ELECTROMAGNETICS
6. AC CIRCUITS

POINTS TO REMEMBER

1. The electric current which does not change its magnitude or direction with time is known as direct current.
2. The electric current whose magnitude changes with time and direction reverses periodically is known as alternating current.

$\therefore e = e_0 \sin \omega t$

Where $e_0 = BAN\omega$ is known as peak value of alternating emf

Also, $e = e_0 \sin \omega t$ and $i = i_0 \sin \omega t$

Where i_0 = peak value of alternating current.

3. Average or mean value:

a) It is the steady current (dc) which when passes through a circuit for half time period of AC sends the same charge as done by the AC in the same time.
b) The average value of AC for the complete cycle is zero.
c) The average value of AC for the half cycle of AC is given by.

$$I_{avg} = \frac{2I_0}{\pi} = 0.636 I_0$$

Similarly, E_{avg} is also $\frac{2E_0}{\pi}$ for half cycle of AC

4. RMS value or Virtual value or Effective value :

The RMS value of AC is the steady current (DC) which when flowing through a given resistance for a given time, produces the same amount of heat as produced by the AC when flowing through the same resistance for the same time.

$$I_V = \frac{I_0}{\sqrt{2}} = 0.707 I_0 \quad \text{and} \quad E_V = \frac{E_0}{\sqrt{2}} = 0.707 E_0$$

RMS value of AC for half cycle is also same as above

5. AC through a pure resistor ;

$E = E_0 \sin \omega t$ and $I = I_0 \sin \omega t$ Where E_0 and I_0 are the peak values of voltage and current of AC respectively.

Since the emf and current raise or fall simultaneously they are in phase with each other.

6. AC through a pure inductor:

$$E = E_0 \sin \omega t \quad \text{and} \quad I = \frac{E_0}{L\omega} \sin\left(\omega t - \frac{\pi}{2}\right)$$

The term $L\omega$ has the units of resistance and it is called as resistance of the inductor or inductive reactance (X_L)

$$I = \frac{E_0}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) \Rightarrow I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where I_0 is the peak value of current. Hence the current is lagging behind emf or emf is leading current by a phase difference of $\frac{\pi}{2}$.

7. AC Through a pure capacitor:

$$E = E_0 \sin \omega t \quad \text{and} \quad I = \frac{E_0}{1/c\omega} \cos \omega t$$

The term $\frac{1}{C\omega}$ is called capacitive reactance where it has the dimensions of resistance. It is also called resistance of capacitor.

$I = \frac{E_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) \quad \Rightarrow I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ Where I_0 is peak value of AC

Hence current leads emf or emf is lagging behind current by $\frac{\pi}{2}$

8. AC through LR circuit :

Let V_L and V_R be the instantaneous voltages across the inductor and resistor respectively.

$V_L = IX_L = IL\omega$ And $V_R = IR$

Where X_L is the inductive reactance when AC flow through a pure resistor,

$\therefore E = I\sqrt{R^2 + (L\omega)^2}$

$Z = \sqrt{R^2 + (L\omega)^2}$ and $Tan\,\phi = \frac{L\omega}{R}$

This is the phase angle by which the emf leads the current in L – R circuit. z is called impedance of LR circuit.

9. AC through C – R circuit :

$V_C = I \times X_C = \frac{I}{C\omega}$ And $V_R = IR$ Where X_C = capacitive reactance

$E = \sqrt{V_R^2 + V_C^2} \Rightarrow E = \sqrt{(IR)^2 + \left(\frac{I}{C\omega}\right)^2}$

$E = I\left(\sqrt{R^2 + \frac{1}{(C\omega)^2}}\right)$ and $Tan\,\phi = \frac{V_C}{V_R} = \frac{1}{C\omega R}$

This is the phase angle by which the emf lags behind the current in C – R circuit

Here $z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$ is called the impedance of C – R circuit.

