forces on the element about any point must also be zero.

It is common to test equilibrium by resolving forces along three mutually perpendicular axes and also by taking moments in three mutually perpendicular planes an to equate these to zero.

2. Pressure

As mentioned above a fluid will exert a normal force on any boundary it is in contact with. Since these boundaries may be large and the force may differ from place to place it is convenient to work in terms of pressure, *p*, which is the force per unit area.

If the force exerted on each unit area of a boundary is the same, the pressure is said to be *uniform*.

$$\text{pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

$$p = \frac{F}{A}$$

Units: Newton's per square metre, Nm^{-2}, $kgm^{-1}s^{-2}$.

(The same unit is also known as a Pascal, *Pa*, i.e. $1Pa = 1\ Nm^{-2}$)

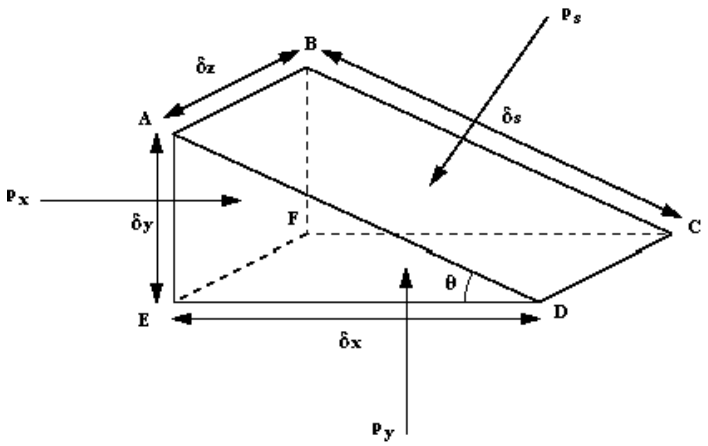
(Also frequently used is the alternative SI unit the *bar,* where $1bar = 10^5\ Nm^{-2}$)

Dimensions: $ML^{-1}T^{-2}$.

3. Pascal's Law for Pressure At A Point

(Proof that pressure acts equally in all directions.)

By considering a small element of fluid in the form of a triangular prism which contains a point P, we can establish a relationship between the three pressures p_x in the x direction, p_y in the y direction and p_s in the direction normal to the sloping face.



Triangular prismatic element of fluid

The fluid is a rest, so we know there are no shearing forces, and we know that all force are acting at right angles to the surfaces .i.e.

p_s acts perpendicular to surface ABCD,

p_x acts perpendicular to surface ABFE and

p_y acts perpendicular to surface FECD.

And, as the fluid is at rest, in equilibrium, the sum of the forces in any direction is zero.

Summing forces in the x-direction:

Force due to p_x,

$$F_{x_A} = p_x \times Area_{ABFE} = p_x \delta x \, \delta y$$

Component of force in the x-direction due to p_s,

$$
\begin{aligned}
F_{x_s} &= -p_s \times Area_{ABCD} \times \sin\theta \\
&= -p_s \delta s \, \delta z \frac{\delta y}{\delta s} \\
&= -p_s \delta y \, \delta z
\end{aligned}
$$

($\sin\theta = \delta y / \delta s$)

Component of force in x-direction due to p_y,

$$F_{x_y} = 0$$

To be at rest (in equilibrium)

$$
\begin{aligned}
F_{xx} + F_{xs} + F_{xy} &= 0 \\
p_x \delta x \delta y + \left(-p_s \delta y \delta z\right) &= 0 \\
p_x &= p_s
\end{aligned}
$$

Similarly, summing forces in the y-direction. Force due to p_y,

$$F_{y_y} = p_y \times Area_{BFCD} = p_y \delta x \delta z$$

Component of force due to p_s,

$$
\begin{aligned}
F_{y_s} &= -p_s \times Area_{ABCD} \times \cos\theta \\
&= -p_s \delta s \, \delta z \frac{\delta x}{\delta s} \\
&= -p_s \delta x \, \delta z
\end{aligned}
$$

($\cos\theta = \delta x / \delta s$)

Component of force due to p_x,

$$F_{y_x} = 0$$

Force due to gravity,

$$\begin{aligned} \text{weight} &= -\text{specific weight} \times \text{volume of element} \\ &= -\rho g \times \frac{1}{2}\delta x\,\delta y\,\delta z \end{aligned}$$

To be at rest (in equilibrium)

$$F_{y_y} + F_{y_s} + F_{y_x} + \text{weight} = 0$$

$$p_y \delta x\,\delta y + \left(-p_s \delta x\,\delta z\right) + \left(-\rho g \frac{1}{2}\delta x\,\delta y\,\delta z\right) = 0$$

The element is small i.e. δx, δy and δz are small, and so $\delta x\,\delta y\,\delta z$ is very small and considered negligible, hence

$$p_y = p_s$$

thus

$$p_x = p_y = p_s$$

Forces on Submerged Surfaces in Static Fluids:

We have seen the following features of statics fluids

- Hydrostatic vertical pressure distribution
- Pressures at any equal depths in a continuous fluid are equal
- Pressure at a point acts equally in all directions (Pascal's law).

- Forces from a fluid on a boundary acts at right angles to that boundary.

Objectives:

We will use these to analyse and obtain expressions for the forces on submerged surfaces. In doing this it should also be clear the difference between:

- Pressure which is a scalar quantity whose value is equal in all directions and,
- Force, which is a vector quantity having both magnitude and direction.

1. Fluid pressure on a surface

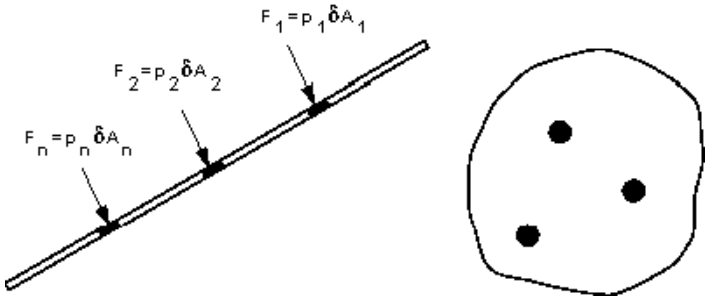
Pressure is defined as force per unit area. If a pressure p acts on a small area δA then the force exerted on that area will be

$$F = p\,\delta A$$

Since the fluid is at rest the force will act at right-angles to the surface.

General submerged plane

Consider the plane surface shown in the figure below. The total area is made up of many elemental areas. The force on each elemental area is always normal to the surface but, in general, each force is of different magnitude as the pressure usually varies.



We can find the total or ***resultant*** force, R, on the plane by summing up all of the forces on the small elements i.e.

$$R = p_1\delta A_1 + p_2\delta A_2 + \ldots\ldots p_n\delta A_n = \sum p\,\delta A$$

This resultant force will act through the centre of pressure, hence we can say

If the surface is a **plane** the force can be represented by one single **resultant force**, acting at right-angles to the plane through the **centre of pressure**.

Horizontal submerged plane

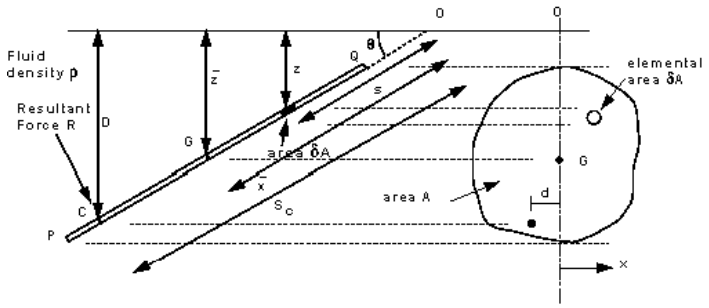
For a horizontal plane submerged in a liquid (or a plane experiencing uniform pressure over its surface), the pressure, p, will be equal at all points of the surface. Thus the resultant force will be given by

$$R = \text{pressure} \times \text{area of plane}$$
$$R = pA$$

Curved submerged surface

If the surface is curved, each elemental force will be a different magnitude and in different direction but still normal to the surface of that element. The resultant force can be found by resolving all forces into orthogonal co-ordinate directions to obtain its magnitude and direction. This will **always** be less than the sum of the individual forces, $\sum p\,\delta A$.

2. Resultant Force and Centre of Pressure on a submerged plane surface in a liquid.



This plane surface is totally submerged in a liquid of density ρ and inclined at an angle of θ to the horizontal. Taking pressure as zero at the surface and measuring down from the surface, the pressure on an element δA, submerged a distance z, is given by

$$p = \rho g z$$

and therefore the force on the element is

$$F = p\,\delta A = \rho g z\,\delta A$$

The resultant force can be found by summing all of these forces i.e.

$$R = \rho g \sum z\,\delta A$$

(assuming ρ and g as constant).

The term $\sum z\,\delta A$ is known as the *1st Moment of Area* of the plane PQ about the free surface. It is equal to $A\bar{z}$ i.e.

$$\sum z\,\delta A = A\bar{z}$$
$$= 1^{st} \text{ moment of area about the line of the free surface}$$

where A is the area of the plane and $\bar{z}$ is the depth (distance from the free surface) to the centroid, G. This can also be written in terms of distance from point O (as $\bar{z} = \bar{x}\sin\theta$)

$$\sum z\,\delta A = A\bar{x}\sin\theta$$
$$= 1^{st} \text{ Moment of area about a line through O} \times \sin\theta$$

Thus:

The resultant force on a plane

$$R = \rho g A\bar{z}$$
$$= \rho g A\bar{x}\sin\theta$$

This resultant force acts at right angles to the plane through the centre of pressure, C, at a depth D. The moment of R about any point will be equal to the sum of the moments of the forces on all the elements δA of the plane about the same point. We use this to find the position of the centre of pressure.

It is convenient to take moments about the point where a projection of the plane passes through the surface, point O in the figure.

$$\text{Moment of } R \text{ about O} = \text{Sum of moments of force on all elements of } \delta A \text{ about O}$$

We can calculate the force on each elemental area:

$$\text{Force on } \delta A = \rho g z \delta A$$
$$= \rho g s \sin\theta \delta A$$

And the moment of this force is:

$$\text{Moment of Force on } \delta A \text{ about O} = \rho g s \sin\theta \delta A \times s$$
$$= \rho g \sin\theta \delta A s^2$$

ρ, g and θ are the same for each element, so the total moment is

$$\text{Sum of moments of forces on all elemets of } \delta A \text{ about O} = \rho g \sin\theta \sum s^2 \delta A$$

We know the resultant force from above $R = \rho g A \bar{x} \sin\theta$, which acts through the centre of pressure at C, so

$$\text{Moment of } R \text{ about O} = \rho g A \bar{x} \sin\theta S_c$$

Equating gives,

$$\rho g A \bar{x} \sin\theta S_c = \rho g \sin\theta \sum s^2 \delta A$$

1. Fluid Dynamics

Objectives

- Introduce concepts necessary to analyse fluids in motion
- Identify differences between Steady/unsteady uniform/non-uniform compressible/incompressible flow
- Demonstrate streamlines and stream tubes

- Introduce the Continuity principle through conservation of mass and control volumes
- Derive the Bernoulli (energy) equation
- Demonstrate practical uses of the Bernoulli and continuity equation in the analysis of flow
- Introduce the momentum equation for a fluid
- Demonstrate how the momentum equation and principle of conservation of momentum is used to predict forces induced by flowing fluids

This section discusses the analysis of fluid in motion - fluid dynamics. The motion of fluids can be predicted in the same way as the motion of solids are predicted using the fundamental laws of physics together with the physical properties of the fluid.

