MAGNETISM

5.VIBRATION MAGNETOMETER

POINTS TO REMEMBER

1. **Vibration magnetometer :**
   a) If a magnet of moment M is suspended freely in a horizontal plane, where the horizontal component of the earth's magnetic field is B_H, the magnet sets itself in the direction of B. If the magnet is given a slight angular displacement about the axis of suspension, the magnet undergoes S.H.M. The restoring torque is due to B_H.

   b) Its time period T is given by $T = 2\pi\sqrt{\frac{I}{MB_H}}$

   where I = moment of inertia of the magnet about the axis of suspension = $\frac{m(\ell^2 + b^2)}{12}$

   c) Its frequency is given by $n = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$

   d) Its time period (or) frequency is independent of length of the suspension.

   e) To compare horizontal components of earth's magnetic field at two different places.

   $$\frac{B_{H_1}}{B_{H_2}} = \frac{T_2^2}{T_1^2}$$

   f) To calculate M and B_H (together with reflection magneto meter)

   $M = \frac{2\pi(d^2 - \ell^2)}{T}\sqrt{\frac{4\pi I \tan\theta}{\mu_0 2d}}$ And $B_H = \frac{2\pi}{T(d^2 - \ell^2)}\sqrt{\frac{\mu_0}{4\pi}\frac{2Id}{\tan\theta}}$

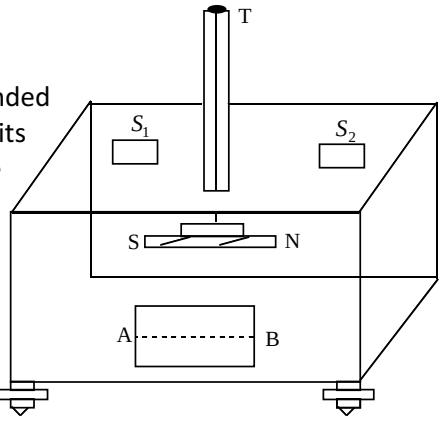
2. **Uses of vibration magnetometer:**
   i) to compare the magnetic moments of two bar magnets.
   ii) to compare the horizontal components of earth's magnetic field at two different places.
   iii) Together with deflection magnetometer vibration magnetometer can be used to
      a) Find the magnetic moment of a magnet and
      b) The horizontal component of earth's field at a given place.

Long Answer Questions :

1. Explain the principle of working of a vibration Magnetometer. How do you determine the magnetic moment ('M') of a bar magnet and the horizontal component of the earth's magnetic field (B_H) using vibrating magnetometer and deflection magnetometer.

Ans: **Principle:**

It works on the principle that when a bar magnet suspended freely in a uniform magnetic field is displaced from its equilibrium position, it executes S.H.M. about the equilibrium position.



Description :

V.M.M. consists of a rectangular wooden box fitted with glass windows on the sides. A long brass (or) glass tube is fixed vertically at the top of the box. At the upper end of the tube, a screw is provided to suspend an unspun silk thread (or) hair of a horse. The lower end of the silk thread carries an arrangement called brass stirrup in which magnet is placed. There are two rectangular slits S_1 and S_2 at the top

of the box to see the vibrations of the magnet placed in the brass stirrup. A plane mirror is fitted at the bottom of the box and a line AB is drawn in a direction parallel to its length. The line is called the index line. This line is useful in setting the magnetometer in magnetic meridian.

Setting :

1) The V.M.M. is leveled with the help of a spirit level.

2) A magnetic compass needle is placed on the reference line AB and the box is rotated so that the needle is parallel to the line. Thus the magnetometer is set along the magnetic meridian.

3) To remove the torsion in the silk thread, a brass bar is placed in the strirrup and adjusts the torsion head till the bar comes to rest parallel to AB. Remove the brass bar and place a magnet in the hanger with North Pole pointing north of the earth. If the magnet comes to rest along the magnetic meridian, then there will be no torsion in the suspension fiber.