10. AC through L – C – R circuit :

a) $V_L = IX_L = IL\omega$; $V_C = IX_C = \frac{I}{C\omega}$ And $V_R = IR$.

b) $E = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$

c) $Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$ is called the impedance of L – C – R circuit.

d) $Tan\,\phi = \frac{V_L - V_C}{V_R} = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$ This is the phase angle by which the emf leads the current.

e) If $L\omega = \frac{1}{C\omega}$, then $Tan\,\phi = 0$ or $\phi = 0$ and hence emf and current are in phase. The circuit in this condition behaves like a pure resistor circuit. This condition is called **resonance** condition.

f) At resonance, $L\omega = \frac{1}{C\omega}$ Or $\omega^2 = \frac{1}{LC}$

$\omega = 2\pi n = \frac{1}{\sqrt{LC}} \Rightarrow n = \frac{1}{2\pi\sqrt{LC}}$

11. Advantages of AC over DC :

a) The generation of AC is more economic than DC
b) AC voltages can be easily stepped up or stepped down using transformers.
c) AC can be transmitted to longer distances with less loss of energy.
d) AC can be easily converted into DC by using rectifiers.

12. Disadvantages :

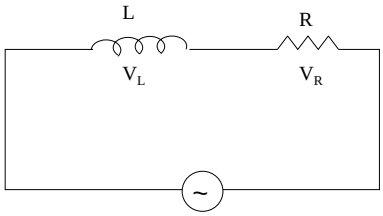
a) AC is more fatal and dangerous than DC
b) AC always flows on the outer layer of the conductor (skin effect) and hence AC requires stranded wires.
c) AC cannot be used in electrolysis like electroplating etc.

LONG ANSWER QUESTIONS

1. Derive expression for impedance and current of an a.c. circuit containing (i) inductance and resistance in series and (ii) capacitance and resistance in series.

Ans. AC through LR circuit :

1. Consider a source of alternating emf connected to a series combination of an inductor L and a resistor R.

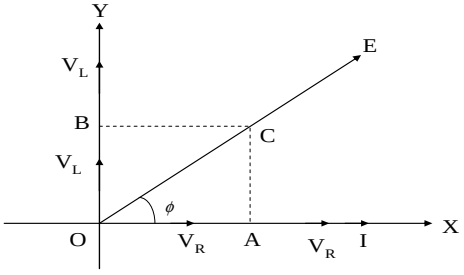


2. Let E be the instantaneous emf and I be the instantaneous current in the circuit.
3. Let V_L and V_R be the instantaneous voltages across the inductor and resistor respectively.
   $V_L = IX_L = IL\omega$ And $V_R = IR$
   Where X_L is the inductive reactance when AC flow through a pure resistor, the voltage and current are in phase unit each other which are represented along OX when AC flows through a pure inductor, voltage leads the current by phase $\frac{\pi}{2}$. Since L and R are in series, the current I is same in both and V_L is represented along OY both and V_L is represented along OY.
4. If OA and OB represents the magnitudes of V_R and V_L then OC represents the effective value of emf in the circuit therefore,

   $OC = OA^2 + OB^2 = \sqrt{V_R^2 + V_L^2}$

   $= \sqrt{(IR)^2 + (IL\omega)^2}$

   $\therefore E = I\sqrt{R^2 + (L\omega)^2}$



$Z = \sqrt{R^2 + (L\omega)^2}$

Where z is called impedance of LR circuit.
Also from the diagram

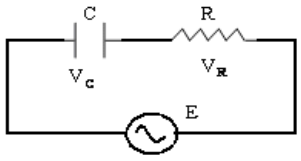
$$Tan\ \phi = \frac{OB}{OA} = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \Rightarrow Tan\ \phi = \frac{L\omega}{R}$$

This is the phase angle by which the emf leads the current in L – R circuit.