It is not difficult to envisage a very complex fluid flow. Spray behind a car; waves on beaches; hurricanes and tornadoes or any other atmospheric phenomenon are all example of highly complex fluid flows which can be analysed with varying degrees of success (in some cases hardly at all!). There are many common situations which are easily analysed.

2. Uniform Flow, Steady Flow

It is possible - and useful - to classify the type of flow which is being examined into small number of groups.

If we look at a fluid flowing under normal circumstances - a river for example - the conditions at one point will vary from those at another point (e.g. different velocity) we have non-uniform flow. If the conditions at one point vary as time passes then we have unsteady flow.

Under some circumstances the flow will not be as changeable as this. He following terms describe the states which are used to classify fluid flow:

- *uniform flow:* If the flow velocity is the same magnitude and direction at every point in the fluid it is said to be *uniform*.
- *non-uniform:* If at a given instant, the velocity is **not** the same at every point the flow is *non-uniform*. (In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform - as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the of the cross-section of the stream of fluid is constant the flow is considered *uniform*.)
- *steady:* A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time.
- *unsteady:* If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. (In practise there is always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.

Combining the above we can classify any flow in to one of four type:

1. *Steady uniform flow*. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.
2. *Steady non-uniform flow.* Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.
3. *Unsteady uniform flow*. At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
4. *Unsteady non-uniform flow.* Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

If you imaging the flow in each of the above classes you may imagine that one class is more complex than another. And this is the case - *steady uniform flow* is by far the most simple of the four. You will then be pleased to hear that this course is restricted to only this class of flow. We will not be encountering any non-uniform or unsteady effects in any of the examples (except for one or two quasi-time dependent problems which can be treated at steady).

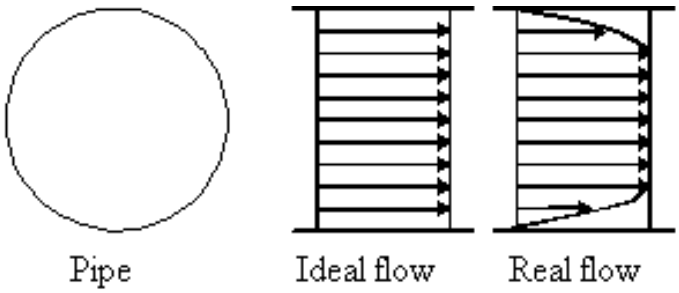
3. Compressible or Incompressible

All fluids are compressible - even water - their density will change as pressure changes. Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density. As you will appreciate, liquids are quite difficult to compress - so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account - even for liquids. Gasses, on the contrary, are very easily compressed, it is essential in most cases to treat these as compressible, taking changes in pressure into account.

4. Three-dimensional flow

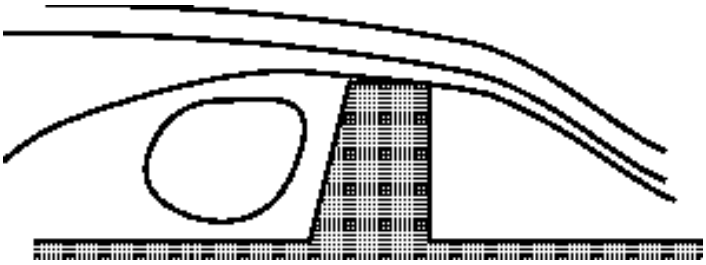
Although in general all fluids flow three-dimensionally, with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one. In these cases changes in the other direction can be effectively ignored making analysis much more simple.

Flow is *one dimensional* if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time only vary in the direction of flow and not across the cross-section. The flow may be unsteady, in this case the parameter vary in time but still not across the cross-section. An example of one-dimensional flow is the flow in a pipe. Note that since flow must be zero at the pipe wall - yet non-zero in the centre - there is a difference of parameters across the cross-section. Should this be treated as two-dimensional flow? Possibly - but it is only necessary if very high accuracy is required. A correction factor is then usually applied.



One dimensional flow in a pipe.

Flow is *two-dimensional* if it can be assumed that the flow parameters vary in the direction of flow and in one direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. An example is flow over a weir foe which typical streamlines can be seen in the figure below. Over the majority of the length of the weir the flow is the same - only at the two ends does it change slightly. Here correction factors may be applied.

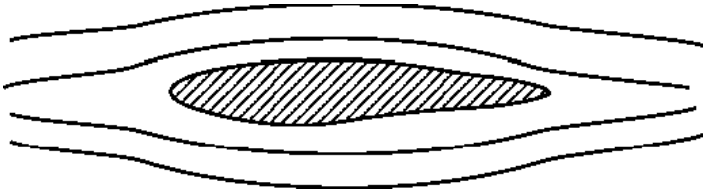


Two-dimensional flow over a weir.

In this course we will **only** be considering steady, incompressible one and two-dimensional flow.

5. Streamlines and streamtubes

In analysing fluid flow it is useful to visualise the flow pattern. This can be done by drawing lines joining points of equal velocity - velocity contours. These lines are know as *streamlines*. Here is a simple example of the streamlines around a cross-section of an aircraft wing shaped body:



Streamlines around a wing shaped body

When fluid is flowing past a solid boundary, e.g. the surface of an aerofoil or the wall of a pipe, fluid obviously does not flow into or out of the surface. So very close to a boundary wall the flow direction must be parallel to the boundary.

- *Close to a solid boundary streamlines are parallel to that boundary*

At all points the direction of the streamline is the direction of the fluid velocity: this is how they are defined. Close to the wall the velocity is parallel to the wall so the streamline is also parallel to the wall.

It is also important to recognise that the position of streamlines can change with time - this is the case in unsteady flow. In steady flow, the position of streamlines does not change.

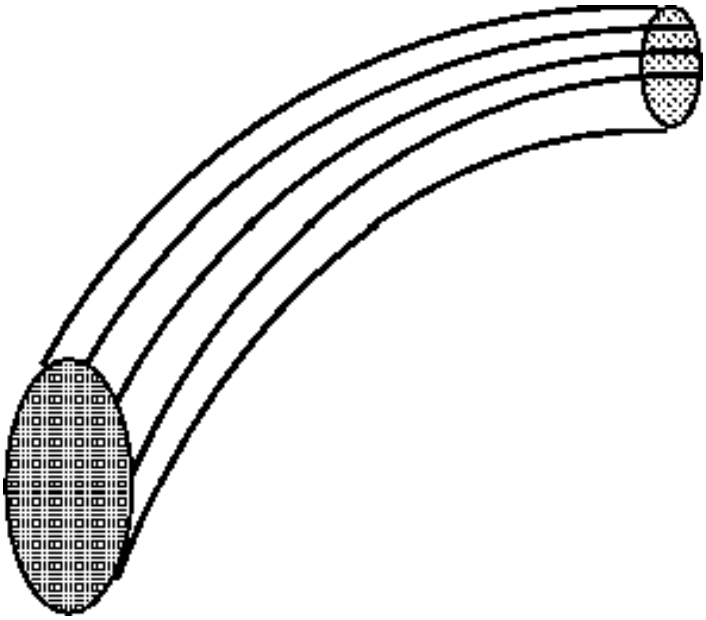
Some things to know about streamlines

- Because the fluid is moving in the same direction as the streamlines, fluid can not cross a streamline.

- Streamlines can not cross each other. If they were to cross this would indicate two different velocities at the same point. This is not physically possible.

- The above point implies that any particles of fluid starting on one streamline will stay on that same streamline throughout the fluid.

A useful technique in fluid flow analysis is to consider only a part of the total fluid in isolation from the rest. This can be done by imagining a tubular surface formed by streamlines along which the fluid flows. This tubular surface is known as a *streamtube*.



A Streamtube

And in a two-dimensional flow we have a streamtube which is flat (in the plane of the paper):



A two dimensional version of the streamtube

The "walls" of a streamtube are made of streamlines. As we have seen above, fluid cannot flow across a streamline, so fluid cannot cross a streamtube wall. The streamtube can often be viewed as a solid walled pipe. A streamtube is **not** a pipe - it differs in unsteady flow as the walls will move with time. And it differs because the "wall" is moving with the fluid

The Bernoulli equation

1. Work and energy

We know that if we drop a ball it accelerates downward with an acceleration $g = 9.81 m/s^2$ (neglecting the frictional resistance due to air). We can calculate the speed of the ball after falling a distance *h* by the formula

$v^2 = u^2 + 2as$ (*a* = *g* and *s* = *h*). The equation could be applied to a falling droplet of water as the same laws of motion apply

A more general approach to obtaining the parameters of motion (of both solids and fluids) is to apply the principle of ***conservation of energy***. When friction is negligible the

---

sum of kinetic energy and gravitational potential energy is constant.

Kinetic energy $= \frac{1}{2}mv^2$

Gravitational potential energy $= mgh$

(*m* is the mass, *v* is the velocity and *h* is the height above the datum).

To apply this to a falling droplet we have an initial velocity of zero, and it falls through a height of h.

Initial kinetic energy $= 0$

Initial potential energy $= mgh$

Final kinetic energy $= \frac{1}{2}mv^2$

Final potential energy $= 0$

We know that

kinetic energy + potential energy = constant

so

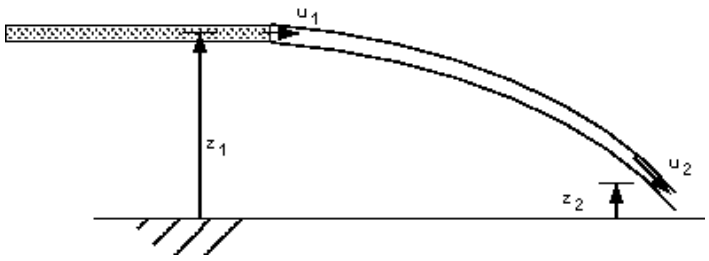
Initial kinetic energy + Initial potential energy = Final kinetic energy + Final potential energy

$$mgh = \frac{1}{2}mv^2$$

so

$$v = \sqrt{2gh}$$

Although this is applied to a drop of liquid, a similar method can be applied to a **continuous jet** of liquid.



The Trajectory of a jet of water

We can consider the situation as in the figure above - a continuous jet of water coming from a pipe with velocity u_1. One particle of the liquid with mass m travels with the jet and falls from height z_1 to z_2. The velocity also changes from u_1 to u_2. The jet is travelling in air where the pressure is everywhere atmospheric so there is no force due to pressure acting on the fluid. The only force which is acting is that due to gravity. The sum of the kinetic and potential energies remains constant (as we neglect energy losses due to friction) so

$$mgz_1 + \frac{1}{2}mu_1^2 = mgz_2 + \frac{1}{2}mu_2^2$$

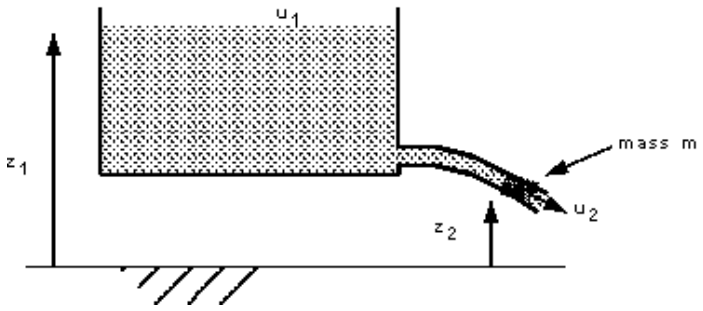
As m is constant this becomes

$$\frac{1}{2}u_1^2 + gz_1 = \frac{1}{2}u_2^2 + gz_2$$

This will give a reasonably accurate result as long as the weight of the jet is large compared to the frictional forces. It is only applicable while the jet is whole - before it breaks up into droplets.

