MAGNETISM

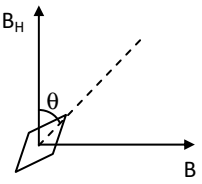
4.DEFLECTION MAGNETOMETER

POINTS TO REMEMBER

Deflection Magnetometer :

1. i) Deflection magnetometer works on the principle of tangent law.

ii) According to tangent law when two uniform magnetic fields act at right angles to each other on a magnetic needle, it comes to rest in the direction of resultant field. $B = B_H \tan \theta$ where B is the magnetic induction due to bar magnet



B_H is the horizontal component of earth's magnetic field

θ is the angle through which the needle is deflected.

iii) The pointer is of aluminium because it is a paramagnetic substance.

iv) The mirror under the pointer helps to take readings without parallax error.

v) The zero of the horizontal scale coincides with the centre of the circular scale.

vi)Deflection magnetometer does not work at the magnetic poles of the earth, because at poles $B_H = 0$.

vii)The deflection magnetometer has minimum relative error when the deflection is 45°.

viii) The deflection is independent of the strength of the magnetic needle.

2. Tan – A position or Gauss – A position or End on position :

i) Arms of the magnetometer are perpendicular to the magnetic meridian i.e. arms lie in East – West direction.

ii) The Aluminium pointer should show $0° - 0°$ reading.

iii) The centre of the magnetic needle should lie on axial line of the bar magnet.

iv) $\frac{\mu_0}{4\pi}\frac{2Md}{\left(d^2 - \ell^2\right)^2} = B_H \tan\theta$ (For long bar magnet)

$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = B_H \tan\theta$ (For short bar magnet)

3. Tan – B position or Gauss B position or broad side on position :

i) arms are in the magnetic meridian i.e, in the north south direction.

ii) The Aluminium pointer should show $0° - 0°$ reading.

iii) The centre of the magnetic needle should lie on axial line of the bar magnet.

iv) $\frac{\mu_0}{4\pi}\frac{M}{\left(d^2 + \ell^2\right)^{\frac{3}{2}}} = B_H \tan\theta$ (For long bar magnet)

$\frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H \tan\theta$ (For short bar magnet)

4. Deflection magnetometer is used to

i) compare magnetic moments

ii) compare earth's magnetic inductions

iii) verify inverse square law and

iv) determine the horizontal component of the earth's magnetic induction.

5. To compare magnetic moments :

i) **Equal distance method :** For Tan A or TanB position $\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$

ii) Null deflection method : For Tan A or TanB position $\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$

6. **To verify inverse square law by Gauss method :** When a short magnet is placed at a distance 'd' in tanA and then tanB positions and θ_A and θ_B are the average deflection then $\frac{\tan\theta_A}{\tan\theta_B}$ =2(verification of inverse square law)
7. Deflection magnometer is more accurate when the deflection is 45°. Therefore the reading should lie between 30° to 60° for accurate results.

Long Answer Questions :

1. State inverse square law and explain how can it be verified by Gauss method?

Ans: Coulomb's Inverse square law:

"The force of attraction (or) repulsion between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them and it acts along the line joining the poles."

Explanation:

Consider two magnetic poles of pole strengths m_1 and m_2 separated by a distance'd' in any medium. Let F be the force of attraction (or) repulsion between them.

$$\therefore F \alpha m_1 m_2$$

$$\alpha \frac{1}{d^2}$$

(or) $F = k\frac{m_1 m_2}{d^2}$ Where $k = \frac{\mu}{4\pi}$ is the constant of proportionality

$\therefore F = \frac{\mu}{4\pi}\frac{m_1 m_2}{d^2}$ Where μ is called the permeability of the medium and $\mu = \mu_0\mu_v$

where μ_0 is the permeability of free space and μ_r is the relative permeability of the medium.

$$\therefore F_{medi} = \frac{\mu_0\mu_r}{4\pi}\frac{m_1 m_2}{d^2}$$

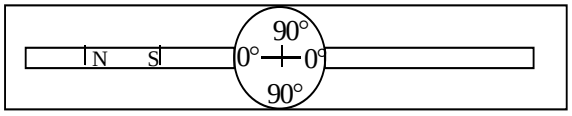
But, for air (or vacuum $\mu_r = 1$

$$\therefore F_{air} = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{d^2}$$

Here $\mu_0 = 4\pi\times10^{-7}$ Newton/amp^2 (or) Henry / meter

Verification of Inverse square law – Gauss Method:

D.M.M is arranged in Tan A position. The wooden arms of the magnetometer must be in East – West direction. The Aluminium pointer should show $0°-0°$ reading.