C– R circuit :

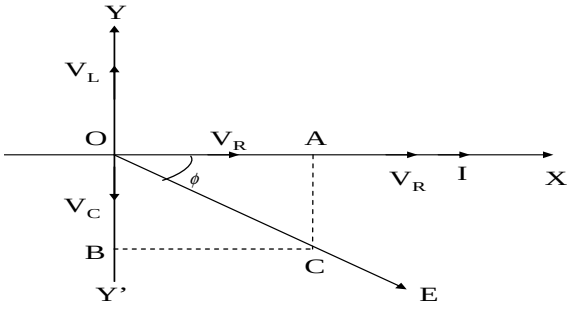
1. Consider a source of alternate emf connected to a series combination of a capacitor C and a resistance R.

$$V_C = I \times X_C = \frac{I}{C\omega} \text{ And } V_R = IR$$



Where X_C=capacitive reactance

2. When AC flows through a pure resistor, the voltage and current are in phase with each other which are represented along OX.
3. When AC flows through a pure capacitor, the emf is logging behind current by a phase $\frac{\pi}{2}$ which is represented along OY'.



$$OC = \sqrt{OA^2 + OB^2}$$

$$E = \sqrt{V_R^2 + V_C^2} \Rightarrow E = \sqrt{(IR)^2 + \left(\frac{I}{C\omega}\right)^2}$$

$$E = I\left(\sqrt{R^2 + \frac{1}{(C\omega)^2}}\right)$$

Here $z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$ is called the impedance of C – R circuit.

Also, from the diagram, $Tan\phi = \frac{V_C}{V_R} = \frac{1}{C\omega R}$

This is the phase angle by which the emf lags behind the current in C – R circuit

2. Obtain an expression for impedance and current in series LCR circuit. Deduce an expression for the resonating frequency of an LCR series resonating circuit.

Ans. An e.m.f $e = E_0 \sin wt$ be applied to a circuit consisting of 'R', 'C' and 'L' in series. As shown in figure let 'i' be the current and 'q' be charge at any instant time 't'.

The potential difference across capacitor is $\frac{q}{C}$ and across inductor is $L\left(\frac{di}{dt}\right)$, both opposite applied e.m.f.

Hence net e.m.f is

$E_0 \sin \omega t - \frac{Ldi}{dt} - \frac{q}{C}$

According to Ohm's law, this net e.m.f is equal to iR.

$\therefore E_0 \sin \omega t - L\frac{di}{dt} - \frac{q}{C} = Ri$

We have $L\frac{di}{dt} + Ri + \frac{q}{C} = E_0 \sin \omega t$

The current 'i' is given by $i = i_0 \sin(\omega t - \phi)$

If $\omega L > \frac{1}{\omega C}$, we have $i = i_0 \sin(\omega t + \phi)$

If $\omega L < \frac{1}{\omega C}$, which is at low frequencies

I_0 is maximum current given by

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Impendence $(Z) = \frac{E_0}{I_0} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At a particular frequency, the total reactance of the circuit is zero,

i.e. $\left(\omega L - \frac{1}{\omega C}\right) = 0$ under this condition impedance Z = R

$\therefore Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\therefore Z = R\left(\therefore \omega L = \frac{1}{\omega C}\right)$

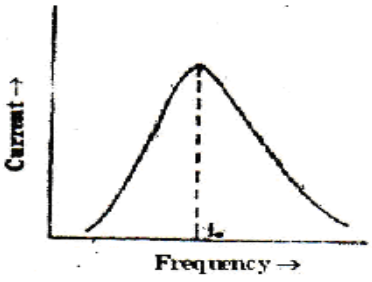
At this frequency, impedance of series LCR circuit is minimum and is equal to R. at this frequency current is maximum. This frequency is called resonant frequency (f_0).

