ELECTROSTATICS
1. COULOMB' LAWS

POINTS TO REMEMBER

1. The Branch of Physics which deals with the charges at rest is called electrostatics.

2. Electric Charge:

a) Charge is the fundamental property of matter, due to which it produces and experiences electric and magnetic effects.
b) The deficiency of electrons is called positive charge.
c) The mass of a substance (body) decreases when it is positively charged.
d) The excess of electron is called negative charge.
e) The mass of the body slightly increases when it is given a negative charge.
f) Repulsion is the sure test for electrification.
g) When a glass rod is rubbed with silk, glass rod gets positive and silk gets negative charge
h) When plastic is rubbed with fur, it gets positive, plastic gets negative.
i) Charges can be isolated
j) Charge is a scalar quantity.
k) Charge is quantized (i.e.) charge is always an integral multiple of electronic charge but not its fraction i.e. Q = ne.
l) The number of electrons present in one coulomb of charge is 6.25×10^{18} .
m) Separation of positive and negative charges in a neutral body using a charged body is called electrostatic induction.
n) Induced and inducing charges are equal.
o) Induced positive and negative charges are equal.
p) Induction always produces attraction.
q) Charge always resides on the outer surface of the conductor (skin effect).
r) Stationary charge produces electric field. These are not affected by magnetic fields.
s) Moving charge produces both electric and magnetic fields.
t) Biggest unit of charge is coulomb.
u) Charge per unit surface area of a conductor is called surface charge density $\sigma = \frac{Q}{A}\, Col/m^2$.

3. Coulomb's Law : The force of attraction (or) repulsion between two charges at rest is directly proportional to the product of magnitude of two charges and inversely proportional to the square of the distance between them and acts along the

$$F = \frac{1}{4\pi\in}\frac{q_1 q_2}{r^2}$$

Where $\in$ is called to absolute permittivity of the medium.

$\in = \in_o \in_r$ where $\in_o$ is called the permittivity of free space and $\in_r$ is the relative permittivity of the medium.

$\in_r = 1$ for air or vacuum.

Hence, for air (or) vacuum

$$F_{air} = \frac{1}{4\pi\in_o}\frac{q_1 q_2}{r^2}$$

$$\frac{1}{4\pi \in_o} = 9\times10^9\ N-m^2/C^2 \text{ (Or)} \in_o = 8.854\times10^{-12}\ C^2/N-m^2 \text{ Or Farad / m.}$$

DF of $\in_o = \left[M^{-1}\ L^{-3}\ T^4\ I^2\right]$

4. Permivitty: Permittivity (or) absolute permittivity of a medium is the characteristic property of the medium around a charge that determines the force between the charges.

5. Relative permittivity: The ratio between the permittivity of the medium $(\in)$ and the permittivity of the space $(\in_o)$ is called relative permittivity of medium $\in = \in_o \in_r$

$\therefore \in_r = \frac{\in}{\in_o}$ This is also called dielectric constant (K) or specific inductive capacity.

6. Super position principle: When a number of charge are present, the total force on a given charge is equal to the vector sum of the forces due to the remaining other charges on the given charge.

If charges $q_1, q_2, q_3, ..., q_n$ located in vacuum at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3 .....\vec{r}_n$ w.r.t the origin, then the total force on q_1 is given by $\bar{F}_1 = \bar{F}_{12} + \bar{F}_{13} + ... + \bar{F}_{1n}$

7. Electric field : The space around an electric charge in which its influence is field is called electric field.

8. Intensity of electric field :

The force experienced by a unit positive charge kept in an electric field is called the intensity of the electric field (E). It is a vector quantity. $E = \frac{1}{4\pi \in_o}\ \frac{q}{r^2}$

9. **Electric lines of force** : The path traced by a unit positive charge placed in a uniform electric field is called electric line of force.

LONG ANSWER QUESTIONS

1. State and explain Coulomb's law in electrostatics. From this law define the unit of charge and intensity of the electric field.

A. Statement :

The force of attraction (or) repulsion between two charges at rest is directly proportional to the product of magnitude of two charges and inversely proportional to the square of the distance between them and acts along the line joining the charges.

Explanation :

Consider two point charge q_1 and q_2 which are at rest. Let r be the distance between the charges and F is the force between them.

According to Coulomb's Law,

$$F\ \alpha\ q_1 q_2$$

$$\alpha\ \frac{1}{d^2}$$

(Or) $F \alpha \frac{q_1 q_2}{r^2}$ (or) $F = k\frac{q_1 q_2}{r^2}$

(Or) $F = \frac{1}{4\pi \in}\ \frac{q_1 q_2}{r^2}$

(The value of constant depends on the system of units used and nature of the medium surrounding the charges.)

Where $\in$ is called to absolute permittivity of the medium.

$\in = \in_o \in_r$ where $\in_o$ is called the permittivity of free space and $\in_r$ is the relative permittivity of the medium.