Working:

Another bar magnet is brought near one of the pole of the suspended magnet and removed immediately. Then the suspended magnet begins to execute SHM about the direction of the field (B_H) as mean position. Its vibrations are counted through the slits. By noting the time for 20 oscillations, the time period for one oscillation can be calculated.

Theory:

Consider a bar magnet of moment M suspended in a uniform earth's magnetic field B_H. Let the magnet is displaced through a small angle θ and released. A restoring torque acting on the bar magnet is given by,

$$\text{Restoring Torque } (\tau) = MB_H Sin\theta$$

If I is the moment of inertia of the bar magnet about the suspension axis and α is the angular acceleration produced, then

$$\text{Deflecting torque} = I\alpha$$

$$\therefore I\alpha = -MB\ Sin\theta$$

When θ is small $Sin\theta \approx \theta$

$$\therefore \alpha = \left(\frac{-MB_H}{I}\right)\theta$$

For a given magnet, the factor $\frac{MB_H}{I}$ is a constant and hence the angular acceleration (α) is directly proportional to angular displacement which is the condition for SHM. Hence the time period of the magnet is given by

$$T = 2\pi\sqrt{\frac{\text{Angular displacement}}{\text{Angular acceleration}}} = 2\pi\sqrt{\frac{\theta}{\alpha}}$$

Experimental determination of M and B_H :

To find M/B_H :

The deflection magnetometer is arranged in tan A position. The wooden arms of the magnetometer should be perpendicular to the magnetic needle (or) they must be in the East – West direction. The Aluminium pointer should show $0^\circ - 0^\circ$ reading, A bar magnet is placed so that the centre of the magnetic needle should lie on the axial line of the bar magnet.

A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d from the centre of the magnetic needle. The centre of the magnetic needle should lie on the axial line of the bar magnet. The deflections of the Aluminium pointer θ_1 and θ_2 are noted. The bar magnet is reversed pole to pole at the same position and the deflections θ_3 and θ_4 are noted. Now the bar magnet is placed on the other arm of the magnetometer at the same distance d and the deflections $\theta_5, \theta_6, \theta_7$ and θ_8 are noted. The average reading θ of these eight deflections is calculated.

From Tangent law,

$$B = B_H Tan\theta$$

$$\frac{\mu_o}{4\pi}\frac{2Md}{\left(d^2 - l^2\right)^2} = B_H Tan\theta$$

$$\therefore \quad \frac{M}{B_H} = \frac{2\pi\left(d^2 - l^2\right)^2 Tan\theta}{\mu_o d}$$

To find MB_H :

The magnet used in D.M.M. is placed in the stirrup of V.M.M. and is allowed to oscillate. The time taken for 20 oscillations and there by time taken for one oscillation (i.e) time period (T) of the magnet can be noted using stop clock. If I is the moment of inertia of the bar magnet, then

$$T = 2\pi\sqrt{\frac{1}{MB_H}} \text{ (or) } T^2 = 4\pi^2\frac{1}{MB_H}$$

$$\therefore \; MB_H = 4\pi^2\frac{1}{T^2}$$

Moment of inertia of the bar magnet is given by

$$I = \frac{m\left(l^2 + b^2\right)}{12}$$

Where m = mass of the magnet

l = length of the magnet

b = breadth of the magnet

Multiplying equations (1) and (2)

$$M^2 = 4\pi^2\frac{1}{T^2}\times\frac{2\pi\left(d^2 - l^2\right)^2 Tan\theta}{\mu_o d}$$

$$\therefore \quad M = \frac{2\pi\left(d^2 - l^2\right)^2}{\mathrm{T}}\sqrt{\frac{2\pi Tan\theta}{\mu_o d}}$$

Dividing equations (1) and (2),

$$B_H = 4\pi^2\frac{1}{T^2}\times\frac{\mu_o d}{2\pi\left(d^2 - l^2\right)^2 Tan\theta}$$

$$B_H = \frac{1}{T\left(d^2 - l^2\right)^2}\sqrt{\frac{2\pi\mu_o d}{Tan\theta}}$$