A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d from the centre of the magnetic needle. The centre of the magnetic needle should lie on the axial line of the bar magnet. The deflections of the Aluminium pointer θ_1 and θ_2 are noted. The bar magnet is reversed pole to pole at the same position and the deflections θ_3 and θ_4 are noted. Now the bar magnet is placed on the other arm of the magnetometer at the same distance d and the deflections θ_5, θ_6, θ_7 and θ_8 are noted. The average reading θ_A of these eight deflections is calculated.

$$\frac{\mu_o}{4\pi}\frac{2M}{d^3} = B_H \text{Tan}\,\theta_A \text{ --------- (1)}$$

D.M.M is arranged in Tan B position. The wooden arms of the magnetometer must be in North – South direction. The Aluminium pointer should show $0^\circ - 0^\circ$ reading.

A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d from the centre of the magnetic needle. The centre of the magnetic needle should lie on the equatorial line of the bar magnet. The deflections of the Aluminium pointer θ_1 and θ_2 are noted. The bar magnet is reversed pole to pole at the same position d and the deflections θ_5, θ_6, θ_7 and θ_8 are noted. The average reading θ_B of these eight deflections is calculated.

$$\frac{\mu_o}{4\pi}\frac{M}{d^3} = B_H Tan\theta_B \text{ ------- (2)}$$

From equations (1) and (2), $\frac{\text{Tan}\,\theta_A}{\text{Tan}\,\theta_B} = 2$ (Nearly)

Precautions:

1) The bar magnet must be placed at the same level as the magnetic needle of the magnetometer

2) All other magnets and magnetic materials are removed from the vicinity of the deflection magnetometer.

2. Describe the principle of working of deflection magnetometer. Explain how the magnetic moments of two short bar magnets can be compared in Tan A position by (i) Equal distance method. (ii) Null Method?

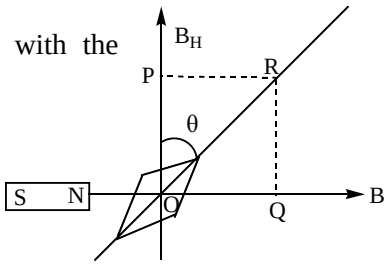
Ans : **Description :**

Deflection Magnetometer consists of a compass box in which a small magnetic needle is pivoted on a frictional less point so as to rotate freely in the horizontal plane. A long Aluminium pointer is attached perpendicular to the needle at its centre. The Aluminium pointer rotates along a circular scale along with the needle. The circular scale is divided into four quadrants, which is graduated in degrees $\left(0^\circ - 90 - 0^\circ - 90^\circ\right)$. A plane mirror is placed below the pointer to take the readings without parallax error. The compass box is an evacuated brass box provided with a glass top.The compass box is placed at the centre of a horizontal wooden board. The arms of the wooden board are provided with two linear scales graduated in mm. The zeroth readings of these scales coincide with the centre of the magnetic needle.

Principle:

The D.M.M. works on the principle of Tangent law.When a magnetic needle is freely suspended it comes to rest in the magnetic meridian plane due to horizontal component of the earth's magnetic field (B_H). When a short bar magnet is brought nearer to the needle it experiences an additional field 'B'.

When a freely suspended magnet is acted upon by these two mutually perpendicular fields, it rests in the resultant field direction as shown in the figure.



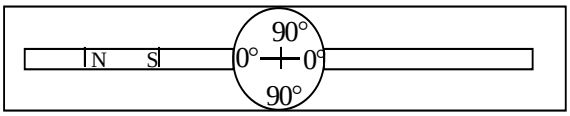
Let θ be the angle made by the resultant field direction with the magnetic meridian.