Flow from a reservoir

We can use a very similar application of the energy conservation concept to determine the velocity of flow along a pipe from a reservoir. Consider the 'idealised reservoir' in the figure below.



An idealised reservoir

The level of the water in the reservoir is z_1. Considering the energy situation - there is no movement of water so kinetic energy is zero but the gravitational potential energy is mgz_1.

If a pipe is attached at the bottom water flows along this pipe out of the tank to a level z_2. A mass m has flowed from the top of the reservoir to the nozzle and it has gained a velocity u_2.

The kinetic energy is now $\frac{1}{2}mu_2^2$ and the potential energy mgz_2. Summarising

Initial kinetic energy $= 0$

Initial potential energy $= mgz_1$

Final kinetic energy $= \frac{1}{2} m u_2^2$

Final potential energy $= mgz_2$

We know that

kinetic energy + potential energy = constant

so

$$mgz_1 = \frac{1}{2} m u_2^2 + mgz_2$$

$$mg(z_1 + z_2) = \frac{1}{2} m u_2^2$$

so

$$u_2 = \sqrt{2g(z_1 - z_2)}$$

We now have a expression for the velocity of the water as it flows from of a pipe nozzle at a height $(z_1 - z_2)$ below the surface of the reservoir. (Neglecting friction losses in the pipe and the nozzle).

Now apply this to this example: A reservoir of water has the surface at 310m above the outlet nozzle of a pipe with diameter 15mm. What is the a) velocity, b) the discharge out of the nozzle and c) mass flow rate. (Neglect all friction in the nozzle and the pipe).

$$\begin{aligned} u_2 &= \sqrt{2g(z_1 - z_2)} \\ &= \sqrt{2 \times g \times 310} \\ &= 78.0 m/s \end{aligned}$$

Volume flow rate is equal to the area of the nozzle multiplied by the velocity

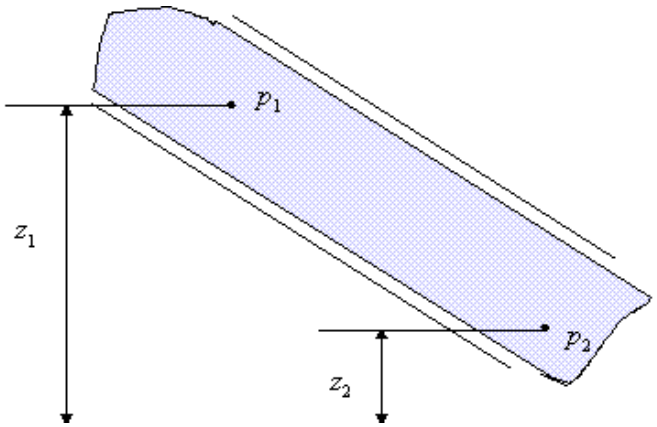
$$Q = Au$$
$$= (\pi \times \tfrac{0.015^2}{4}) \times 78.0$$
$$= 0.01378\, m^3/s$$

The density of water is $1000 kg/m^3$ so the mass flow rate is

$$\text{mass flow rate} = \text{density} \times \text{volume flow rate}$$
$$= \rho Q$$
$$= 1000 \times 0.01378$$
$$= 13.78\, kg/s$$

In the above examples the resultant pressure force was always zero as the pressure surrounding the fluid was the everywhere the same - atmospheric. If the pressures had been different there would have been an extra force acting and we would have to take into account the work done by this force when calculating the final velocity.

We have already seen in the hydrostatics section an example of pressure difference where the velocities are zero.



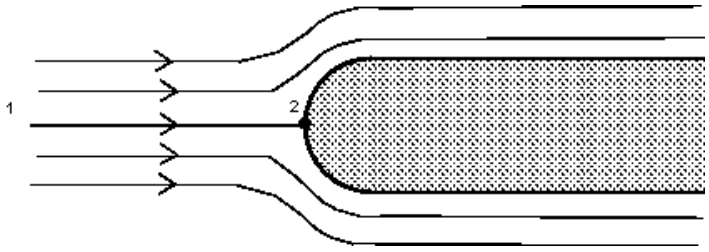
The pipe is filled with stationary fluid of density ρ has pressures p_1 and p_2 at levels z_1 and z_2 respectively. What is the pressure difference in terms of these levels?

Applications of the Bernoulli Equation

The Bernoulli equation can be applied to a great many situations not just the pipe flow we have been considering up to now. In the following sections we will see some examples of its application to flow measurement from tanks, within pipes as well as in open channels.

1. Pitot Tube

If a stream of uniform velocity flows into a blunt body, the stream lines take a pattern similar to this:



Streamlines around a blunt body

Note how some move to the left and some to the right. But one, in the centre, goes to the tip of the blunt body and stops. It stops because at this point the velocity is zero - the fluid does not move at this one point. This point is known as the *stagnation point*.

From the Bernoulli equation we can calculate the pressure at this point. Apply Bernoulli along the central streamline from a point upstream where the velocity is u_1 and the pressure p_1 to the stagnation point of the blunt body where the velocity is zero, $u_2 = 0$. Also $z_1 = z_2$.

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

$$\frac{p_1}{\rho}+\frac{u_1^2}{2}=\frac{p_2}{\rho}$$

$$p_2=p_1+\frac{1}{2}\rho u_1^2$$

This increase in pressure which bring the fluid to rest is called the *dynamic pressure*.

Dynamic pressure = $\frac{1}{2}\rho u_1^2$

or converting this to head (using $h=\frac{p}{\rho g}$)

Dynamic head = $\frac{1}{2g}u_1^2$

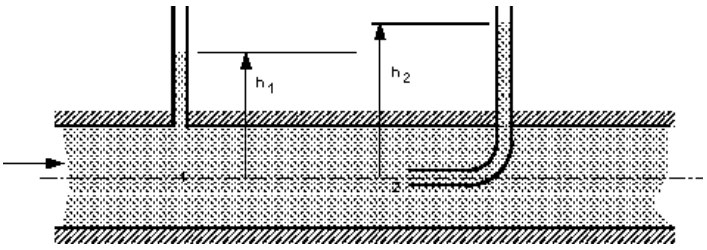
The total pressure is know as the *stagnation pressure* (or *total pressure*)

Stagnation pressure = $p_1 + \frac{1}{2}\rho u_1^2$

or in terms of head

Stagnation head = $\frac{p_1}{\rho g} + \frac{1}{2g}u_1^2$

The blunt body stopping the fluid does not have to be a solid. I could be a static column of fluid. Two piezometers, one as normal and one as a Pitot tube within the pipe can be used in an arrangement shown below to measure velocity of flow.



A Piezometer and a Pitot tube

Using the above theory, we have the equation for p_2 ,

$$p_2 = p_1 + \frac{1}{2}\rho u_1^2$$

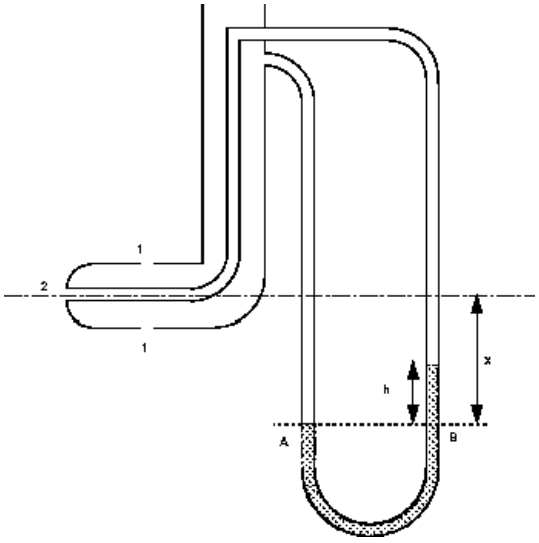
$$\rho g h_2 = \rho g h_1 + \frac{1}{2}\rho u_1^2$$

$$u = \sqrt{2g(h_2 - h_1)}$$

We now have an expression for velocity obtained from two pressure measurements and the application of the Bernoulli equation.

2. Pitot Static Tube

The necessity of two piezometers and thus two readings make this arrangement a little awkward. Connecting the piezometers to a manometer would simplify things but there are still two tubes. The *Pitot static* tube combines the tubes and they can then be easily connected to a manometer. A Pitot static tube is shown below. The holes on the side of the tube connect to one side of a manometer and register the *static head, (h_1)*, while the central hole is connected to the other side of the manometer to register, as before, the *stagnation head (h_2)*.



A Pitot-static tube

Consider the pressures on the level of the centre line of the Pitot tube and using the theory of the manometer,

$$p_A = p_2 + \rho g X$$
$$p_B = p_1 + \rho g(X - h) + \rho_{man} g h$$
$$p_A = p_B$$
$$p_2 + \rho g X = p_1 + \rho g(X - h) + \rho_{man} g h$$

We know that $p_2 = p_{static} = p_1 + \frac{1}{2}\rho u_1^2$, substituting this in to the above gives

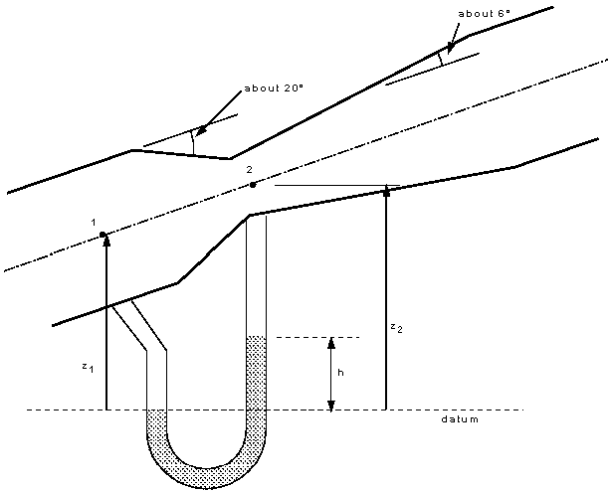
$$p_1 + hg(\rho_{man} - \rho) = p_1 + \frac{\rho u_1^2}{2}$$

$$u_1 = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$

The Pitot/Pitot-static tubes give velocities at points in the flow. It does not give the overall discharge of the stream, which is often what is wanted. It also has the drawback that it is liable to block easily, particularly if there is significant debris in the flow.

3. Venturi Meter

The Venturi meter is a device for measuring discharge in a pipe. It consists of a rapidly converging section which increases the velocity of flow and hence reduces the pressure. It then returns to the original dimensions of the pipe by a gently diverging 'diffuser' section. By measuring the pressure differences the discharge can be calculated. This is a particularly accurate method of flow measurement as energy loss are very small.