At resonance, $\omega L - \frac{1}{\omega C} = 0$

$\omega^2 = \frac{1}{LC}$ or $\omega = \frac{1}{\sqrt{LC}}$

$2\pi f_0 = \frac{1}{\sqrt{LC}}$

$f_0 = \left(\frac{1}{2\pi\sqrt{LC}}\right)$



The frequency response curve of LCR is shown in figure.

SHORT ANSWER QUESTIONS

1. Explain instantaneous, maximum, and rms values of a current.

Sol: i) The strength of the current at any instant of time't' is known as instantaneous current. It is given by, $I = I_0 \sin \omega t$

ii) In the above equation I_0 is known maximum current. While ω is angular velocity.

iii) Root mean square (R.M.S) value:

The RMS value of AC is the steady current (DC) which when flowing through a given resistance for a given time, produces the same amount of heat as produced by the AC when flowing through the same resistance for the same time.

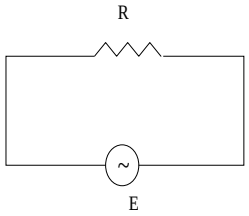
2. Discus the flow of a.c. through a pure resistor.

Sol: Consider a pure resistance of resistance R(zero inductance) is connected to a source of an alternate emf **.**The instantaneous value of alternating emf is given by,

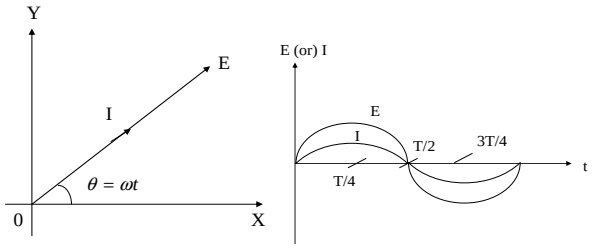
$E = E_0 \sin \omega t$

The current through the resistance at any instant is given by,

$\therefore I = I_0 \sin \omega t \quad \left(\because I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t \right)$



Where E_0 and I_0 are the peak values of voltage and current of AC respectively.



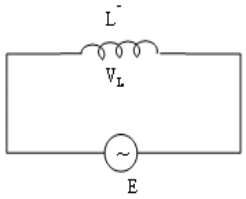
Since the emf and current raise or fall simultaneously they are in phase with each other

There is no phase difference between the E.m.f. and current in a resistor.

3. Obtain an expression for the current in a inductance when an a.c. e.m.f is applied.

Ans. Consider a pure inductor which has no resistance connected to a source of emf E. The instantaneous emf is given by

$E = E_0 \sin \omega t$



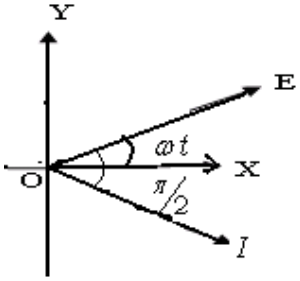
Let I be the current through the circuit and $\frac{dI}{dt}$ be the rate of change of current in the circuit at any instant. The net emf in the circuit is given by

$$E - \frac{L dI}{dt} = 0 \quad \Rightarrow \quad E = \frac{L dI}{dt}$$

Since there is no resistance in the circuit there is no P.D. in the circuit.

$$E = \frac{L dI}{dt}$$

or $dI = \frac{E}{L} dt = \frac{E_0}{L} \sin \omega t \, dt$



Integrating the above equation,

$\int dI = \frac{E_0}{L} \int \sin \omega t \, dt$

$I = \frac{E_0}{L\omega} (-\cos \omega t)$

$I = \frac{E_0}{L\omega} \sin \left(\omega t - \frac{\pi}{2} \right)$

The term $L\omega$ has the units of resistance and it is called as resistance of the inductor or inductive reactance (X_L)

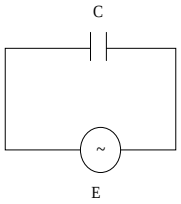
$I = \frac{E_0}{X_L} \sin \left(\omega t - \frac{\pi}{2} \right) \Rightarrow I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$

Where I_0 is the peak value of current. Hence the current is lagging behind emf or emf is leading current by a phase difference of $\frac{\pi}{2}$.

