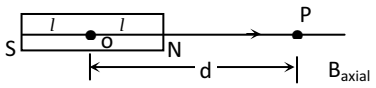
MAGNETISM

3.MAGNETIC INDUCTION

POINTS TO REMEMBER

1. Magnetic induction at a point on the axial line of a bar magnet :



Let 'P' is a point which is at a distance 'd' from the centre of a bar magnet on its axial line.

The magnetic induction field at point 'P' is given by $B_{axial} = \frac{\mu_0}{4\pi}\frac{2Md}{\left(d^2 - \ell^2\right)^2}$

i) For a short bar magnet $\ell^2 << d^2$. Hence ℓ^2 can be neglected.

$\therefore B_{axial} = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$

ii) The direction of B is from south to north along the axial line.

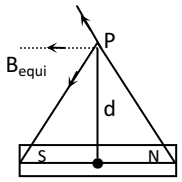
iii) Direction of $\vec{B}$ is parallel to magnetic moment $\vec{M}$.

v) Intensity of magnetic field at the same point is given by, $H_{axial} = \frac{1}{4\pi}\frac{2Md}{\left(d^2 - \ell^2\right)^2}$

For a short bar magnet $\ell^2 << d^2$. Hence ℓ^2 can be neglected. $H_{axial} = \frac{1}{4\pi}\frac{2M}{d^3}$

2. Magnetic induction at a point of the equatorial line of a bar magnet :

Let 'P' is a point on the equatorial line of a bar magnet which is at a distance'd' from the centre of the bar magnet.



The magnetic induction field at point 'P' is given by

$B_{eqi} = \frac{\mu_0}{4\pi}\frac{M}{\left(d^2 + \ell^2\right)^{3/2}}$

c) For a short bar magnet $\ell^2 << d^2$. Hence ℓ^2 can be neglected, $B_{eqi} = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

iii) The direction of B is from North to South parallel to its axis.

iv) Direction of $\vec{B}$ is antiparallel to $\vec{M}$.

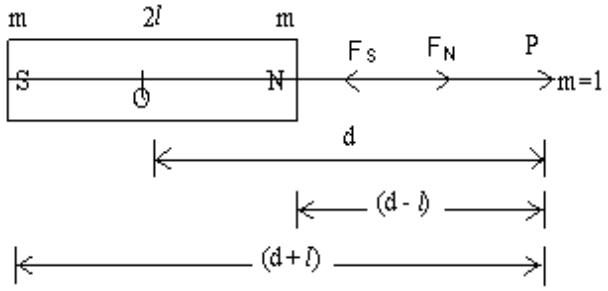
v) Intensity of magnetic field at point 'P' is given by, $H = \frac{1}{4\pi}\frac{M}{d^3}$

SHORT ANSWER QUESTIONS:

1. Derive an expression for the magnetic induction on the equatorial line of a bar magnet?

Ans: Consider a bar magnet of length '2*l*' and pole strength 'm'. Consider a point P on the axial line of the bar magnet at a distance 'd' from the centre 'O' of the bar magnet.

Let a unit north is placed at a point P. It experiences a repulsive force F_N due to the North Pole and an attractive pole F_S due to the South Pole.



$F_N = \frac{\mu_0}{4\pi}\frac{m\times l}{(d-l)^2}$ Along $\overrightarrow{NP}$ And $F_S = \frac{\mu_0}{4\pi}\frac{m\times l}{(d+l)^2}$ Along $\overrightarrow{PS}$

The resultant force on unit North Pole is given by

$$F_{net} = F_N - F_S = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right] = \frac{\mu_0 m}{4\pi}\left[\frac{(d+l)^2-(d-l)^2}{(d-l)^2(d+l)^2}\right]$$

$$F = \frac{\mu_0}{4\pi} m\left[\frac{4dl}{\left(d^2-l^2\right)^2}\right]$$

But M = 2*l*m is called the magnetic moment .Net force (or) magnetic induction (B) is given by

$$\therefore F = \frac{\mu_0}{4\pi}\frac{2Md}{\left(d^2-l^2\right)^2}$$

For a short bar magnet (i.e. if *l* is very much less than d, *l* < < < d then

$$B \approx \frac{\mu_0}{4\pi}\frac{2M}{d^3} \text{ or } H = \frac{1}{4\pi}\frac{2M}{d^3}$$