$\in_r = 1$ for air or vacuum.

Hence, for air (or) vacuum

$$F_{air} = \frac{1}{4\pi \in_o} \frac{q_1 q_2}{r^2}$$

$\frac{1}{4\pi \in_o} = 9 \times 10^9 \ N - m^2 / C^2$ (Or) $\in_o = 8.854 \times 10^{-12} \ C^2 / N - m^2$ Or Farad / m.

i) The S.I. unit of electrical charge is coulomb.

Unit charge (Coulomb) :

$$F = \frac{1}{4\pi \in_o \in_r} \frac{q_1 q_2}{r^2}$$

$\in_r = 1; q_1 = q_2 = q = 1$ C are $\in_r = 1$ nr then $F = 9 \times 10^9$ N.

A charge when placed at a distance of 1m from a similar charge in air (a) vacuum repel each other with a force of $9 \times 10^9 N$ is called a unit charge (or) one coulomb.

Intensity of electric field :

The force experienced by a unit positive charge kept in an electric field is called the intensity of the electric field (E). It is a vector quantity.

$$E = \frac{1}{4\pi \in_o} \frac{q}{r^2} \text{ N / c}$$

The force acting on a charge q placed in a uniform electric field of intensity E is given by

$\bar{F} = \bar{E} q$ *Newton*

If q is +ve,F & E are parallel

If q is –ve, F & E are anti- parallel

SHORT ANSWER QUESTIONS

1. State and explain Coulomb's inverse square law in electricity.

A. Statement :

The force of attraction (or) repulsion between two charges at rest is directly proportional to the product of magnitude of two charges and inversely proportional to the square of the distance between them and acts along the line joining the charges.

Explanation :

Consider two point charge q_1 and q_2 which are at rest. Let r be the distance between the charges and F is the force between them.

According to Coulomb's Law,

$F \propto q_1 q_2$

$$\alpha \frac{1}{d^2}$$

(Or) $F \alpha \frac{q_1 q_2}{r^2}$ (or) $F = k \frac{q_1 q_2}{r^2}$

(Or) $F = \frac{1}{4\pi \in} \frac{q_1 q_2}{r^2}$

(The value of constant depends on the system of units used and nature of the medium surrounding the charges.)

Where $\in$ is called to absolute permittivity of the medium.

$\in = \in_o \in_r$ where $\in_o$ is called the permittivity of free space and $\in_r$ is the relative permittivity of the medium.

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Hence, for air (or) vacuum

$$F_{air} = \frac{1}{4\pi \in_o} \frac{q_1 q_2}{r^2}$$

$\frac{1}{4\pi \in_o} = 9 \times 10^9 \ N - m^2 / C^2$ (Or) $\in_o = 8.854 \times 10^{-12} \ C^2 / N - m^2$ Or Farad / m.

2. Define intensity of electric field at a point. Derive an expression for the intensity of the electric field at a point due to a point charge:

A. **<u>Intensity of electric field (E):</u>** Intensity of electric field at a point in the electric field is numerically equal to the force acting on unit positive charge placed at that point. It is a vector quantity.

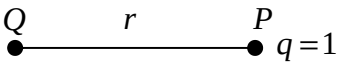
Unit : Newton / coulomb (N/C) (or) volt / meter.

<u>Expression</u>:

Let Q be the point charge situated at point 'O' in air (or) vaccum. Consider a point P which is at a distance r from the charge Q. The intensity of the electric field at P can be calculated by imaging a unit positive charge at P.

According to Coulomb's law the force acting on this unit charge in given by,

$$\boldsymbol{F = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$$



By definition it is equal to the intensity of the electric field (E). It acts ways from the charge.

$\therefore$ $\boldsymbol{E}$ in vector Form, $\boldsymbol{\overline{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^3} \bar{r}}$

If a charge $\boldsymbol{Q_0}$ is placed at P the force acting on it is F = $\boldsymbol{EQ_0}$

$$\boldsymbol{E = \frac{F}{Q_0} N / C}.$$

3. State and explain the principle of superposition in Electro–statics concerning the force due to multiple charges.

A. **<u>Statement:</u>** It states that if a charge 'q' is simultaneously acted upon by electrical force due to multiple charges $\boldsymbol{q_1, q_2, q_3, ...q_n}$, then the net force acting on the charge is the vector sum of individual forces acting on 'q' due to individual charges $\boldsymbol{q_1, q_2, q_3, ...q_n}$ independent each force acting at a time. Consider a system of multiple charges $\boldsymbol{q_1, q_2, q_3, ...q_n}$ as shown in

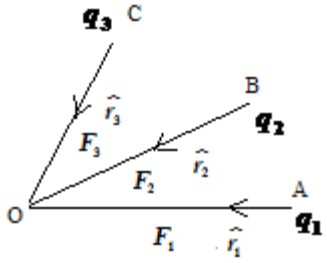


figure. The net force on charge q at '0' due to multiple charges $q_1, q_2, q_3, ...q_n$.