4. Obtain an expression for the current in a capacitor when an a.c. e.m.f. is applied.

Ans. Consider a capacitor of capacity C connected to a source of alternating emf E. The instantaneous emf is given by.

$$E = E_0 \sin \omega t$$

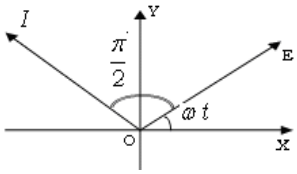


The net emf in the circuit is given by,

$E - \frac{q}{C} = 0$ (or) $E = \frac{q}{C}$

(or) q = EC = $CE_0 \sin \omega t$

$\therefore I = \frac{dq}{dt} = CE_0 \omega \cos \omega t$ (or) $I = \frac{E_0}{1/c\omega} \cos \omega t$



The term $\frac{1}{C\omega}$ is called capacitive reactance where it has the dimensions of resistance. It is also called resistance of capacitor.

$$I = \frac{E_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) \quad \Rightarrow I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where I_0 is peak value of AC

Hence current leads emf or emf is lagging behind current by $\frac{\pi}{2}$

VERY SHORT ANSWER QUESTIONS

1. What is the r.m.s value of the voltage in an A.C. circuit?

Ans. **r.m.s. value (or effective or virtual value) of A.C. Voltage** (e_{rms}): It is the square root of the average of squares of all the of all the instantaneous values of voltage over one complete cycle.

$e_{rms} = \frac{e_0}{\sqrt{2}} = 0.707\, e_0$ Here e_0 = maximum or peak value of A.C. in one complete cycle.

2. What is the phase difference between a.c. e.m.f. and current in the following?
Pure resistor, pure inductor and pure capacitor:

Ans. a) In pure resistor a.c. circuit, there is no phase difference between a.c. e.m.f. and current.

b) In pure inductor a.c. circuit, current lags behind the e.m.f $\frac{\pi}{2}$ radian.

c) In pure capacitor a.c. circuit, current leads emf by $\frac{\pi}{2}$ radian.

3. What is the impedance in the following series circuits?
i) L and R ii) C and R

Ans. i) In L-R series circuit, impedance $Z = \sqrt{R^2 + (L\omega)^2}$

Where $L\omega$ is inductive reactance (X_L)

ii) In C-R series circuit, impedance $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

where $\frac{1}{\omega C}$ is capacitive reactance (X_c)

4. State the expression for the reactance of (i) an inductor and (ii) a capacitor.

Ans. i) Inductive reactance $(X_L) = \omega L$

ii) Capacitive reactance $(X_C) = \dfrac{1}{\omega C}$

SOLVED PROBLEMS

1. The equation of an alternating emf is e =100 sin (100π)t. Find the peak value of its emf and its frequency.

Sol: $e = 100\sin(100\pi t)$

Comparing it with $e = e_0 \sin \pi t$

$e_0 = 100V$

$\omega = 100\pi$ rad/s

$2\pi f = 100\pi$

Frequency $f = \dfrac{100\pi}{2\pi} = 50Hz$

2. The equation of the alternating voltage supplied to our houses is $e = (311\sin 100\pi t)$ volt. A hot wire instrument is used to measure the alternating voltage. What reading does it show?

Sol: Rms voltage, $E = \dfrac{\text{peak voltage}}{\sqrt{2}} = \dfrac{e_0}{\sqrt{2}}$.

Comparing $e = 311\sin 100\pi t$ with $e = e_0 \sin \omega t$, we get

$e_0 = 311$ V

$e = \dfrac{e_0}{\sqrt{2}} = \dfrac{311}{\sqrt{2}} = 219.9$ V.