Then from the figure,

$$Tan\theta = \frac{OQ}{OP} = \frac{B}{B_H} \text{ (or) } B = B_H Tan\theta$$

Tan A (or) Gauss A (or) End - on position :

The wooden arms of the magnetometer should be perpendicular to the magnetic needle (or) they must be in the East - West direction. The Aluminium pointer should show $0° - 0°$ reading. A bar magnet is placed so that the centre of the magnetic needle should lie on the axial line of the bar magnet.



$$\therefore \; B = \frac{\mu_o}{4\pi} \frac{2Md}{\left(d^2 - l^2\right)^2}$$

From Tangent law, $B = B_H Tan\theta$

$$\frac{\mu_o}{4\pi} \frac{2Md}{\left(d^2 - l^2\right)} = B_H Tan\theta$$

For a short bar magnet,

$$\frac{\mu_o}{4\pi} \frac{2M}{d^3} = B_H Tan\theta$$

Comparision of magnetic moments : Tan A Position – Equal distance method :

D.M.M is arranged in Tan A position. The wooden arms of the magnetometer must be in East – West direction. The Aluminium pointer should show $0° - 0°$ reading.

A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d from the centre of the magnetic needle. The centre of the magnetic needle should lie on the bar magnet. The deflections of the Aluminium pointer θ_1 and θ_2 are noted. The bar magnet is reversed pole to pole at the same position and the deflections θ_3 and θ_4 are noted. Now the bar magnet is placed on the other arm of the magnetometer at the same distance d and the deflections θ_5, θ_6, θ_7 and θ_8 are noted. The average reading θ_1 of these eight deflections is calculated.

$$\frac{\mu_o}{4\pi} \frac{2M_1}{d^3} = B_H Tan\,\theta_1 \text{ ---------- (1)}$$

Now the above procedure is repeated with another short bar magnet of moment M_2 for the same distance d and the average reading θ_{11} of the eight deflections is calculated

$$\therefore \frac{\mu_o}{4\pi}\frac{2M_2}{d^3} = B_H Tan\theta_{11} \text{ ------ (2)}$$

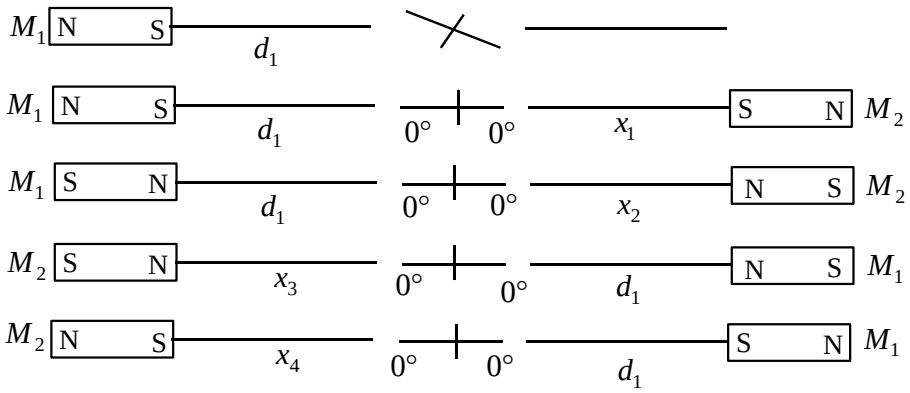
From equations (1) and (2), $\frac{M_1}{M_2} = \frac{Tan\theta_1}{Tan\theta_{11}}$

Comparision of Magnetic Moments : Tan A position – Null method :

D.M.M is arranged in Tan A position. The wooden arms of the magnetometer must be in East – West direction. The Aluminium pointer should show $0° - 0°$ reading.

A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d_1 from the centre of the magnetic needle. The centre of the magnetic needle should lie on the axial line of the bar magnet. Hence the magnetic needle gets deflected.

A second short bar magnet of moment M_2 is placed on the other arm of the magnetometer co-axially with the first bar magnet. The like poles of the two bar magnets should face the magnetic needle. The distance of the second magnet is adjusted until the Aluminium pointer shows $0° - 0°$ (i.e.) null deflection and the distance of the second magnet x_1 from the centre of the needle is measure. The poles of both the magnets are reversed and by keeping the first magnet at a distance d_1, the distance of the 2nd magnet x_2 for null deflection is measured.