The force on q due to q_1 is

$F_1 = \frac{1}{4\pi\in_0}\frac{qq_2}{r_1^2}r_1$ *where* r_1 is the unit vector along $\overrightarrow{AO}$.

The force on q due to q_2 is

$F_2 = \frac{1}{4\pi\in_0}\frac{qq_2}{r_2^2}r_2$ *where* r_2 is the unit vector along $\overrightarrow{BO}$

The force on q due to q_3 is

$F_3 = \frac{1}{4\pi\in_0}\frac{qq_3}{r_3^2}r_3$ *where* r_3 is the unit vector along $\overrightarrow{CO}$

$\therefore$ The net force on q is $\overline{F} = \overline{F_1} + \overline{F}_2 + \overline{F_3} + ... + \overline{F_n} = \sum \overline{F_n}$

4. Explain modern theory of electrification. State the law of conservation of charge.

A. Atoms consists of positive charges concentrated at nucleus and negative charges called electrons are revolving round the nucleus. The magnitude of positive and negative charges is equal. So atoms are electrically neutral.

When an body loses some electrons, total positive charge is more than total negative charge. So substances which loose electrons will become positively charged. When an body gains some electrons, then total negative charge in it are more than total positive charges. So substances which gain electrons will become negatively charged.

Law of conservation of charge: This law states that the total charge in an isolated system remains constant. Thus the electrical charge can neither be created nor be destroyed.

5. Define permittivity and dielectric constant of a medium. Derive an expression for the electric field strength at a given distance from a point charge.

A. **Permittivity (ε):** Permittivity is a physical property of a medium which determines in interaction between the charges. So force between two charges depends on the permittivity of medium in which the charges were kept.

Permittivity of free space $\varepsilon_0 = \frac{1}{36\pi\times10^9}C^2N^{-1}m^{-2} = 8.85\times10^{-11}C^2N^{-1}m^{-2}$

Intensity of electric field :

The force experienced by a unit positive charge kept in an electric field is called the intensity of the electric field (E). It is a vector quantity.

$E = \frac{1}{4\pi\in_o}\frac{q}{r^2}$

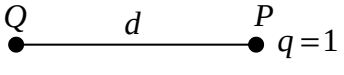
The force acting on a charge q placed in a uniform electric field of intensity E is given by

$\overline{F} = \overline{E}q$ *Newton* - If q is +ve,F & E are parallel

- If q is –ve, F & E are anti- parallel

Explanation :

Consider a unit positive charge placed at a distance d from a charge q in air (or) vacuum. The force acting on the unit positive charge is given by



$F = \frac{1}{4\pi \in_o} \frac{Q \times 1}{d^2}$ This force acting on the unit positive charge gives the intensity of electric field E at the point 'P'. $E = \frac{1}{4\pi \in_o} \frac{q}{r^2}$

This is a vector quantity which is directed along the force experienced by a positive charge.

Its unit is Newton / coulomb or volt / metre.

VERY SHORT ANSWER QUESTIONS

1. IC of charge is equal to the charge of n number of electron in magnitude. What is the value of n ?

A. Charge on electron e = 1.6×10^{-19}C

$$\boldsymbol{Q = ne \Rightarrow n = \frac{Q}{e}}$$

$$\boldsymbol{\therefore n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}} \text{ Electrons}$$

2. What is meant by the statement that 'charge is quantized'?

A. The minimum charge that may be transferred from one body to the other is equal to the charge of electron (1.6×10^{-19}C). The charge is available in multiples of charge on electron i.e., Q = $\pm ne$. Henc chaerge is said to be quantized.

3. What happens to the weight of a body when it is charged positively?

A. When a body is positively charged, it looses weight due to the loss of electrons.

4. a) What happens to the force between two charges if the distance between them is a) halved, b) doubled?

A. According to inverse square law, $\boldsymbol{F \propto \frac{1}{d^2}}$

a) $\boldsymbol{\frac{F_2}{F_1} = \left(\frac{d_1}{d_2}\right) \Rightarrow \frac{F_2}{F_1} = \left(\frac{d_1}{d_1/2}\right)^2 \Rightarrow F_2 = 4F_1}$

The force between the charges increases by four times

b) $\boldsymbol{\frac{F_2}{F_1} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{d_1}{2d_1}\right)^2 \Rightarrow F_2 = \frac{F_1}{4}}$

The force is reduced to $\boldsymbol{\frac{1}{4}th}$ of its original value.

5. The electric lines of force do not intersect. Why?

A. The tangent drawn to electric line of force will give the direction of electric field at the point. If the electric lines of force intersect then at the point of intersection electric field will act in two different directions, which is not possible. Hence electric lines of force do not intersect.