3. Write the equation of an alternating emf of 120 V, 60 Hz.

Sol: 120 V represents the rms emf i.e., $e = 120$ V

The peak emf, $e_0 = \sqrt{2}\ e = \sqrt{2} \times 120 = 169.7$ V

$\omega = 2\pi f = 2\pi(60) = 120\pi$ rad/s

$\therefore$ The equation of the alternating emf is $e = e_0 \sin \omega t$

$e = (169.7\sin 120\ \pi t)$ volt.

4. An alternating emf.e =300 sin $(100\pi)t$ volt is applied to a pure resistance of 100 Ω. Calculate the rms current through the circuit

Sol: Comparing the alternating emf., e = 300 sin (100π)t volt with $e = e_0 \sin \omega t$,

peak emf, $e_0 = 300$ volt.

The peak current = $\dfrac{e_0}{R} = \dfrac{300}{100} = 3A$

The rms current= $\dfrac{\text{peak current}}{\sqrt{2}} = \dfrac{3}{\sqrt{2}} = 2.121A$.

5. A capacitance of 0.4 μF is connected to an alternating emf of 100 cycles/second. What is the capacitive reactance?

Sol: $C = 0.4\mu F = 0.4\times10^{-6} F;\ f = 100Hz$

$X_C = \dfrac{1}{C\omega} = \dfrac{1}{C(2\pi f)} = \dfrac{1}{0.4\times10^{-6}\times2\pi\times100} = 3978\Omega$

6. An inductor of inductance 2 mH is connected to an alternating emf of 50 Hz. Find the inductive reactance.

Sol: $L = 2mH = 2\times10^{-3} H;\ f = 50\,Hz$

The inductive reactance, $X_L = L\omega = L(2\pi f) = 2\times10^{-3}\times2\pi\times50 = 0.6284\Omega$

5. An inductance of 1 H, a capacitance of 2 μF and a resistance of 20 Ω are connected in series across an alternating emf of 220 V, 50 Hz. Find (i) the impedance of the circuit, (ii) the rms current through the inductance, capacitance and resistance, (iii) the peak current and (iv) the peak emf.

Sol: rms emf, $e = 220V$; $f = 50Hz$

i) The impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

R= 20Ω; $X_L = L\omega = L(2\pi f) = 1(2\pi\times50) = 314.2\Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{C(2\pi f)} = \frac{1}{2\times10^{-6}\times2\pi\times50} = 1591\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + (314.2 - 1591)^2} = 1277\Omega$$

ii) The rms current $I = \frac{e}{Z} = \frac{220}{1277} = 0.1723A.$

iii) The peak current, $I_0 = I\sqrt{2} = 0.2436A$

iv) The peak emf, $e_0 = e\sqrt{2} = 220\times\sqrt{2} = 311.1V$

6. A resistance of 10 Ω, a capacitance of 0.1 μ F and an inductance of 2 mH are connected in series across a source of alternating emf of variable frequency. At what frequency does maximum current flow?

Sol: $$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

$L = 2mH = 2\times10^{-3}H$; $C = 0.1\mu F = 0.1\times10^{-6}F$

$$f_0 = \frac{1}{2\pi\sqrt{2\times10^{-3}\times0.1\times10^{-6}}} = \frac{1\times10^5}{2.828\pi} = 11.25kHz$$

7. When an inductor, a capacitor and a resistor are connected in series across a source of alternating emf, the p.d.'s across the inductor, capacitor and resistor are 70 V, 30 V and 30 V respectively. Find the emf of the source.