A Venturi meter

Applying Bernoulli along the streamline from point 1 to point 2 in the narrow *throat* of the Venturi meter we have

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

By the using the continuity equation we can eliminate the velocity u_2,

$$Q = u_1 A_1 = u_2 A_2$$
$$u_2 = \frac{u_1 A_1}{A_2}$$

Substituting this into and rearranging the Bernoulli equation we get

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = \frac{u_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$
$$u_1 = \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

To get the theoretical discharge this is multiplied by the area. To get the actual discharge taking in to account the losses due to friction, we include a coefficient of discharge

$$Q_{ideal} = u_1 A_1$$
$$Q_{actual} = C_d Q_{ideal} = C_d u_1 A_1$$
$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

This can also be expressed in terms of the manometer readings

$$p_1 + \rho g z_1 = p_2 + \rho_{man} g h + \rho g (z_2 - h)$$
$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Thus the discharge can be expressed in terms of the manometer reading::

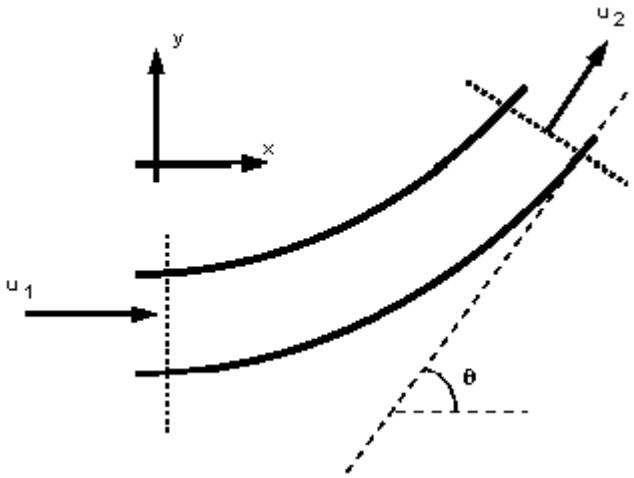
Application of the Momentum Equation

In this section we will consider the following examples:

1. Force due to the flow of fluid round a pipe bend.
2. Force on a nozzle at the outlet of a pipe.
3. Impact of a jet on a plane surface.

1. The force due the flow around a pipe bend

Consider a pipe bend with a constant cross section lying in the horizontal plane and turning through an angle of θ°.



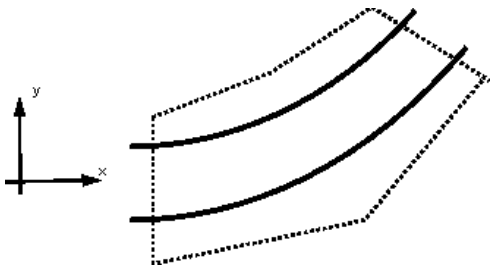
Flow round a pipe bend of constant cross-section

Why do we want to know the forces here? Because the fluid changes direction, a force (very large in the case of water supply pipes,) will act in the bend. If the bend is not fixed it will move and eventually break at the joints. We need to know how much force a support (thrust block) must withstand.

Step in Analysis:

1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the **total** force
4. Calculate the **pressure** force
5. Calculate the **body** force
6. Calculate the **resultant** force

1 Control Volume



The control volume is draw in the above figure, with faces at the inlet and outlet of the bend and encompassing the pipe walls.

2 Co-ordinate axis system

It is convenient to choose the co-ordinate axis so that one is pointing in the direction of the inlet velocity. In the above figure the x-axis points in the direction of the inlet velocity.

3 Calculate the **total** force

In the x-direction:

$$F_{Tx} = \rho Q\left(u_{2x} - u_{1x}\right)$$
$$u_{1x} = u_1$$
$$u_{2x} = u_2 \cos\theta$$
$$F_{Tx} = \rho Q\left(u_2 \cos\theta - u_1\right)$$

In the y-direction:

$$F_{Ty} = \rho Q\left(u_{2y} - u_{1y}\right)$$
$$u_{1y} = u_1 \sin 0 = 0$$
$$u_{2y} = u_2 \sin\theta$$
$$F_{Ty} = \rho Q u_2 \sin\theta$$

4 Calculate the **pressure** force

$$F_P = \text{pressure force at 1} \text{ - pressure force at 2}$$
$$F_{Px} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$
$$F_{Py} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

5 Calculate the **body** force

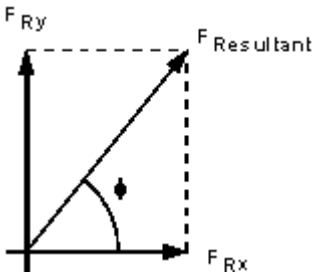
There are no body forces in the x or y directions. The only body force is that exerted by gravity (which acts into the paper in this example - a direction we do not need to consider).

6 Calculate the **resultant** force

$$F_{Tx} = F_{Rx} + F_{Px} + F_{Bx}$$
$$F_{Ty} = F_{Ry} + F_{Py} + F_{By}$$

$$F_{Rx} = F_{Tx} - F_{Px} - 0 = \rho Q\left(u_2 \cos\theta - u_1\right) - p_1 A_1 + p_2 A_2 \cos\theta$$
$$F_{Ry} = F_{Ty} - F_{Py} - 0 = \rho Q u_2 \sin\theta + p_2 A_2 \sin\theta$$

And the resultant force **on the fluid** is given by



$$F_R = \sqrt{F_{Rx}^2 - F_{Ry}^2}$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$$

the force **on the bend** is the same magnitude but in the opposite direction

$$R = -F_R$$

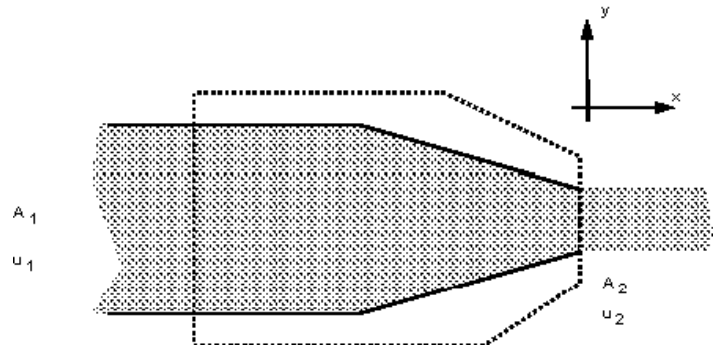
2. Force on a pipe nozzle

Force on the nozzle at the outlet of a pipe. Because the fluid is contracted at the nozzle forces are induced in the nozzle. Anything holding the nozzle (e.g. a fireman) must be strong enough to withstand these forces.

The analysis takes the same procedure as above:

1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the **total** force
4. Calculate the **pressure** force
5. Calculate the **body** force
6. Calculate the **resultant** force

1 & 2 Control volume and Co-ordinate axis are shown in the figure below.



Notice how this is a one dimensional system which greatly simplifies matters.

3 Calculate the **total** force

$$F_T = F_{Tx} = \rho Q\left(u_2 - u_1\right)$$

By continuity, $Q = A_1 u_1 = A_2 u_2$, so

$$F_{Tx} = \rho Q^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

4 Calculate the **pressure** force

$$F_P = F_{Px} = \text{pressure force at 1 - pressure force at 2}$$

We use the Bernoulli equation to calculate the pressure

$$\frac{p_1}{\rho g} + \frac{u_1}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2}{2g} + z_2 + h_f$$

Is friction losses are neglected, $h_f = 0$

the nozzle is horizontal, $z_1 = z_2$

and the pressure outside is atmospheric, $p_2 = 0$,

and with continuity gives

$$p_1 = \frac{\rho Q^2}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

5 Calculate the **body** force

The only body force is the weight due to gravity in the y-direction - but we need not consider this as the only forces we are considering are in the x-direction.

6 Calculate the **resultant** force

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$
$$F_{R_x} = F_{T_x} - F_{P_y} - 0$$

$$F_{T_x} = \rho Q^2\left(\frac{1}{A_2} - \frac{1}{A_1}\right) - \frac{\rho Q^2}{2}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$$

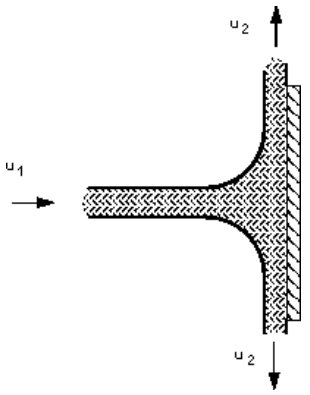
So the fireman must be able to resist the force of

$$R = -F_{T_x}$$

3. Impact of a Jet on a Plane

We will first consider a jet hitting a flat plate (a plane) at an angle of 90, as shown in the figure below.

We want to find the reaction force of the plate i.e. the force the plate will have to apply to stay in the same position.

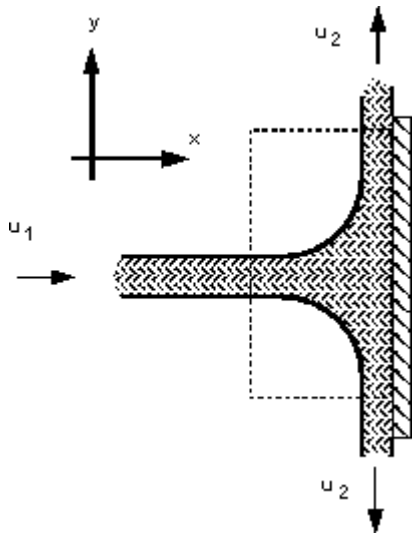


A perpendicular jet hitting a plane.

The analysis take the same procedure as above:

1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the **total** force
4. Calculate the **pressure** force
5. Calculate the **body** force
6. Calculate the **resultant** force

1 & 2 Control volume and Co-ordinate axis are shown in the figure below.



3 Calculate the **total** force

$$F_{T_x} = \rho Q\left(u_{2_x} - u_{1_x}\right)$$
$$= -\rho Q u_{1_x}$$

As the system is symmetrical the forces in the y-direction cancel i.e.

$$F_{T_y} = 0$$

4 Calculate the **pressure force.**

The pressure force is zero as the pressure at both the inlet and the outlets to the control volume are atmospheric.

5 Calculate the **body** force

As the control volume is small we can ignore the body force due to the weight of gravity.

6 Calculate the **resultant** force

$$\begin{aligned} F_{T_x} &= F_{R_x} + F_{P_x} + F_{B_x} \\ F_{R_x} &= F_{T_x} - 0 - 0 \\ &= -\rho Q u_{1x} \end{aligned}$$

Exerted **on the fluid.**

The force **on the plane** is the same magnitude but in the opposite direction

$$R = -F_{R_x}$$

Laminor and Turbulant Flow:

1. Real fluids

The flow of real fluids exhibits viscous effect, that is they tend to "stick" to solid surfaces and have stresses within their body.

You might remember from earlier in the course Newtons law of viscosity:

$$\tau \propto \frac{du}{dy}$$

This tells us that the shear stress, □, in a fluid is proportional to the velocity gradient - the rate of change of velocity across the fluid path. For a "Newtonian" fluid we can write:

$$\tau = \mu \frac{du}{dy}$$

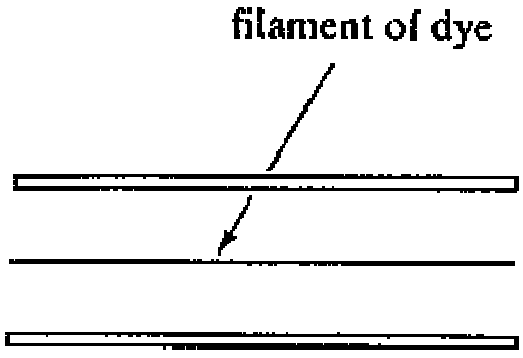
where the constant of proportionality, □ is known as the coefficient of viscosity (or simply viscosity). We saw that for some fluids - sometimes known as exotic fluids - the value of □ changes with stress or velocity gradient. We shall only deal with Newtonian fluids.