6. An uncharged body if kept in contact for some time with a charged body gets repelled. Why?

A. When a charged body is made in contact with another uncharged body for some time, the charge on the first body is shared between both of them due to which they repel each other.

7. Charged particle of mass m and charge q is placed in an electric field of intensity E. Write down the equation for force and acceleration of the particle.

A. When a charge q is placed in an electric field of strength E, the force on it is given by

(i) F = qE But F = ma $\Rightarrow \boldsymbol{a = \frac{F}{m}}$ and $\boldsymbol{a = \frac{qE}{m}}$

8. A charge q of mass m starting from rest is allowed to move between two plates having a potential difference of V volt what is the final velocity of it?

A; From $\boldsymbol{\frac{1}{2}mu^2 = QV}$ $\boldsymbol{\therefore\ u = \sqrt{\frac{2QV}{m}}}$

9. Write down that units for

a) Permittivity b) charge c) Electric field strength d) Potential and e) Capacitance.

A. a) Permittivity: farad / metre.
b) Charge: coulomb.
c) Electric field strength: Newton/ coulomb (or) volt /metre
d) Potential: volt
e) Capacitance is 'farad'.

10. Repulsion is the sure test of electrification than attraction. Why?

A. Positively charged body can attract both negatively charged and neutral bodies, but positively charged body can only repel another positively charged body. Hence repulsion is sure test of electrification.

11. What is the difference between electric lines of force and magnetic lines of induction?

A. The electric lines of force always leave or end on the surface of a charged body. Magnetic lines of induction are closed curves starting from North Pole and ending on the South Pole of the magnet and inside the magnet they move from south to north.

12. Define permittivity and relative permittivity.

A. **Permittivity :** The property of the medium which determines the force between the charges is called permittivity or absolute permittivity of the medium. It is denoted by $\boldsymbol{\varepsilon}$.

Relative permittivity : It is defined as the ratio of force of attraction or repulsion between two charges in air or vacuum to the force of attraction or repulsion between the same charges separated by the same distance in the medium.

SOLVED PROBLEMS

1. Calculate the number of electrons in 1 coulomb of charge

Sol: $e = 1.6\times10^{-19}C$

If 'n' is the number of electrons required to make 1C of charge, then

$1C = n\left[1.610^{-19}C\right]$

Or $n = \frac{1}{1.610^{-19}} = 6.25\times10^{18}$

2. Two alpha particles are separated by a distance of 10^{-13} m. Calculate the force between them in free space.

Sol: $q_1 = +2\times1.6\times10^{-19}C$ and $q_2 = +2\times1.6\times10^{-19}C$; $r = 10^{-13}$ m.

Now,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{\left(9\times10^9\right)\left(+21.6\times10^{-19}\right)\left(+2\times1.6\times10^{-19}\right)}{\left(10^{-13}\right)^2} = +92.39\times10^{-3}\,N$$

The force has a magnitude of $92.39\times10^{-3}\,N$ and is a repulsive force.

3. The force between two charges $2\mu C$ and $4\mu C$ is 24 N, when they are separated by a certain distance in free space. Calculate the force if (a) distance between them is doubled and (b) distance is halved

Sol: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = 24N$

a) When the distance is doubled,

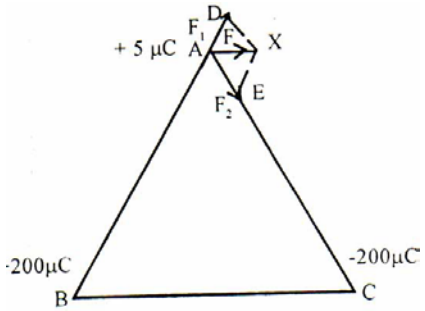
$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(2r)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{4r^2} = \frac{1}{4}\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\right] = \frac{F}{4} \quad = 24/4 \ = 6 \text{ N}$$

b) When the distance is halved

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left(\frac{r}{2}\right)^2} = 4\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\right] = 4\times24 = 96N$$

4. Two charges of +200 μC and -200 μC are placed at the corners B and C of an equilateral triangle ABC of side 0.1 m. What is the force on a charge of 5 μC placed at A?

Sol: Force on 5 μC charge at A due to the 200 μC charge at B is given by



$$F_1 = \frac{\left(9\times10^9\right)\left(200\times10^{-6}\right)\left(5\times10^{-6}\right)}{\left(0.1\right)^2} = 900N$$ In the direction of BA

Force on 5 μC charge at A due to the -200 μC charge at C is

$$F_2 = \frac{(9\times10^9)(200\times10^{-6})(5\times10^{-6})}{(0.1)^2} = 900N$$ In the direction of AC.