Now the magnets are interchanged on the wooden arms of the magnetometer. Keeping the first magnet at a distance d_1, the distances of the 2nd magnet x_3 and x_4 for null deflections are noted as above.

The average distance of the 2nd bar magnet for null deflection d_2 is calculated.

For the first magnet, $B_1 = \frac{\mu_o}{4\pi}\frac{2M_1}{d_1^3}$

For the second magnet, $B_2 = \frac{\mu_o}{4\pi}\frac{2M_2}{d_2^3}$

Hence for null deflection $B_1 = B_2$

$$\frac{\mu_o}{4\pi}\frac{2M_1}{d_1^3} = \frac{\mu_o}{4\pi}\frac{2M_2}{d_2^3}$$

$\therefore \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$ From equations (1) and (2), $\frac{\text{Tan}\,\theta_A}{\text{Tan}\,\theta_B} = 2$

(Nearly)

Precautions :

1) The bar magnet must be placed at the same level as the magnetic needle of the magnetometer

2) All other magnets and magnetic materials are removed from the vicinity of the deflection magnetometer.

3. Define 'Magnetic moment' of a bar magnet. Explain the method of comparing the magnetic moments of two short bar magnets in tan B positions in (i) Equal distance method, (ii) Null method.

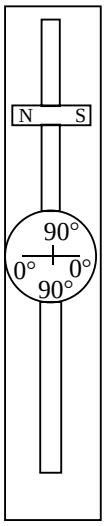
Ans: **Magnetic Moment :** The product the pole strength and the length of the bar magnet is known as Magnetic moment M = 2*l* x m

Description :

Deflection Magnetometer consists of a compass box in which a small magnetic needle is pivoted on a frictional less point so as to rotate freely in the horizontal plane. A long Aluminium pointer is attached perpendicular to the needle at its centre. The Aluminium pointer rotates along a circular scale along with the needle. The circular scale is divided into four quadrants, which is graduated in degrees $(0^\circ - 90 - 0^\circ - 90^\circ)$. A plane mirror is placed below the pointer to take the readings without parallax error. The compass box is an evacuated brass box provided with a glass top.

The compass box is placed at the centre of a horizontal wooden board. The arms of the wooden board are provided with two linear scales graduated in mm. The zeroth readings of these scales coincide with the centre of the magnetic needle.

The D.M.M. works on the principle of Tangent law. $B = B_H Tan\theta$



Tan B (or) Gauss B (or) Broad – side position :

The wooden arms of the magnetometer should be in the direction of the magnetic needle (i.e) along North – South direction. The Aluminium pointer should show $0^\circ - 0^\circ$ reading. A bar magnet is placed so that the centre of the magnetic needle should lie on the equatorial line of the bar magnet.

$$\therefore\ B = \frac{\mu}{4\pi}\ \frac{M}{\left(d^2 + l^2\right)^{3/2}}$$

From Tangent law, $B = B_H Tan\theta$

$$\therefore\ \frac{\mu_o}{4\pi}\ \frac{M}{\left(d^2 + l^2\right)^{3/2}} = B_H \mathrm{Tan}\,\theta$$

For a short bar magnet, $\frac{\mu_o}{4\pi}\ \frac{M}{d^3} = B_H \mathrm{Tan}\,\theta$

Comparision of Magnetic Moments :Tan B Position – Equal distance method :

D.M.M is arranged in Tan B position. The wooden arms of the magnetometer must be in North – South direction. The Aluminium pointer should show $0^\circ - 0^\circ$ reading.