In his lecture we shall look at how the forces due to momentum changes on the fluid and viscous forces compare and what changes take place.

2. Laminar and turbulent flow

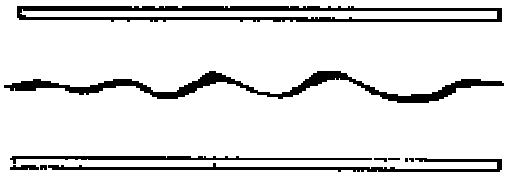
If we were to take a pipe of free flowing water and inject a dye into the middle of the stream, what would we expect to happen?

This



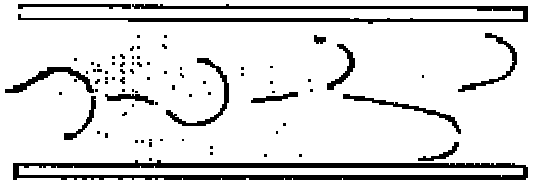
Laminar (viscous)

this



Transitional

or this



Turbulent

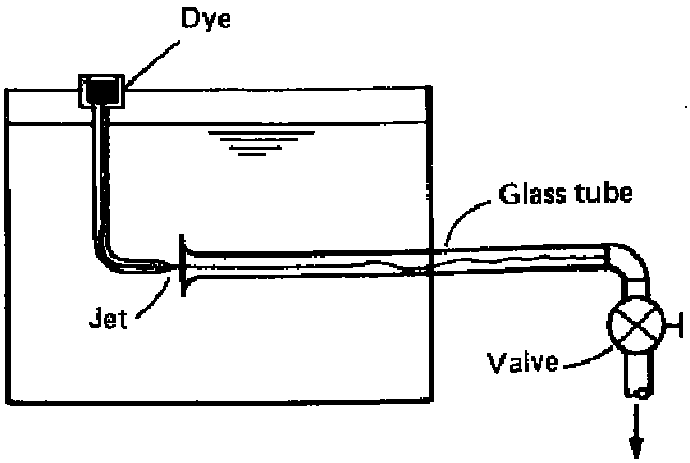
Actually both would happen - but for different flow rates. The top occurs when the fluid is flowing fast and the lower when it is flowing slowly.

The top situation is known as **turbulent** flow and the lower as **laminar** flow.

In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls.

But what is fast or slow? And at what speed does the flow pattern change? And why might we want to know this?

The phenomenon was first investigated in the 1880s by Osbourne Reynolds in an experiment which has become a classic in fluid mechanics.



He used a tank arranged as above with a pipe taking water from the centre into which he injected a dye through a needle. After many experiments he saw that this expression

$$\frac{\rho u d}{\mu}$$

where □ = density, u = mean velocity, d = diameter and □ = viscosity

would help predict the change in flow type. If the value is less than about 2000 then flow is laminar, if greater than 4000 then turbulent and in between these then in the transition zone.

This value is known as the Reynolds number, Re:

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

Laminar flow: Re < 2000

Transitional flow: 2000 < Re < 4000

Turbulent flow: Re > 4000

What are the units of this Reynolds number? We can fill in the equation with SI units:

$$\rho = kg/m^3, \quad u = m/s, \quad d = m$$
$$\mu = Ns/m^2 = kg/ms$$

$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{kg}{m^3}\frac{m}{s}\frac{m}{1}\frac{m}{kg} = 1$$

i.e. it has **no units**. A quantity that has no units is known as a **non-dimensional** (or dimensionless) quantity. Thus the Reynolds number, Re, is a non-dimensional number.

We can go through an example to discover at what velocity the flow in a pipe stops being laminar.

If the pipe and the fluid have the following properties:

water density □ = 1000 kg/m^3

pipe diameter d = 0.5m

(dynamic) viscosity, □ = 0.55x10^3 Ns/m^2

We want to know the maximum velocity when the Re is 2000.

$$\mathrm{Re} = \frac{\rho u d}{\mu} = 2000$$
$$u = \frac{2000\mu}{\rho d} = \frac{2000 \times 0.55 \times 10^{-3}}{1000 \times 0.5}$$
$$u = 0.0022 m/s$$

If this were a pipe in a house central heating system, where the pipe diameter is typically 0.015m, the limiting velocity for laminar flow would be, 0.0733 m/s.

Both of these are very slow. In practice it very rarely occurs in a piped water system - the velocities of flow are much greater. Laminar flow does occur in situations with fluids of greater viscosity - e.g. in bearing with oil as the lubricant.

At small values of Re above 2000 the flow exhibits small instabilities. At values of about 4000 we can say that the flow is truly turbulent. Over the past 100 years since this experiment, numerous more experiments have shown this phenomenon of limits of Re for many different Newtonian fluids - including gasses.

What does this abstract number mean?

We can say that the number has a physical meaning, by doing so it helps to understand some of the reasons for the changes from laminar to turbulent flow.

$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

It can be interpreted that when the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent. When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.

In summary:

<u>Laminar flow</u>

- Re < 2000
- 'low' velocity
- Dye does not mix with water
- Fluid particles move in straight lines
- Simple mathematical analysis possible
- Rare in practice in water systems.

Transitional flow

- 2000 > Re < 4000
- 'medium' velocity
- Dye stream wavers in water - mixes slightly.

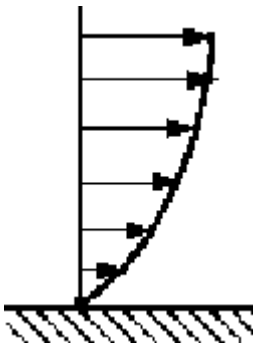
Turbulent flow

- Re > 4000
- 'high' velocity
- Dye mixes rapidly and completely
- Particle paths completely irregular
- Average motion is in the direction of the flow
- Cannot be seen by the naked eye
- Changes/fluctuations are very difficult to detect. Must use laser.
- Mathematical analysis very difficult - so experimental measures are used
- Most common type of flow.

3. Pressure loss due to friction in a pipeline.

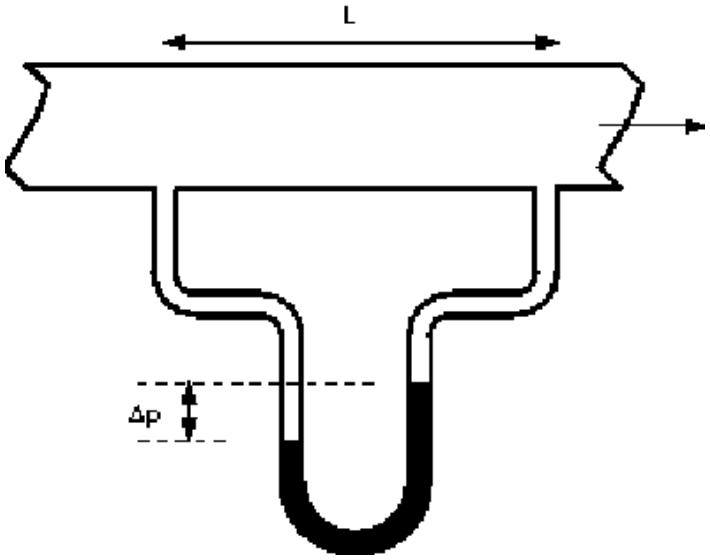
Up to this point on the course we have considered ideal fluids where there have been no losses due to friction or any other factors. In reality, because fluids are viscous, energy is lost by flowing fluids due to friction which must be taken into account. The effect of the friction shows itself as a pressure (or head) loss.

In a pipe with a real fluid flowing, at the wall there is a shearing stress retarding the flow, as shown below.



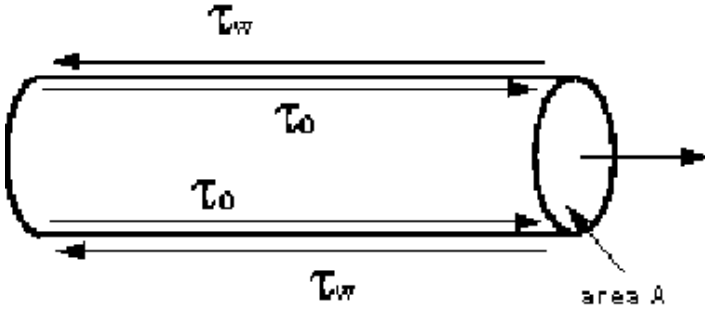
If a manometer is attached as the pressure (head) difference due to the energy lost by the fluid overcoming the shear stress can be easily seen.

The pressure at 1 (upstream) is higher than the pressure at 2.



We can do some analysis to express this loss in pressure in terms of the forces acting on the fluid.

Consider a cylindrical element of incompressible fluid flowing in the pipe, as shown



The pressure at the upstream end is *p*, and at the downstream end the pressure has fallen by □p to (p-□p).

The driving force due to pressure (F = Pressure x Area) can then be written

driving force = Pressure force at 1 - pressure force at 2

$$pA - (p - \Delta p)A = \Delta p\, A = \Delta p \frac{\pi d^2}{4}$$

The retarding force is that due to the shear stress by the walls

$$\begin{aligned} &= \text{shear stress} \times \text{area over which it acts} \\ &= \tau_w \times \text{area of pipe wall} \\ &= \tau_w \pi d L \end{aligned}$$

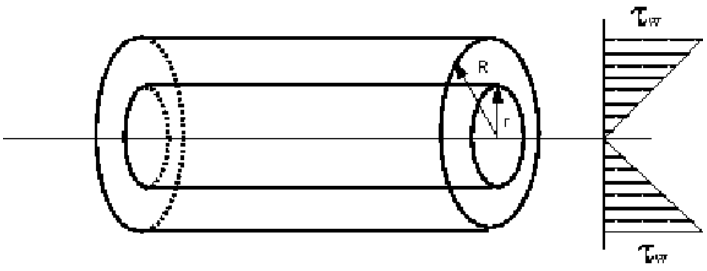
As the flow is in equilibrium,

driving force = retarding force

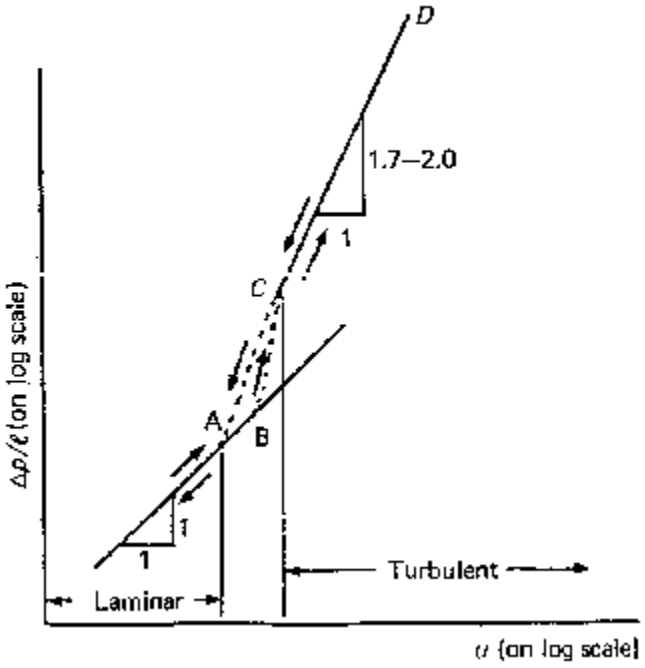
$$\Delta p \frac{\pi d^2}{4} = \tau_w \pi d L$$

$$\Delta p = \frac{\tau_w 4 L}{d}$$

Giving an expression for pressure loss in a pipe in terms of the pipe diameter and the shear stress at the wall on the pipe.