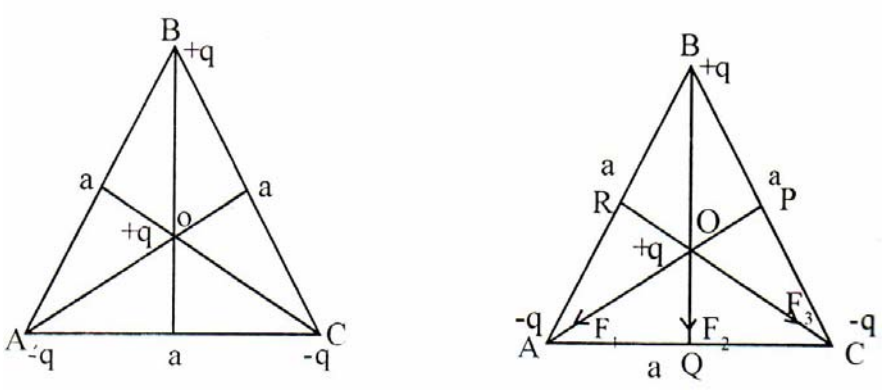
The angle between F_1 and F_2 is 120^0 and hence the resultant force F is given by $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} = 900N$

The direction is given by the angle α made by F with F_2 (AC)

$$\tan\alpha = \frac{F_1\sin120^0}{1+F_2\cos120^0} = \sqrt{3} \quad \text{Or} \quad \alpha = 60^0$$

5. Three charges –q, +q and –q are placed at the corners of an equilateral triangle of side a. Find the resultant electric force on a charge +q placed at the centroid O of the triangle.

Sol:



Let the force acting on +q charge at O due to –q at A be F_1, +q at B be F_2 and –q at C be F_3 and $AO = BO = CO = \frac{a}{\sqrt{3}}$

In magnitude, $F_1 = F_2 = F_3 = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q^2}{\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{3q^2}{4\pi\varepsilon_0 a^2}$

Now angle between F_1 and F_3 (OA and OC) is $\theta = 120^0$ and hence, the resultant of F_1 and F_3 in magnitude is, $F' = F_1 = F_3 = \frac{3q^2}{4\pi\varepsilon_0 a^2}$.

The direction of F' is along BOQ (from previous example). That is F_2 and F' are in the same direction,

Hence, the resultant force on +q at O is

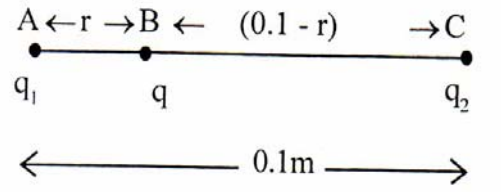
$$F = F' + F_2 = \frac{3q^2}{4\pi\varepsilon_0 a^2} + \frac{3q^2}{4\pi\varepsilon_0 a^2} = \frac{3q^2}{2\pi\varepsilon_0 a^2}$$

This is in the direction of BOQ.

6. Two positive charges $q_1 = 4\times10^{-6}C$ and $q_2 = 9\times10^{-6}C$ are placed 10 cm apart in air. Find the position of a third charge placed between them, such that there will be no resultant force on it.

Sol: $q_1 = +4\times10^{-6}C$ and $q_2 = +9\times10^{-6}C$

Separation = 0.1 m



To have the resultant force on a q to be zero, the two forces F_1 and F_2 are equal in magnitude

Force on q due to q_1, $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{r^2}$ and

Force on q due to q_2, $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{(0.1-r)^2}$

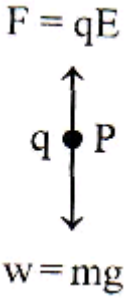
$$F_1 = F_2 \quad \Rightarrow \frac{q_1 q}{r^2} = \frac{q_2 q}{(0.1-r)^2} \quad \Rightarrow \frac{q_1}{r^2} = \frac{q_2}{(0.1-r)^2} \Rightarrow ; \frac{4\times10^{-6}}{r^2} = \frac{9\times10^{-6}}{(0.1-r)^2}$$

$$r = \frac{0.2}{5} = 0.04m = 4cm \text{ .'}$$

7. A charged particle P has a mass of 10^{-6} kg and carries a charge of $4.9\times10^{-6}C$. Calculate the intensity of the electric field to be applied on it, in vertically upward direction. So as to keep it at rest.

Sol: $m = 10^{-16}kg \quad ; \quad q = 4.9\times10^{-19}C \quad ; \quad g = 9.8ms^{-2}$

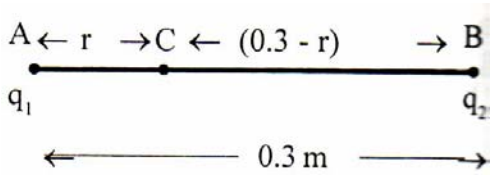
mg = qE $\quad \Rightarrow \quad E = \frac{mg}{q}$



The intensity of the electric fiel $\quad E = \frac{10^{-16}\times9.8}{4.9\times10^{-19}} = 2\times10^{3}NC^{-1}$

8. Two point charges 4 μC and 9 μC are separated by 30 cm. Find the point where the strength of the field is zero

Sol:



Field strength at C due to q_1 is $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$ along AC

Field strength at C due to q_2 is $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(0.3-r)^2}$ along BC.