A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d from the centre of the magnetic needle. The centre of the magnetic needle should lie on the equatorial line of the bar magnet. The deflections of the Aluminium pointer θ_1 and θ_2 are noted. The bar magnet is reversed pole to pole at the same position and the deflections θ_3 and θ_4 are noted. Now the bar magnet is placed on the other arm of the magnetometer at the same distance d and the deflections θ_5, θ_6, θ_7 and θ_8 are noted. The average reading θ_1 of these eight deflections is calculated.

$$\frac{\mu_o}{4\pi}\frac{M_1}{d^3} = B_H \text{Tan } \theta_1 \text{ --------- (1)}$$

Now the above procedure is repeated with another short bar magnet of moment M_2 for the same distance d and the average reading θ_{11} of the eight deflections is calculated.

$$\therefore \frac{\mu_o}{4\pi}\frac{M_2}{d^3} = B_H Tan\ \theta_{11} \text{ ------ (2)}$$

From equations (1) and (2),

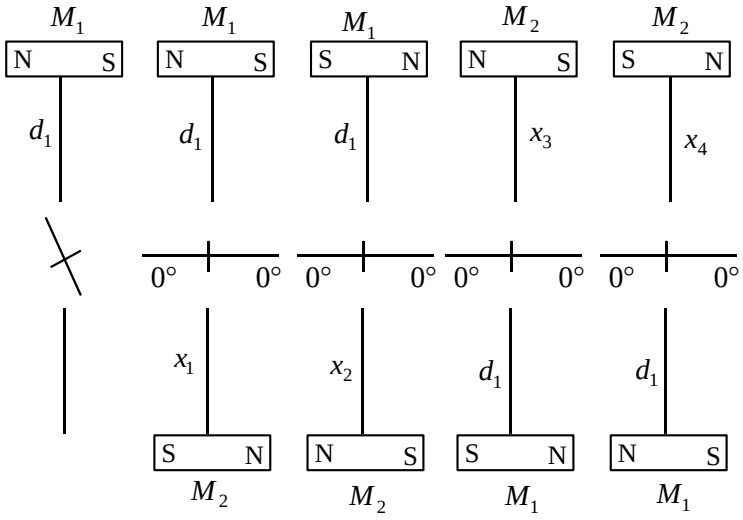
$$\frac{M_1}{M_2} = \frac{Tan\theta_1}{Tan\theta_{11}}$$

Comparision of Magnetic Moments :Tan B Position – Null Method :

D.M.M is arranged in Tan B position. The wooden arms of the magnetometer must be in North – South direction. The Aluminium pointer should show $0° - 0°$ reading.

A short bar magnet of moment M_1 is placed on one of the arms of the magnetometer at a distance d_1 from the centre of the magnetic needle. The centre of the magnetic needle should lie on the equatorial line of the bar magnet. Hence the magnetic needle gets deflected.

A second short bar magnet of moment M_2 is placed on the other arm of the magnetometer parallel to the first bar magnet. The unlike poles of the two bar magnets should lie on one side. The distance of the second magnet is adjusted until the Aluminium pointer shows $0° - 0°$ (i.e.) null deflection of the second magnet x_1 from the centre of the needle is measured. The poles of both the magnets are reversed and by keeping the first magnet at a distance d_1, the distance of the 2nd magnet x_2 for null deflection is measured.



Now the magnets are interchanged on the wooden arms of the magnetometer. Keeping the first magnet at a distance d_1, the distances of the 2nd magnet x_3 and x_4 for null deflections are noted as above.

The average distance of the 2nd bar magnet for null deflection d_2 is calculated

For the first magnet, $$B_1 = \frac{\mu_o}{4\pi}\frac{2M_1}{d_1^3}$$

For the second magnet, $$B_2 = \frac{\mu_o}{4\pi}\frac{2M_2}{d_2^3}$$

Hence for null deflection $B_1 = B_2$

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$$

From equations (1) and (2), $\frac{\text{Tan}\,\theta_A}{\text{Tan}\,\theta_B} = 2$ (Nearly)

Precautions:

1) The bar magnet must be placed at the same level as the magnetic needle of the magnetometer

2) All other magnets and magnetic materials are removed from the vicinity of the deflection magnetometer.

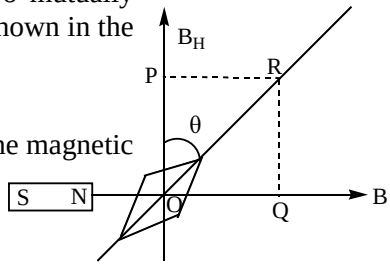
Short Answer Questions :

1. State and explain "Tangent Law" in magnetism? (March2011)

Ans: The D.M.M. works on the principle of Tangent law.