The shear stress will vary with velocity of flow and hence with Re. Many experiments have been done with various fluids measuring the pressure loss at various Reynolds numbers. These results plotted to show a graph of the relationship between pressure loss and Re look similar to the figure below:



This graph shows that the relationship between pressure loss and Re can be expressed as

laminar	$\Delta p \propto u$
turbulent	$\Delta p \propto u^{1.7 \text{ (or } 2.0)}$

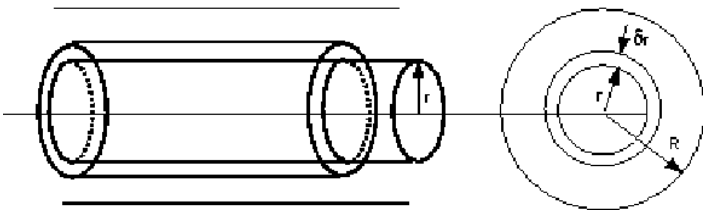
As these are empirical relationships, they help in determining the pressure loss but not in finding the magnitude of the shear stress at the wall τ_w on a particular fluid. If we knew τ_w we could then use it to give a general equation to predict the pressure loss.

4. Pressure loss during laminar flow in a pipe

In general the shear stress τ_w. is almost impossible to measure. But for laminar flow it is possible to calculate a theoretical value for a given velocity, fluid and pipe dimension.

In laminar flow the paths of individual particles of fluid do not cross, so the flow may be considered as a series of concentric cylinders sliding over each other - rather like the cylinders of a collapsible pocket telescope.

As before, consider a cylinder of fluid, length L, radius **r**, flowing steadily in the centre of a pipe.



We are in equilibrium, so the shearing forces on the cylinder equal the pressure forces.

$$\tau\ 2\pi r\ L = \Delta p\ A = \Delta p \pi r^2$$

$$\tau = \frac{\Delta p}{L}\frac{r}{2}$$

By Newtons law of viscosity we have $\tau = \mu \frac{du}{dy}$, where y is the distance from the wall. As we are measuring from the pipe centre then we change the sign and replace y with r distance from the centre, giving

$$\tau = -\mu \frac{du}{dr}$$

Which can be combined with the equation above to give

$$\frac{\Delta p}{L}\frac{r}{2} = -\mu \frac{du}{dr}$$

$$\frac{du}{dr} = -\frac{\Delta p}{L}\frac{r}{2\mu}$$

In an integral form this gives an expression for velocity,

$$u = -\frac{\Delta p}{L}\frac{1}{2\mu}\int r\,dr$$

Integrating gives the value of velocity at a point distance r from the centre

$$u_r = -\frac{\Delta p}{L}\frac{r^2}{4\mu} + C$$

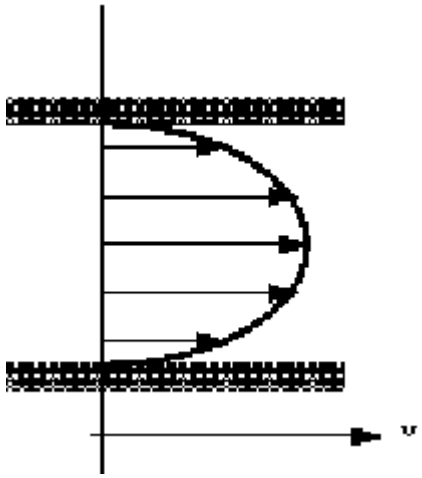
At $r = 0$, (the centre of the pipe), $u = u_{max}$, at $r = R$ (the pipe wall) $u = 0$, giving

$$C = \frac{\Delta p}{L}\frac{R^2}{4\mu}$$

so, an expression for velocity at a point r from the pipe centre when the flow is laminar is

$$u_r = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

Note how this is a parabolic profile (of the form $y = ax^2 + b$) so the velocity profile in the pipe looks similar to the figure below



What is the discharge in the pipe?

$$Q = u_m A$$

$$u_m = \int_0^R u_r \, dr$$

$$= \frac{\Delta p}{L} \frac{1}{4\mu} \int_0^R \left(R^2 - r^2 \right) dr$$

$$= \frac{\Delta p}{L} \frac{R^2}{8\mu} = \frac{\Delta p \, d^2}{32 \mu L}$$

So the discharge can be written

$$Q = \frac{\Delta p \, d^2}{32 \mu L} \frac{\pi d^2}{4}$$

$$= \frac{\Delta p}{L} \frac{\pi d^2}{128 \mu}$$

This is the Hagen-Poiseuille equation for laminar flow in a pipe. It expresses the discharge Q in terms of the pressure gradient ($\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$), diameter of the pipe and the viscosity of the fluid.

We are interested in the pressure loss (head loss) and want to relate this to the velocity of the flow. Writing pressure loss in terms of head loss h_f, i.e. $p = \rho g h_f$

$$u = \frac{\rho g h_f \, d^2}{32 \mu L}$$

$$h_f = \frac{32 \mu L u}{\rho g d^2}$$

This shows that pressure loss is directly proportional to the velocity when flow is laminar.

It has been validated many time by experiment.

It justifies two assumptions:

1. fluid does not slip past a solid boundary
2. Newtons hypothesis.

1. Dimensional Analysis

In engineering the application of fluid mechanics in designs make much of the use of empirical results from a lot of experiments. This data is often difficult to present in a readable form. Even from graphs it may be difficult to interpret. Dimensional analysis provides a strategy for choosing relevant data and how it should be presented.

This is a useful technique in all experimentally based areas of engineering. If it is possible to identify the factors involved in a physical situation, dimensional analysis can form a relationship between them.

The resulting expressions may not at first sight appear rigorous but these qualitative results converted to quantitative forms can be used to obtain any unknown factors from experimental analysis.

2. Dimensions and units

Any physical situation can be described by certain familiar properties e.g. length, velocity, area, volume, acceleration etc. These are all known as dimensions.

Of course dimensions are of no use without a magnitude being attached. We must know more than that something has a length. It must also have a standardised unit - such as a meter, a foot, a yard etc.

Dimensions are properties which can be measured. Units are the standard elements we use to quantify these dimensions.

In dimensional analysis we are only concerned with the nature of the dimension i.e. its quality not its quantity. The following common abbreviation are used:

length = L

mass = M

time = T

force = F

temperature = □

In this module we are only concerned with L, M, T and F (not □). We can represent all the physical properties we are interested in with L, T and one of M or F (F can be represented by a combination of LTM). These notes will always use the LTM combination.

The following table (taken from earlier in the course) lists dimensions of some common physical quantities:

Quantity	SI Unit	.	Dimension
velocity	m/s	ms^{-1}	LT^{-1}
acceleration	m/s^2	ms^{-2}	LT^{-2}
force	N $kg\ m/s^2$	$kg\ ms^{-2}$	$M\ LT^{-2}$
energy (or work)	Joule J N m, $kg\ m^2/s^2$	$kg\ m^2s^{-2}$	ML^2T^{-2}
power	Watt W N m/s $kg\ m^2/s^3$	Nms^{-1} $kg\ m^2s^{-3}$	ML^2T^{-3}
pressure (or stress)	Pascal P, N/m^2, $kg/m/s^2$	Nm^{-2} $kg\ m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
density	kg/m^3	$kg\ m^{-3}$	ML^{-3}
specific weight	N/m^3		

	kg/m^2/s^2	kg m^{-2}s^{-2}	ML^{-2}T^{-2}
relative density	a ratio no units	.	1 no dimension
viscosity	N s/m^2 kg/m s	N sm^{-2} kg m^{-1}s^{-1}	M L^{-1}T^{-1}
surface tension	N/m kg /s^2	Nm^{-1} kg s^{-2}	MT^{-2}

1. 3. Dimensional Homogeneity

Any equation describing a physical situation will only be true if both sides have the same dimensions. That is it must be **dimensionally homogenous**.

For example the equation which gives for over a rectangular weir (derived earlier in this module) is,

$$Q = \frac{2}{3} B\sqrt{2g}H^{3/2}$$

The SI units of the left hand side are m^3s^{-1}. The units of the right hand side must be the same. Writing the equation with only the SI units gives

$$\begin{aligned} m^3s^{-1} &= m\left(ms^{-2}\right)^{1/2} m^{3/2} \\ &= m^3s^{-1} \end{aligned}$$

i.e. the units are consistent.

To be more strict, it is the dimensions which must be consistent (any set of units can be used and simply converted using a constant). Writing the equation again in terms of dimensions,

$$L^3T^{-1} = L\left(L\,T^{-2}\right)^{1/2} L^{3/2}$$
$$= L^3T^{-1}$$

Notice how the powers of the individual dimensions are equal, (for L they are both 3, for T both -1).

This property of dimensional homogeneity can be useful for:

1. Checking units of equations;
2. Converting between two sets of units;
3. Defining dimensionless relationships (see below).

4. Results of dimensional analysis

The result of performing dimensional analysis on a physical problem is a single equation. This equation relates all of the physical factors involved to one another. This is probably best seen in an example.

If we want to find the force on a propeller blade we must first decide what might influence this force.

It would be reasonable to assume that the force, *F*, depends on the following physical properties:

diameter, *d*

forward velocity of the propeller (velocity of the plane), *u*

fluid density, □

revolutions per second, *N*

fluid viscosity, □

Before we do any analysis we can write this equation:

F = □ *(d, u,* □*, N,* □ *)*

or

$0 = \Box_1 (F, d, u, \Box, N, \Box)$

where $\Box$ and $\Box_1$ are unknown functions.

These can be expanded into an infinite series which can itself be reduced to

$F = \Box d^m u^p \Box^{\Box} N^r \Box^{\Box}$

where K is some constant and m, p, q, r, s are unknown constant powers.

From dimensional analysis we

1. obtain these powers
2. form the variables into several dimensionless groups

The value of K or the functions $\Box$ and $\Box_1$ must be determined from experiment. The knowledge of the dimensionless groups often helps in deciding what experimental measurements should be taken.

5. Buckingham's
$\Box$ theorems

Although there are other methods of performing dimensional analysis, (notably the *indicial* method) the method based on the Buckingham $\Box$ theorems gives a good generalised strategy for obtaining a solution. This will be outlined below.

There are two theorems accredited to Buckingham, and know as his $\Box$ theorems.

1^{st} $\Box$ theorem:

A relationship between **m** variables (physical properties such as velocity, density etc.) can be expressed as a relationship between **m-n** *non-dimensional* groups of variables (called $\Box$ groups), where **n** is the number of fundamental dimensions (such as mass, length and time) required to express the variables.

So if a physical problem can be expressed:

$\Box (Q_1 , Q_2 , Q_3 , \ldots\ldots, Q_m) = 0$

then, according to the above theorem, this can also be expressed

$\Box (\Box_1 , \Box_2 , \Box_3 , \ldots\ldots, Q_{m-n}) = 0$

In fluids, we can normally take n = 3 (corresponding to M, L, T).

2^{nd} □ theorem

Each □ group is a function of **n** *governing* or *repeating variables* plus one of the remaining variables.