Precautions:

1) The magnet should be made to vibrate in the horizontal plane only

2) The amplitude of the vibration of the magnet must be small

3) Precautions of D.M.M. are made properly

Short Answer Questions

1. **Derive an expression for the period of oscillation of a bar magnet in vibration magnetometer.**

Sol: Consider a bar magnet of moment M suspended in a uniform earth's magnetic field B_H. Let the magnet is displaced through a small angle θ and released. A restoring torque acting on the bar magnet is given by,

$$\text{Restoring Torque } (\tau) = MB_H \sin\theta$$

If I is the moment of inertia of the bar magnet about the suspension axis and α is the angular acceleration produced, then

$$\text{Deflecting torque} = \mathrm{I}\alpha$$

$$\therefore \mathrm{I}\alpha = -MB\sin\theta$$

When θ is small $\sin\theta = \theta$

$$\therefore \alpha = \left(\frac{-MB_H}{I}\right)\theta$$

For a given magnet, the factor $\frac{MB_H}{I}$ is a constant and hence the angular acceleration (α) is directly proportional to angular displacement which is the condition for SHM. Hence the time period of the magnet is given by

$$T = 2\pi\sqrt{\frac{\text{Angular displacement}}{\text{Angular acceleration}}} = 2\pi\sqrt{\frac{\theta}{\alpha}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB_H}}$$

$\therefore$ Time period of vibrating magnet $\therefore T = 2\pi\sqrt{\frac{I}{MB_H}}$

Where I is moment of inertia of the magnet used $I = \frac{m(l^2 + b^2)}{12}$

Very Short Answer Questions:

1. **On what factors does the period of oscillation of a bar magnet in a uniform magnetic field depend?**

Ans: The time period of oscillation of bar magnet in uniform magnetic field depends on (i) Moment of inertia (I) of the magnet (ii) Moment of the magnet (M)and (iii) the magnetic induction(B). $T = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{MB}}}$

2. State the principle of working of a vibration magnetometer and write the expression for the time period of an oscillating magnet .(May 2009)

Ans. It works on the principle that when a bar magnet suspended freely in a uniform magnetic field is displaced from its equilibrium position, it executes S.H.M. about the equilibrium position.

$T = 2\pi\sqrt{\frac{\mathrm{I}}{\mathrm{MB}}}$

SOLVED PROBLEMS

1. A bar magnet of moment of inertia $1\times10^{-2}\ kg\ m^2$ vibrates in a magnetic field in induction $0.36\times10^{-4}\ tesla$. The time period of vibration is 10s. Find the magnetic moment of the bar magnet.

Sol: $I = 1\times10^{-2}\ kg\ m^2$, T = 10s, $B = 0.36\times10^{-4}\ Tesla$

Time period of vibration of a magnet is, $T = 2\pi\sqrt{\frac{I}{MB}}$ *Or* $M = 4\pi^2\frac{I}{T^2B}$

$$M = 4\pi^2\frac{1\times10^{-2}}{(10)^2\times0.36\times10^{-4}} = 110\,Am^2$$

UNSOLVED PROBLEMS

1. A magnetic needle pivoted through its centre of mass and is free to rotate in a plane containing uniform magnetic field $200\times10^{-4}T$ .When it is displaced slightly from the equilibrium it makes 2 oscillations per second. If the moment of inertia of the needle about the axis of oscillation is $0.75\times10^{-5}\ kg\ m^2$, find the magnetic moment of the needle $(\pi^2 = 10)$

Sol: $B_H = 200\times10^{-4}\ \ T = 2\times10^2 T\ ;\ \ n = 2Hz\ ; I = 0.75\times10^{-5}\ kgm^2 = 75\times10^{-7}\ kgm^2$

$$n = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}} \Rightarrow 2 = \frac{1}{2\times3.14}\sqrt{\frac{M\times2\times10^{-2}}{75\times10^{-7}}} \Rightarrow M = 0.06\,Am^2$$