Sol: $E_R = 30V, E_L = 70; E_C = 30V$

$E = \sqrt{E_R^2 + (E_L - E_C)^2}.\left(\because Z = \sqrt{R^2 + (X_L - X_C)^2}\right)$

$E = \sqrt{30^2 + (70-30)^2} = \sqrt{30^2 + 40^2} = 50V.$

UNSOLVED PROBLEMS

1. The equation of an alternating e.m.f. is $e = 169.7\sin(120\pi t)$ **volt. Find the peak value of its e.m.f. and its frequency.**

Sol: $E = 169.7\sin(120\pi t)V$

$E_0 = 169.7\,volt = 169.7V$

$\omega = 102\pi\,rads^{-1}$

$$n = \frac{\omega}{2\pi} = \frac{120\pi}{2\pi} = 60Hz$$

2. Write the equations of a sinusoidal voltage of r.m.s. value 220V and frequency 50Hz.

Sol: $V_{rms} = 220V$

$\Rightarrow V_0 = V_{rms}\times\sqrt{2} = 220\sqrt{2}V$

N = 50 Hz

$w = 2\pi n = 2\pi(50) = 100\pi\,rad\,s^{-1}$

$V = V_0 \sin \omega t$

$\Rightarrow V = 220\sqrt{2}\sin(100\pi)t \quad = 311.1\sin(100\pi t)\, volt$

3. An alternating e.m.f. of $e = 300\sin 100\pi t$ is applied across the terminals of an inductor of inductance 20 mH. Find (i) the inductive reactance, (ii) the peak current and (iii) the r.m.s. current

Sol: $e = 300\sin 100\pi t$

$e_0 = 300V \quad ; \quad \omega = 100\pi\, rads^{-1} \quad ; \quad L = 20mH = 20\times10^{-3}H$

$X_L = \omega L = 100\times20\times10^{-3} = 2\pi = 2\times3.14 = 6.284\Omega$

$I_0 = \dfrac{e_0}{X_L} = \dfrac{300}{2\pi} = \dfrac{150}{\pi}A = \dfrac{150}{3.14} = 7.7A$

$I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{150}{\pi\sqrt{2}} \quad \Rightarrow \; I_{rms} = \dfrac{75\times1.414}{3.14} = 33.7A$

3. An alternating e.m.f. of $e = 200\sin 100\pi t$ is applied across a capacitor of capacitance $2\mu F$. Find (i). The capacitive reactance (ii). The peak current and (iii) the r.m.s current.

Sol: $e = 200\sin 100\pi t \quad ; \quad e_0 = 200V \quad ; \quad \omega = 10\pi\, rads^{-1},$

$c = 2\mu F = 2\times10^{-6}F$

Capacitive reactance $= X_c = \dfrac{1}{\omega c} = \dfrac{1}{100\pi\times2\times10^{-6}} = \dfrac{5\times10^3}{\pi} - \dfrac{5000}{\pi} \quad = 1591\Omega$

Peak current $= I_0 = \dfrac{e_0}{X_0} = \dfrac{200}{\dfrac{5000}{\pi}} = \dfrac{\pi}{25}A$

$I_{rms} = \dfrac{I_0}{\sqrt{2}} = \dfrac{\pi}{25\sqrt{2}}A$ = 0.008889A

4. An alternating sinusoidal e.m.f. of 220V, 50Hz is connected to circuit of resistance 1Ω and inductance 0.01H. What is the r.m.s. current and the phase difference between the e.m.f. and the current?

Sol: E = 220V; n = 50Hz

$\omega = 2\pi n = 2\pi\times50 = 100\pi\, rad\, s^{-1}$

$R = 1\Omega$

$L = 0.01 = 10^{-2}H$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(1)^2 + (L\omega)^2} \quad = \sqrt{11}\Omega$

Peak current $= I_0 = \dfrac{E}{Z} = \dfrac{220}{\sqrt{11}} = 20\sqrt{11}A$ = 66.7A

$\tan\phi = \dfrac{X}{R} = \dfrac{\pi}{1} = \dfrac{3.14}{1} \Rightarrow \tan\phi = \tan 72^0 \; \Rightarrow \phi = 72^0$

4. A 0.03H inductor, a 10Ω resistor and a $2\mu F$ capacitor are connected in series. At what frequency will they resonate? What will be the phase difference between the e.m.f. and current at resonance/