As these two air in opposite directions, the resultant E will be zero, if

$$E_1 = E_2 . \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(0.3-r^2)} \quad \Rightarrow \quad \frac{4\times10^{-6}}{r^2} = \frac{9\times10^{-6}}{(0.3-r)^2}$$

$$r = 12cm$$

The resultant intensity will be zero at a distance of 0.12 m or 12 cm from $4\mu C$ between the two charges.

9. Find the force experienced by a chloride ion having 4 electrons removed, when placed in an electric field of intensity 2 NC^{-1}

Sol: Charge on the chloride ion $q = +4e = +4\times1.6\times10^{-19}C$ and $E = 2NC^{-1}$

The force experienced by chloride ion is

$$F = qE = +\left(4\times1.6\times10^{-19}\right)(2) \quad = +12.8\times10^{-19}N$$

The chloride ion experiences a force of 12.8×10^{-19} N magnitude, in the direction of the field.

10. When a charge of 1 μC is placed in an electric field, it experiences a force of $2\times10^{-3}N$. Find the intensity of the field. (March2010)

Sol: $q = 1\times10^{-6}C; \quad F = 2\times10^{-3}N$.

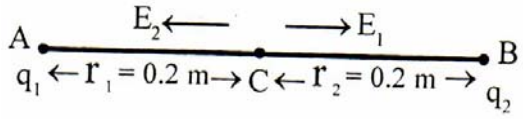
Force experienced F = q E

$$\therefore 2\times10^{-3} = \left(1\times10^{-6}\right)E$$

$$E = \frac{2\times10^{-3}}{1\times10^{-6}} = 2\times10^{3}\,NC^{-1}$$

11. Two point charges 5 μC and 8 μC are kept at a distance of 0.4 m from each other. Find the field mid way between them (a) when both charges are positive and (b) when 5 μC is positive and 8 μC is negative.

Sol: a) $q_1 = +5\times10^{-6}C$ and $q_2 = +8\times10^{-6}C$ and are at a distance of 0.4 m.



The mid point C will be at 0.2 m distance from each charge.
The field strength at C due to q_1 at A will be

$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}$ and along AC …….(1)

The field strength at C due to $+q_2$ at B will be

$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_2^2}$ along BC.

Evidently E_2 is greater than E_1 as $q_2 > q_1$ and $r_1 = r_2 = 0.2m$.
Resultant field strength at C is

$E = E_2 - E_1$

$= = 9\times10^{9}\left[\frac{8}{0.04} - \frac{5}{0.04}\right]\times10^{-6}$

$= 6.75\times10^{5}\,NC^{-1}$ along BC.

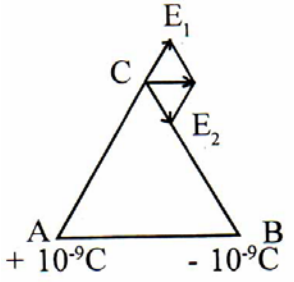
b) $q_1 = +5\times10^{-6}C$; $q_2 = -8\times10^{-6}C$ Now E_1 and E_2 will be in the same direction as E_1.
Resultant field strength at C is

$E = E_1 + E_2$

$= 9\times10^{9}\left[\frac{8\times10^{-6}}{0.04} + \frac{5\times10^{-6}}{0.04}\right] = = 29.25\times10^{5}\,NC^{-1}$ along AC.

12. Two electric charges of $+10^{-9}$ C and 10^{-9} C are placed at the corners A and B of an equilateral triangle ABC of side 5 cm. What is the electric intensity at C?

Sol: The electric intensity at C due to the charges at A is



$E_1 = \frac{9\times10^{9}\times10^{-9}}{(0.05)^2} = 3600\,N/C$ along AC

The electric intensity at C due to the charge at B is

$$E_2 = \frac{9\times10^9\times10^{-9}}{(0.05)^2} = 3600 N/C \text{ along CB.}$$

$\therefore E_1$ and E_2 are equal and the angle between them is 120^0

The resultant intensity at C,

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos120^0} = \sqrt{E_1^2 + E_1^2 + 2E_1^2\cos120^0} = E_1 = 3600 N/C.$$

This is in a direction parallel to AB.