---

6. Choice of repeating variables

Repeating variables are those which we think will appear in all or most of the □ groups, and are a influence in the problem. Before commencing analysis of a problem one must choose the repeating variables. There is considerable freedom allowed in the choice.

Some rules which should be followed are

1. From the 2^{nd} theorem there can be **n** (= 3) repeating variables.
2. When combined, these repeating variables variable must contain all of dimensions (M, L, T) (That is not to say that each must contain M,L and T).
3. A combination of the repeating variables must not form a dimensionless group.
4. The repeating variables do not have to appear in all □ groups.
5. The repeating variables should be chosen to be measurable in an experimental investigation. They should be of major interest to the designer. For example, pipe diameter (dimension L) is more useful and measurable than roughness height (also dimension L).

In fluids it is usually possible to take □**, u** and **d** as the threee repeating variables.

This freedom of choice results in there being many different □ groups which can be formed - and all are valid. There is not really a wrong choice.

7. An example

Taking the example discussed above of force *F* induced on a propeller blade, we have the equation

$0 = \square (F, d, u, \square, N, \square)$

n = 3 and m = 6

There are m - n = 3 □ groups, so

$\square (\square_1 , \square_2 , \square_3) = 0$

The choice of □, *u, d* as the repeating variables satisfies the criteria above. They are measurable, good design parameters and, in combination, contain all the dimension M,L and T. We can now form the three groups according to the 2nd theorem,

$$\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} F \quad \pi_2 = \rho^{a_2} u^{b_2} d^{c_2} N \quad \pi_3 = \rho^{a_3} u^{b_3} d^{c_3} \mu$$

As the □ groups are all dimensionless i.e. they have dimensions $M^0L^0T^0$ we can use the principle of dimensional homogeneity to equate the dimensions for each □ group.

For the first □ group, $\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} F$

In terms of SI units $1 = \left(kgm^{-3}\right)^{a_1} \left(ms^{-1}\right)^{b_1} (m)^{c_1} kgms^{-2}$

And in terms of dimensions

$$M^0 L^0 T^0 = \left(M L^{-3}\right)^{a_1} \left(L T^{-1}\right)^{b_1} (L)^{c_1} M L T^{-2}$$

For each dimension (M, L or T) the powers must be equal on both sides of the equation, so

for M: $0 = a_1 + 1$

$a_1 = -1$

for L: $0 = -3a_1 + b_1 + c_1 + 1$

$0 = 4 + b_1 + c_1$

for T: $0 = -b_1 - 2$

$b_1 = -2$

$c_1 = -4 - b_1 = -2$

Giving □$_1$ as

$$\pi_1 = \rho^{-1} u^{-2} d^{-2} F$$

$$\pi_1 = \frac{F}{\rho u^2 d^2}$$

And a similar procedure is followed for the other □ groups. Group $\pi_2 = \rho^{a_2} u^{b_2} d^{c_2} N$

$$M^0 L^0 T^0 = \left(M L^{-3}\right)^{a_1} \left(L T^{-1}\right)^{b_1} (L)^{c_1} T^{-1}$$

For each dimension (M, L or T) the powers must be equal on both sides of the equation, so

for M: $0 = a_2$

for L: $0 = -3a_2 + b_2 + c_2$

$0 = b_2 + c_2$

for T: $0 = -b_2 - 1$

$b_2 = -1$

$c_2 = 1$

Giving π_2 as

$$\pi_2 = \rho^0 u^{-1} d^1 N$$

$$\pi_2 = \frac{Nd}{u}$$

And for the third, $\pi_3 = \rho^{a_3} u^{b_3} d^{c_3} \mu$

$$M^0 L^0 T^0 = \left(M L^{-3}\right)^{a_3} \left(L T^{-1}\right)^{b_3} (L)^{c_3} M L^{-1} T^{-1}$$

For each dimension (M, L or T) the powers must be equal on both sides of the equation, so

for M: $0 = a_3 + 1$

$a_3 = -1$

for L: $0 = -3a_3 + b_3 + c_3 - 1$

$b_3 + c_3 = -2$

for T: $0 = -b_3 - 1$

$b_3 = -1$

$c_3 = -1$

Giving π_3 as

$$\pi_3 = \rho^{-1} u^{-1} d^{-1} \mu$$

$$\pi_3 = \frac{\mu}{\rho u d}$$

Thus the problem may be described by the following function of the three non-dimensional □ groups,

□ *(□₁ , □₂ , □₃) = 0*

$$\phi\left(\frac{F}{\rho u^2 d^2}, \frac{Nd}{u}, \frac{\mu}{\rho u d}\right) = 0$$

This may also be written:

$$\frac{F}{\rho u^2 d^2} = \phi\left(\frac{Nd}{u}, \frac{\mu}{\rho u d}\right)$$

8. Wrong choice of physical properties.

If, when defining the problem, extra - unimportant - variables are introduced then extra □ groups will be formed. They will play very little role influencing the physical behaviour of the problem concerned and should be identified during experimental work. If an important / influential variable was missed then a □ group would be missing. Experimental analysis based on these results may miss significant behavioural changes. It is therefore, very important that the initial choice of variables is carried out with great care.

9. Manipulation of the □ groups

Once identified manipulation of the □ groups is permitted. These manipulations do not change the number of groups involved, but may change their appearance drastically.

Taking the defining equation as: □ *(□₁ , □₂ , □₃ …… □$_{m-n}$) = 0*

Then the following manipulations are permitted:

1. Any number of groups can be combined by multiplication or division to form a new group which replaces one of the existing. E.g. □$_1$ and □$_2$ may be combined to form □$_{1a}$ = □$_1$ / □$_2$ so the defining equation becomes
   □ *(□$_{1a}$, □$_2$, □$_3$ …… □$_{m-n}$) = 0*
2. The reciprocal of any dimensionless group is valid. So □ *(□₁ ,1/ □₂ , □₃ …… 1/□$_{m-n}$) = 0* is valid.
3. Any dimensionless group may be raised to any power. So □ *((□₁)^2, (□₂)$^{1/2}$, (□₃)^3…… □$_{m-n}$) = 0* is valid.
4. Any dimensionless group may be multiplied by a constant.

5. Any group may be expressed as a function of the other groups, e.g.
   $\Pi_2 = \Pi\ (\Pi_1, \Pi_3 \ldots\ldots \Pi_{m-n})$

In general the defining equation could look like

$\Pi\ (\Pi_1, 1/\Pi_2, (\Pi_3)^i \ldots\ldots 0.5\Pi_{m-n}) = 0$

10. Common Π groups

During dimensional analysis several groups will appear again and again for different problems. These often have names. You will recognise the Reynolds number $\Pi ud/\Pi$. Some common non-dimensional numbers (groups) are listed below.

Reynolds number $\mathrm{Re} = \dfrac{\rho u d}{\mu}$ inertial, viscous force ratio

Euler number $\mathrm{En} = \dfrac{p}{\rho u^2}$ pressure, inertial force ratio

Froude number $\mathrm{Fn} = \dfrac{u^2}{gd}$ inertial, gravitational force ratio

Weber number $\mathrm{We} = \dfrac{\rho u d}{\sigma}$ inertial, surface tension force ratio

Mach number $\mathrm{Mn} = \dfrac{u}{c}$ Local velocity, local velocity of sound ratio

11. Examples

The discharge Q through an orifice is a function of the diameter d, the pressure difference p, the density Π, and the viscosity Π, show that $Q = \dfrac{d^2 p^{1/2}}{\rho^{1/2}} \phi\left(\dfrac{d\rho^{1/2} p^{1/2}}{\mu}\right)$, where Π is some unknown function.

Write out the dimensions of the variables

$\Pi\ \Pi$ ML^{-3} *u:* LT^{-1}

d: L $\Pi\ \Pi$ $ML^{-1}T^{-1}$

p:(force/area) $ML^{-1}T^{-2}$

We are told from the question that there are 5 variables involved in the problem: *d, p,* □, □ and *Q*.

Choose the three recurring (governing) variables; *Q, d,* □̃

From Buckingham's □ theorem we have m-n = 5 - 3 = 2 non-dimensional groups.

$$\phi(Q,d,\rho,\mu,p)=0$$
$$\phi(\pi_1,\pi_2)=0$$
$$\pi_1 = Q^{a_1} d^{b_1} \rho^{c_1} \mu$$
$$\pi_2 = Q^{a_2} d^{b_2} \rho^{c_2} p$$

For the first group, π_1:

$$M^0 L^0 T^0 = \left(L^3 T^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} ML^{-1}T^{-1}$$

M] $0 = c_1 + 1$

$c_1 = -1$

L] $0 = 3a_1 + b_1 - 3c_1 - 1$

$-2 = 3a_1 + b_1$

T] $0 = -a_1 - 1$

$a_1 = -1$

$b_1 = 1$

$$\pi_1 = Q^{-1} d^{1} \rho^{-1} \mu$$
$$= \frac{d\mu}{\rho Q}$$

And the second group π_2 :

(note p is a pressure (force/area) with dimensions $ML^{-1}T^{-2}$)

$$M^0 L^0 T^0 = \left(L^3 T^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} MT^{-2}L^{-1}$$

M] $0 = c_2 + 1$

$c_2 = -1$

L] $0 = 3a_2 + b_2 - 3c_2 - 1$

$-2 = 3a_2 + b_2$

T] $0 = -a_2 - 2$

$a_2 = -2$

$b_2 = 4$

$$\pi_2 = Q^{-2} d^4 \rho^{-1} p$$
$$= \frac{d^4 p}{\rho Q^2}$$

So the physical situation is described by this function of non-dimensional numbers,

$$\phi(\pi_1, \pi_2) = \phi\left(\frac{d\mu}{Q\rho}, \frac{d^4 p}{\rho Q^2}\right) = 0$$

or

$$\frac{d\mu}{Q\rho} = \phi_1\left(\frac{d^4 p}{\rho Q^2}\right)$$

The question wants us to show : $Q = \frac{d^2 p^{1/2}}{\rho^{1/2}} \phi\left(\frac{d\rho^{1/2} p^{1/2}}{\mu}\right)$

Take the reciprocal of square root of π_2: $\frac{1}{\sqrt{\pi_2}} = \frac{\rho^{1/2} Q}{d^2 p^{1/2}} = \pi_{2a}$,

Convert π_1 by multiplying by this new group, π_{2a}

$$\pi_{1a} = \pi_1 \pi_{2a} = \frac{d\mu}{Q\rho} \frac{\rho^{1/2} Q}{d^2 p^{1/2}} = \frac{\mu}{d\rho^{1/2} p^{1/2}}$$

then we can say

$$\phi\left(1/\pi_{1a},\pi_{2a}\right)=\phi\left(\frac{d\rho^{1/2}p^{1/2}}{\mu},\frac{d^2p^{1/2}}{Q\rho^{1/2}}\right)=0$$

or

$$Q=\frac{d^2p^{1/2}}{\rho^{1/2}}\phi\left(\frac{d\rho^{1/2}p^{1/2}}{\mu}\right)$$

1. Similarity

Hydraulic models may be either true or distorted models. True models reproduce features of the prototype but at a scale - that is they are *geometrically* similar.

2. Geometric similarity

Geometric similarity exists between model and prototype if the ratio of all corresponding dimensions in the model and prototype are equal.

$$\frac{L_{\text{model}}}{L_{\text{prototype}}}=\frac{L_{\text{m}}}{L_{\text{p}}}=\lambda_L$$

where λ_L is the scale factor for length.