Sol: L = 0.03H $= 3\times10^{-2}H \quad ; \quad R = 10\Omega \quad ; \quad C = 2\mu F = 2\times10^{-6}F$

For resonance,

$\omega L = \dfrac{1}{\omega C} \Rightarrow \omega^2 = \dfrac{1}{LC}$

$\Rightarrow \omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(3\times10^{-2})(2\times10^{-6})}} \quad = \dfrac{10^4}{\sqrt{6}}\, rad\, \sec^{-1}$

$\Rightarrow n = \frac{\omega}{2\pi} = \frac{10^4}{\sqrt{6} \times 6.28}$ = 649.7 Hz.

$\tan\phi = \frac{X}{R} = \frac{X_L - X_c}{R} = \frac{\left(\omega L - \frac{1}{\omega c}\right)}{R}$

Since $\omega L = \frac{1}{\omega c} \Rightarrow \tan\phi = 0 \quad \Rightarrow \phi = 0^0$

4. A series LCR circuit has L = 1 mH, $C = 0.1\mu F$ and $R = 10\Omega$. It is connected across a source of alternating emf of 5V but of variable frequency. Find (i) the frequency at which the impedance is minimum, (ii) the p.d. across the capacitor, (iii) p.d. across the inductor, (iv) the p.d. across the resistor and (v) the current at resonance.

Sol: $L = 1mH = 10^{-3} H$; $C = 0.1\mu F = 10^{-7} F$ **;** $R = 10\Omega$; E = 5V

Impedance is minimum when $\omega L = \frac{1}{\omega C}$

$\Rightarrow \omega^2 = \frac{1}{LC} \rightarrow (2\pi f)^2 = \frac{1}{LC}$

$\Rightarrow 4\pi^2 f^2 = \frac{1}{LC}$

$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-3} \times 10^{-7}}} = 1591Hz$

Impedance at resonance = $Z = \sqrt{R^2 + X^2} = \sqrt{(10)^2 + (X_L - X_C)^2} = 10\Omega$

i) Current at resonance $I = \frac{E}{z} = \frac{5}{10} = 0.5A$

ii) P.D. across capacitor $V_c = IX_c = 0.5 \times \frac{1}{wc} = \frac{1}{2} \times \frac{1}{(2\pi f) \times 10^{-7}} = \frac{10^7}{4\pi \times \frac{10^5}{2\pi}} = 50V$

iii) P.D. across inductor $V_L = IX_L = I(\omega L) = 0.5 \times (2\pi f) \times 10^{-3}$

$= \frac{1}{2} \times 2\pi \times \frac{10^5}{2\pi} \times 10^{-3} = \frac{100}{2} = 50V$

iv) P.D. across resistor $= V_R = IR = 0.5 \times 10 = 5V$

5. An inductor, a capacitor and a resistor are connected in series across a source of alternating e.m.f. The p.d.'s across the inductor, capacitor, and resistor are 90V, 60V, 40V respectively. Find the e.m.f. of the source.

Sol: $V_L = 90V$; $V_C = 60V$; $V_R = 40V$

$E = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{40^2 + (90 - 60)^2} = \sqrt{1600 + 900}$ = 50V

ASSESS YOURSELF

1. When the emf is maximum how is the current in a pure inductor?

Sol: Minimum, since the phase difference between emf and current through a pure inductor is 90^0

2. When the emf is maximum how is the current in a pure resistor?

Ans. Maximum, since the phase difference between emf and current through a pure resistor is 0^0.

3. When the emf is minimum how is the current in a pure capacitor?

Ans. Maximum, since the phase difference between and current through a pure capacitor is 90^0

4. In a series LCR circuit, at resonance what is the nature of the circuit?

Ans. It behaves like a pure resistor circuit.

5. In a series LCR circuit, what is the relation between the potential difference across the inductor and capacitor?

Ans. They are equal and opposite.