13. An infinite number of charges each of magnitude q are placed on x-axis at distance of 1,2,4,8,…….meter from the origin. Find intensity of the electric field at origin

Sol: The electric intensities due to all the charges are in the same direction. The resultant intensity of the electric field at the origin will be

$$E = E_1 + E_2 + E_3 + ...$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + ...\right] = \frac{q}{4\pi\varepsilon_0}\left[1 + \frac{1}{4} + \frac{1}{16} + ...\right]$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{4}}\right] = \frac{q}{4\pi\varepsilon_0}\times\frac{4}{3} = \frac{q}{3\pi\varepsilon_0}.E \text{ is from right to left.}$$

UNSOLVED PROBLEMS

1. Calculate the force between two similar charges of magnitude 2C each separated by a distance of 2 km.

A. $q_1 = q_2 = 2C$

d = 2 km = 2000 m ; F = ?

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2} = \frac{9\times10^9\times2\times2}{(2000)^2} \Rightarrow F = 9\times10^3\, N$$

2. The force experienced by a charge of 2 μ C in an electric field is 3×10^{-3}N. Calculate the intensity of the electric field.

A. q = $2\mu C = 2\times10^{-6} C$; F= 3×10^{-3}N.

$$E = \frac{F}{q} \Rightarrow \qquad E = \frac{3\times10^{-3}}{2\times10^{-6}} = 1.5\times10^3\, NC^{-1}$$

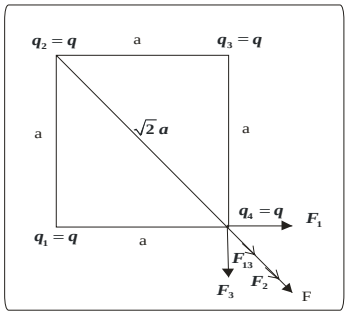
3. 12 J of work should be done in moving 0.01 Coulomb charges between two points in an electric field. What is the potential difference between the points?

A. Q = 0.01C.

$$V_1 - V_2 = \frac{W}{Q} = \frac{12}{0.01} = 1200\ Volts$$

4. Four charges of equal magnitude 2 μC are kept at the four corners of a square of side 5 cm. Find the force experienced by any one of them.

A. q = 2×10^{-6}C, a = 5 cm = 5×10^{-2} m

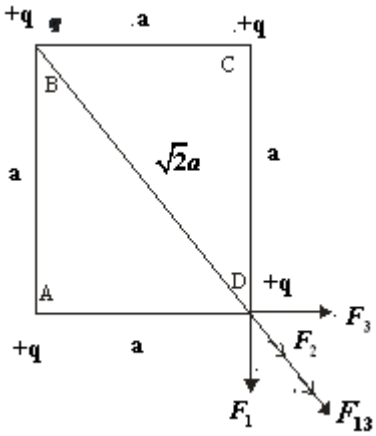


$$F = \frac{q^2}{4\pi\varepsilon_0 a^2}\left[\sqrt{2}+\frac{1}{2}\right] = 9\times10^9 \times \left(\frac{2\times10^{-6}}{5\times10^{-2}}\right)^2\left[\sqrt{2}+\frac{1}{2}\right]$$

F = 27.56 N

5. Four charges of +q,+q, +q, and +q are placed at the corners A,B,C and D of a square. Find the resultant force on the charge at D.

A. $F_1 = F_3 = \frac{1}{4\pi\varepsilon_0}\frac{(q)(q)}{a^2}$



As angle between F_1 & F_3 is 90°

Resultant of F_1 & F_3 = $F_{13} = \sqrt{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\right)$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{(q)(q)}{\left(\sqrt{2}a\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^3}{\left(2a^2\right)}$$

$$F_{res} = F_2 + F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(2a^2\right)} + \sqrt{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\right) =$$

$$\frac{q^2}{8\pi\in_0 a^2}\left(1+2\sqrt{2}\right)$$

6. Two charges $2\mu C$ and $1\mu C$ are placed at a distance of 10 cm. Where a third charge should be placed between them so that it does not experience any force?

A. Let the third charge q be placed at a distance x from $1\mu C$.

Now, $\frac{1\times q}{4\pi\varepsilon_0 x^2} = \frac{2\times q}{4\pi\varepsilon_0 (10-x)^2} \Rightarrow x = \frac{10}{\sqrt{2}+1}$ = 4.142 cm from $1\mu C$

7. Calculate the ratio of electric and gravitational forces between two protons. Mass of proton = 1.672×10^{-27}kg and charge is 1.6×10^{-19}C and G= 6.67×10^{-11} Nm^2kg^2.

A. Electro static force between two protons

$$E_E = \frac{9\times10^9 Q_1Q_2}{d^2} \Rightarrow F_E = \frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{d^2}$$

$m_1 = m_2 = 1.672\times10^{-27}kg$; G = 6.67×10^{-11} Nm^2/kg^{-2}.