When a magnetic needle is freely suspended it comes to rest in the magnetic meridian plane due to horizontal component of the earth's magnetic field (B_H). When a short bar magnet is brought nearer to the needle it experiences an additional field 'B'.

When a freely suspended magnet is acted upon by these two mutually perpendicular fields, it rests in the resultant field direction as shown in the figure.



Let θ be the angle made by the resultant field direction with the magnetic meridian.

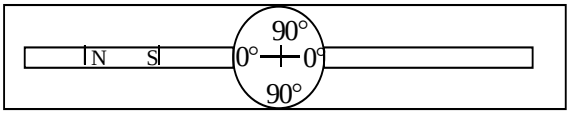
Then from the figure, $\text{Tan}\,\theta = \frac{OQ}{OP} = \frac{B}{B_H}$

(or) $B = B_H Tan\,\theta$

2. Explain 'Tan A' and 'Tan B' positions of a deflection magnetometer. (March 2010,March2009)

Ans : Tan A (or) Gauss A (or) End - on position :

The wooden arms of the magnetometer should be perpendicular to the magnetic needle (or) they must be in the East - West direction. The Aluminium pointer should show $0° - 0°$ reading. A bar magnet is placed so that the centre of the magnetic needle should lie on the axial line of the bar magnet.



$$\therefore B = \frac{\mu_o}{4\pi}\frac{2Md}{\left(d^2 - l^2\right)^2}$$

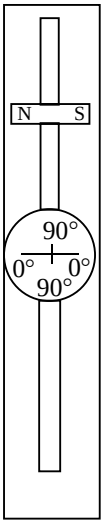
From Tangent law, $B = B_H \tan\theta$

$$\frac{\mu_o}{4\pi}\frac{2Md}{\left(d^2 - l^2\right)} = B_H\ Tan\,\theta$$

For a short bar magnet,

$$\frac{\mu_o}{4\pi}\frac{2M}{d^3} = B_H Tan\theta$$

Tan B (or) Gauss B (or) Broad – side position :



The wooden arms of the magnetometer should be in the direction of the magnetic needle (i.e) along North – South direction. The Aluminium pointer should show $0°-0°$ reading. A bar magnet is placed so that the centre of the magnetic needle should lie on the equatorial line of the bar magnet.

$$\therefore\ B = \frac{\mu}{4\pi}\frac{M}{\left(d^2+l^2\right)^{3/2}}$$

From Tangent law, $B = B_H \text{Tan}\,\theta$

$$\therefore\ \frac{\mu_o}{4\pi}\frac{M}{\left(d^2+l^2\right)^{3/2}} = B_H Tan\ \theta$$

For a short bar magnet, $\frac{\mu_o}{4\pi}\frac{M}{d^3} = B_H Tan\ \theta$

Very Short Answer Questions

1. **In equal distance method of a deflection magnetometer, the deflections produced by two bar magnets are** 30^0 and 60^0. What is the ratio of their magnetic moments?

Ans: For equal distance method, $\frac{M_1}{M_2} = \frac{Tan\theta_1}{Tan\theta_2} = \frac{Tan\,30^0}{Tan\,60^0} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \Rightarrow M_1 : M_2 = 1:3$

Solved Problems

1. **Two short bar magnets having magnetic moments in the ratio 27 : 8 are placed on opposite arms of a deflection magnetometer. Find the distance of stronger magnet if the weaker magnet is at a distance of 10cm from the centre of the magnetometer for null deflection.**

Sol: $M_1 : M_2 =$ **27** : **8** and $d_1 = ?$ and $d_2 = 10cm = 0.1m$

$$\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3} \quad \Rightarrow \quad \frac{27}{8} = \frac{d_1^3}{(0.1)^3} \Rightarrow d_1 = \frac{3\times 0.1}{2} = 15cm$$

2. **The ratio of magnetic moments of two bar magnets is 4:5. If the deflection due to first bar magnet in the deflection magnetometer is 45^0, then what is the deflection due to second bar magnet?**