For area

$$\frac{A_{\text{model}}}{A_{\text{prototype}}}=\frac{L_{\text{m}}^2}{L_{\text{p}}^2}=\lambda_L^2$$

All corresponding angles are the same.

3. Kinematic similarity

Kinematic similarity is the similarity of time as well as geometry. It exists between model and prototype

1. If the paths of moving particles are geometrically similar
2. If the rations of the velocities of particles are similar

Some useful ratios are:

Velocity $\frac{V_m}{V_p} = \frac{L_m / T_m}{L_p / T_p} = \frac{\lambda_L}{\lambda_T} = \lambda_u$

Acceleration $\frac{a_m}{a_p} = \frac{L_m / T_m^2}{L_p / T_p^2} = \frac{\lambda_L}{\lambda_T^2} = \lambda_a$

Discharge $\frac{Q_m}{Q_p} = \frac{L_m^3 / T_m}{L_p^3 / T_p} = \frac{\lambda_L^3}{\lambda_T} = \lambda_Q$

This has the consequence that streamline patterns are the same.

4. Dynamic similarity

Dynamic similarity exists between geometrically and kinematically similar systems if the ratios of all forces in the model and prototype are the same.

Force ratio $\frac{F_m}{F_p} = \frac{M_m a_m}{M_p a_p} = \frac{\rho_m L_m^3}{\rho_p L_p^3} \times \frac{\lambda_L}{\lambda_T^2} = \lambda_\rho \lambda_L^2 \left(\frac{\lambda_L}{\lambda_T}\right)^2 = \lambda_\rho \lambda_L^2 \lambda_u^2$

This occurs when the controlling dimensionless group on the right hand side of the defining equation is the same for model and prototype.

5. Models

When a hydraulic structure is build it undergoes some analysis in the design stage. Often the structures are too complex for simple mathematical analysis and a hydraulic model is build. Usually the model is less than full size but it may be greater. The real structure is known as the prototype. The model is usually built to an exact geometric scale of the prototype but in some cases - notably river model - this is not possible. Measurements can be taken from the model and a suitable scaling law applied to predict the values in the prototype.

To illustrate how these scaling laws can be obtained we will use the relationship for resistance of a body moving through a fluid.

The resistance, *R*, is dependent on the following physical properties:

□□ ML^{-3} *u:* LT^{-1} *l:(length) L* □: $ML^{-1}T^{-1}$

So the defining equation is □ *(R, □, u, l, □) = 0*

Thus, m = 5, n = 3 so there are 5-3 = 2 □ groups

$$\pi_1 = \rho^{a_1} u^{b_1} l^{c_1} R \quad \pi_2 = \rho^{a_2} u^{b_2} d^{c_2} \mu$$

For the □$_1$ group $M^0 L^0 T^0 = \left(M L^{-3}\right)^{a_1} \left(L T^{-1}\right)^{b_1} (L)^{c_1} M L T^{-2}$

Leading to □$_1$ as

$$\pi_1 = \frac{R}{\rho u^2 l^2}$$

For the □$_2$ group $M^0 L^0 T^0 = \left(M L^{-3}\right)^{a_2} \left(L T^{-1}\right)^{b_2} (L)^{c_2} M L^{-1} T^{-1}$

Leading to □$_1$ as

$$\pi_2 = \frac{\mu}{\rho u l}$$

Notice how 1/□$_2$ is the Reynolds number. We can call this □$_{2a.}$

So the defining equation for resistance to motion is

□ (□$_1$, □$_{2a}$) = 0

We can write

$$\frac{R}{\rho u^2 l^2} = \phi\left(\frac{\rho u l}{\mu}\right)$$

$$R = \rho u^2 l^2 \phi\left(\frac{\rho u l}{\mu}\right)$$

This equation applies whatever the size of the body i.e. it is applicable to a to the prototype and a geometrically similar model. Thus for the model

$$\frac{R_m}{\rho_m u_m^2 l_m^2} = \phi\left(\frac{\rho_m u_m l_m}{\mu_m}\right)$$

and for the prototype

$$\frac{R_p}{\rho_p u_p^2 l_p^2} = \phi\left(\frac{\rho_p u_p l_p}{\mu_p}\right)$$

Dividing these two equations gives

$$\frac{R_m / \rho_m u_m^2 l_m^2}{R_p / \rho_p u_p^2 l_p^2} = \frac{\phi\left(\rho_m u_m l_m / \mu_m\right)}{\phi\left(\rho_p u_p l_p / \mu_p\right)}$$

At this point we can go no further unless we make some assumptions. One common assumption is to assume that the Reynolds number is the same for both the model and prototype i.e.

$$\rho_m u_m l_m / \mu_m = \rho_p u_p l_p / \mu_p$$

This assumption then allows the equation following to be written

$$\frac{R_m}{R_p} = \frac{\rho_m u_m^2 l_m^2}{\rho_p u_p^2 l_p^2}$$

Which gives this scaling law for resistance force:

$$\lambda_R = \lambda_\rho \lambda_u^2 \lambda_L^2$$

That the Reynolds numbers were the same was an essential assumption for this analysis. The consequence of this should be explained.

$$\mathrm{Re}_m = \mathrm{Re}_p$$

$$\frac{\rho_m u_m l_m}{\mu_m} = \frac{\rho_p u_p l_p}{\mu_p}$$

$$\frac{u_m}{u_p} = \frac{\rho_p}{\rho_m}\frac{\mu_m}{\mu_p}\frac{l_p}{l_m}$$

$$\lambda_u = \frac{\lambda_\mu}{\lambda_\rho \lambda_L}$$

Substituting this into the scaling law for resistance gives

$$\lambda_R = \lambda_\rho \left(\frac{\lambda_\mu}{\lambda_\rho}\right)^2$$

So the force on the prototype can be predicted from measurement of the force on the model. But only if the fluid in the model is moving with same Reynolds number as it would in the prototype. That is to say the R_p can be predicted by

$$R_p = \frac{\rho_p u_p^2 l_p^2}{\rho_m u_m^2 l_m^2} R_m$$

provided that $u_p = \frac{\rho_m}{\rho_p}\frac{\mu_p}{\mu_m}\frac{l_m}{l_p}u_m$

In this case the model and prototype are **dynamically similar**.

Formally this occurs when the controlling dimensionless group on the right hand side of the defining equation is the same for model and prototype. In this case the controlling dimensionless group is the Reynolds number.

6. Dynamically similar model examples

Example 1

An underwater missile, diameter 2m and length 10m is tested in a water tunnel to determine the forces acting on the real prototype. A 1/20th scale model is to be used. If the maximum allowable speed of the prototype missile is 10 m/s, what should be the speed of the water in the tunnel to achieve dynamic similarity?

For dynamic similarity the Reynolds number of the model and prototype must be equal:

$$\mathrm{Re}_m = \mathrm{Re}_p$$

$$\left(\frac{\rho u d}{\mu}\right)_m = \left(\frac{\rho u d}{\mu}\right)_p$$

So the model velocity should be

$$u_m = u_p \frac{\rho_p}{\rho_m}\frac{d_p}{d_m}\frac{\mu_m}{\mu_p}$$

As both the model and prototype are in water then, $\rho_m = \rho_p$ and $\mu_m = \mu_p$ so

$$u_m = u_p \frac{d_p}{d_m} = 10\frac{1}{1/20} = 200 m/s$$

Note that this is a **very** high velocity. This is one reason why model tests are not always done at exactly equal Reynolds numbers. Some relaxation of the equivalence requirement is often acceptable when the Reynolds number is high. Using a wind tunnel may have been possible in this example. If this were the case then the appropriate values of the □ and □ ratios need to be used in the above equation.

Example 2

A model aeroplane is built at 1/10 scale and is to be tested in a wind tunnel operating at a pressure of 20 times atmospheric. The aeroplane will fly at 500km/h. At what speed should the wind tunnel operate to give dynamic similarity between the model and prototype? If the drag measure on the model is 337.5 N what will be the drag on the plane?

From earlier we derived the equation for resistance on a body moving through air:

$$R = \rho u^2 l^2 \phi\left(\frac{\rho u l}{\mu}\right) = \rho u^2 l^2 \phi(\mathrm{Re})$$

For dynamic similarity $\mathrm{Re}_m = \mathrm{Re}_p$, so

$$u_m = u_p \frac{\rho_p}{\rho_m}\frac{d_p}{d_m}\frac{\mu_m}{\mu_p}$$

The value of □ does not change much with pressure so $□_m = □_p$

The equation of state for an ideal gas is $p = □RT$. As temperature is the same then the density of the air in the model can be obtained from

$$\frac{p_m}{p_p} = \frac{\rho_m RT}{\rho_p RT} = \frac{\rho_m}{\rho_p}$$

$$\frac{20p_p}{p_p} = \frac{\rho_m}{\rho_p}$$

$$\rho_m = 20\rho_p$$

So the model velocity is found to be

$$u_m = u_p \frac{1}{20}\frac{1}{1/10} = 0.5u_p$$

$$u_m = 250km/h$$

The ratio of forces is found from

$$\frac{R_m}{R_p} = \frac{\left(\rho u^2 l^2\right)_m}{\left(\rho u^2 l^2\right)_p}$$

$$\frac{R_m}{R_p} = \frac{20}{1}\frac{(0.5)^2}{1}\frac{(0.1)^2}{1} = 0.05$$

So the drag force on the prototype will be

$$R_p = \frac{1}{0.05}R_m = 20 \times 337.5 = 6750N$$

7. Models with free surfaces - rivers, estuaries etc.

When modelling rivers and other fluid with free surfaces the effect of gravity becomes important and the major governing non-dimensional number becomes the Froude (Fn) number. The resistance to motion formula above would then be derived with *g* as an extra dependent variables to give an extra □ group. So the defining equation is:

□ *(R, □, u, l, □, g) = 0*

From which dimensional analysis gives:

$$R = \rho u^2 l^2 \phi\left(\frac{\rho u l}{\mu}, \frac{u^2}{gl}\right)$$

$$R = \rho u^2 l^2 \phi(\mathrm{Re}, \mathrm{Fn})$$

Generally the prototype will have a very large Reynolds number, in which case slight variation in Re causes little effect on the behaviour of the problem. Unfortunately models are sometimes so small and the Reynolds numbers are large and the viscous effects take effect. This situation should be avoided to achieve correct results. Solutions to this problem would be to increase the size of the model - or more difficult - to change the fluid (i.e. change the viscosity of the fluid) to reduce the Re.

8. Geometric distortion in river models

When river and estuary models are to be built, considerable problems must be addressed. It is very difficult to choose a suitable scale for the model and to keep geometric similarity. A model which has a suitable depth of flow will often be far to big - take up too much floor space. Reducing the size and retaining geometric similarity can give tiny depth where viscous force come into play. These result in the following problems:

1. accurate depths and depth changes become very difficult to measure;
2. the bed roughness of the channel becomes impracticably small;
3. laminar flow may result - (turbulent flow is normal in river hydraulics.)

The solution often adopted to overcome these problems is to abandon strict geometric similarity by having different scales in the horizontal and the vertical. Typical scales are 1/100 in the vertical and between 1/200 and 1/500 in the horizontal. Good overall flow patterns and discharge characteristics can be produced by this technique, however local detail of flow is not well modelled.

In these model the Froude number (u^2/d) is used as the dominant non-dimensional number. Equivalence in Froude numbers can be achieved between model and prototype even for distorted models. To address the roughness problem artificially high surface roughness of wire mesh or small blocks is usually used.