Gravitational force between them $F_G = \frac{Gm_1m_2}{d^2}$

$$\therefore \frac{F_E}{F_G} = \frac{9\times10^9\times Q_1Q_2}{G\times m_1\times m_2} = 1.23\times10^{36}.$$

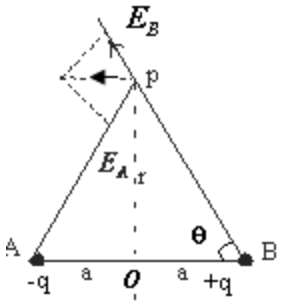
8. Two equal charges q of opposite sign are separated by a small, distance 2a. Find the electric intensity E at a point on the perpendicular bisector of the line joining the two charges at a very large distance r from the line. Find the direction of E also

(Note: Actually, the arrangement of the two equal and opposite charges is called an electric dipole; the line joining the two charges is called the axial line is called the equatorial line.)

A. Consider an electric dipole of two equal but opposite charges –q and +q separated by distance '2a'. 'P' is a point at distance 'r' from centre of the dipole 'o'.

Electric intensity at 'P' due to '-q' = $\left|\overrightarrow{E_A}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2}$ Directed along $\overrightarrow{AP}$

Electric intensity at 'P' due to '+q' = $\left|\overrightarrow{E_B}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2}$ Directed along $\overrightarrow{BP}$



$$E_{res} = E_A\cos\theta + E_B\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2+a^2\right)}\cos\theta + \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^r+a^2\right)}\cos\theta$$

$$= 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q}{\left(r^2+a^2\right)}\cos\theta$$

From $\Delta le\ OPB,\quad \cos\theta = \frac{OB}{PB} = \frac{a}{\left(r^2+a^2\right)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2+a^2\right)^{3/2}} \left(\because P = 2qa\right)$

If r>> a then $E_{eq}\ \frac{1}{4\pi\varepsilon_0}.\left(\frac{2qa}{r^3}\right) = \frac{1}{4\pi\in_0}.\frac{P}{r^3}$

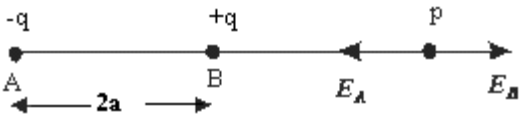
Direction of E is along a line parallel to the line joining the charges.

9. In the same arrangement of charges of the above problem, find the electric field E on the axial line (line joining the two charges) at a very large distance r from the centre from the centre of the dipole. Find the direction of E also.

A. Consider a electric dipole consisting of two equal but opposite charges –q and +q at distance 2*l* 'p' is a point at a distance 'r' from the centre of the dipole '0'. The electric intensity 'E' at P due to the dipole is the vector sum of the field due to charge –q at A and +q at B.

Electric intensity at 'P' due to q = $E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r^2+a^2\right)}$ along PA

Electric field intensity at 'P' due to +q



$E_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(r-a\right)^2}$ Along BP

$$E_{res} = E_B - E_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\left(r-a\right)^2} - \frac{q}{\left(r+a\right)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2+a^2+2ar-r^2+2ar-a^2}{\left(r^2-a^2\right)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2\times 2ar}{\left(r^2-a^2\right)^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{\left(r^2-a^2\right)^2} \qquad \left(\because P = 2aq\right)$$

Where r>>> a then a^2 is neglected then

$$E_{axial} = \frac{1}{4\pi\in_0}.\left(\frac{4qa}{r^3}\right)$$

10. An infinite number of charges each q are placed on the x – axis at distances of 1, 2, 4, 8….meter from the origin. If the charges are alternately positive and negative find the electrical field (intensity of electric field) at origin.

A.

$$\vec{E}_{res} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + ... + \infty$$

$$\Rightarrow \vec{E}_{res} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{1^2} - \frac{q}{2^2} + \frac{q}{4^2} - \frac{q}{8^2} + \frac{1}{16^2} - \frac{q}{(32)^2} + ...\infty\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-\left(-\frac{1}{4}\right)}\right] = \frac{q}{5\pi\in_0}$$

ASSESS YOURSELF

1. Usually it is the negative charge that is transferred when two bodies are rubbed together. Can you explain why?

Sol: The electrons are very light and loosely bound to the atoms than the positive charges.

2. Both mass and charge are scalars and hence got the additive property. However, in adding charges it is not enough to just add the amounts of charges. Why?

Sol: Unlike mass, charges are of two different kinds, positive and negative.

3. At the out set it appears as though the principle of superposition is similar to the additive property. But, it is not so. Can you guess the important difference?

Sol: The electrostatic forces are to be added vectorially

4. Why is an arrow mark necessary on the electric line of force?

Sol: The direction of the electric field.

5. The concept of electric field is due to Farady and now it is one of the central concepts in Physics. Can you see the electric fields? How can you detect the electric fields?

Sol: No. By the force acting on a charge.

6. Consider two charges +q and –q placed at B and C of an equilateral triangle ABC. For this system, the total charges is zero. But the electric field (intensity) at A which is equidistant from B and C is not zero. Why?

Sol: Charges are scalars, but the electrical intensities are vectors and add vectorially.