Sol: $\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$ and $M_1 : M_2 = 4:5$

$$\therefore \frac{4}{5} = \frac{Tan\ 45^0}{Tan\,\theta_2} = \frac{1}{Tan\,\theta_2} \Rightarrow Tan\,\theta_2 = \frac{5}{4} = 1.25 \Rightarrow \theta_2 = \tan^{-1}(1.25) = 51.34^0$$

Unsolved Problem

1. A short magnet produced a deflection of 45^0 when placed at a some distance in Tan A position of the deflection magnetometer. If another magnet of same length and pole strength is kept at the same distance in Tan B position, what is the deflection produced?

Sol: Tan A position:

$$B_a = B_H \, Tan\,\theta \Rightarrow \frac{\mu_0}{4\pi}\frac{2M_1}{d_1^3} = B_H \tan 45^0 \; .....(1)$$

Tan B position:

Since $m_1 = m_2$, length is same then $M_2 = M_1$ & $d_2 = d_1$

$$B_H \tan\theta = \frac{\mu_0}{4\pi}\frac{M_2}{d_2^3} .....(2)$$

From (1) and (2) , $\quad \frac{Tan\,\theta}{Tan\ 45^0} = \frac{M_2}{d_2^3} \times \frac{d_1^3}{2M_1} = \frac{1}{2}$

$$\Rightarrow Tan\,\theta = \frac{1}{2} = 0.5000 = Tan\ 26^0\,34' \quad \Rightarrow \theta = 26^0\ 34'$$

2. A short bar magnet placed end on to a compass needle with its centre at 0.15 m from the centre of the needle deflects the needle through 45^0. By how far the magnet should be moved away so that the deflection may fall to 30^0?

Sol: In end on position or Tan A position, $d_1 = 0.15m\,; \theta_1 = 45^0\,;\ \theta_2 = 30^0\,; d_2 = ?$

In Tan A position, $\quad \frac{M_1}{M_2} = \frac{Tan\,\theta_1}{Tan\,\theta_2} = \frac{d_1^3}{d_2^3}$

$$\Rightarrow d_2^3 = \frac{d_1^3\, Tan\,\theta_2}{Tan\,\theta_1} = (0.15)^3 \times \frac{Tan\,45^0}{Tan\ 30^0} = 0.15^3 \times \sqrt{3} = 0.15 \times 3^{1/3} = 0.18m$$

$\therefore$ Distance to be moved $= d_2 - d_1$ = 0.18 – 0.18 = 0.03 m

3. Two bar magnets 'A' and B are placed on the arms of a deflection magnetometer. When their distances from the centre of the needle are 20cm and 40cm respectively, the needle lies in the magnetic meridian. If the moment of the magnet 'A' is $100Am^2$, calculate the moment of the magnet 'B'.

Sol: Distance of magnet A = 20 cm ; Distance of magnet B = 40 cm

Magnetic moment of magnet A = $100A-m^2$; Magnetic moment of magnet B =?

In null deflection method, $\quad \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3} \Rightarrow M_2 \Rightarrow 100 \times \left(\frac{40}{20}\right)^3 = 100 \times 2^3 = 800\, A.m^2$

4. A magnetometer is in Tan A position. A magnet produces in the needle a deflection of 30^0. How many such similar magnets must be tied together such that like poles are touching each other to increase the deflection to 60^0?

(Hint: the magnetic moments add up when the bar magnets are tied together such that like poles are touching each other)

Sol: Let magnetic moment of the magnet $= M_1$; $\theta_1 = 30^0$; $\theta_2 = 60^0$

In tan A position $\frac{M_1}{M_2} = \frac{Tan\,\theta_1}{Tan\ \theta_2} \Rightarrow M_2 = M_1 \frac{\tan\theta_1}{\tan\theta_2} = M_1 \frac{\tan 60^0}{\tan 30^0} = 3M_1$

$\therefore M_2 = 3M_1$

To produce a deflection of 60^0 , three similar magnets must be placed with like poles adjacent to each other.

ASSESS YOURSELF:

1. What is the net force acting on the magnetic needle shown in the figure?

Ans: Zero