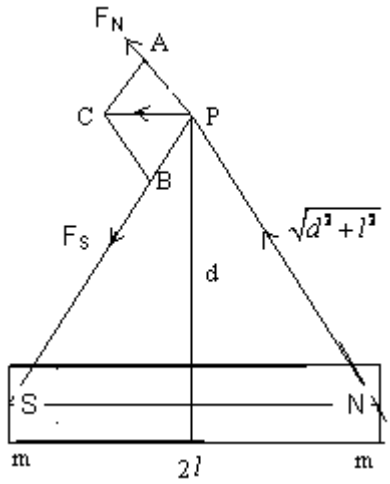
The direction of magnetic induction is from South Pole $\overrightarrow{SN}$ along the axis.

2. Derive an expression for the magnetic induction at a point on the equatorial line of a bar magnet.?

Ans: Equatorial line of a magnet is the perpendicular bisector of the axis of the magnet. Let us Consider a bar magnet of length 2*l* and pole strength 'm'. Consider a

point P on the equatorial line of the bar magnet at a distance ‘d’ form the centre O of the bar magnet.

Let a unit North Pole is placed at the point P. It experience a repulsive force F_N due to the north and attractive force F_S due to the south pole of the bar magnet.



$$F_N = \frac{\mu_0}{4\pi}\frac{m\times 1}{\left(d^2+l^2\right)} \; and \;\; F_S = \frac{\mu_0}{4\pi}\frac{m\times 1}{\left(d^2+l^2\right)}$$

The resultant force acting on the unit North Pole can be determined by drawing a parallelogram with F_N and F_S as adjacent sides as shown. The diagonal PC represents the resultant force.

In the diagram ΔPAC and ΔPNS are similar triangles.

$$\therefore \frac{PC}{NS} = \frac{PA}{PN} \quad \text{(or)} \quad PC = \frac{PA}{PN}\times NS$$

Or PC $= \dfrac{F_N \times 2l}{\sqrt{d^2+l^2}}$ (or) $PC = \dfrac{\mu_0}{4\pi}\dfrac{m}{\left(d^2+l^2\right)}\dfrac{2l}{\left(d^2+l^2\right)^{1/2}}$

But M = 2*l*m = magnetic moment. The magnetic induction on the equatorial line is given by

$$B = \frac{\mu_0}{4\pi}\frac{M}{\left(d^2+l^2\right)^{3/2}}$$

For a short bar magnet i.e. if *l* << d; then

$$B \approx \frac{\mu_0}{4\pi}\frac{M}{d^3} \quad \text{(or)} \quad H = \frac{1}{4\pi}\frac{M}{d^3}$$

The direction of magnetic induction on the equatorial line is from north to south $\overline{NS}$ parallel to the axis

Very Short Answer Questions:

1. Give the expressions for the magnetic induction due to a bar magnet on (i) the axial line and (ii) equatorial line.? (March 2010)

Ans: If the length is '2l'and pole strength is 'm' for a bar magnet, the magnetic induction at a point on the axial line at a distance 'd' from the centre of the magnet is given by,

$$B_{arial} = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2}$$

For a short magnet, $B_{arial} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$

On the equatorial line of the bar magnet, $B_{equatorial} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}$

For a short magnet, $B_{equatorial} = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$.

2. Where do you find the null points in the combined field due to a bar magnet and the earth, when (i) the N-pole of the magnet is kept towards the north of the earth, (ii) the S-Pole of the magnet is kept towards the north of the earth?

Ans: (i) When the N-pole of the magnet is kept towards the north pole of earth, the null points in the combined field due to the bar magnet and the earth are obtained on the equatorial line of the magnet on either side at equal distance from the centre of the magnet..

ii) When the S-pole of the magnet is kept towards the north pole of earth, the null points in the combined field due to the bar magnet the bar magnet and the earth are obtained on the axial line of the magnet on either side at equal distance from the centre of the magnet.

SOLVED PROBLEMS

1. Find the magnetic induction at a distance of 20cm on the equatorial line of a short bar magnet with a magnetic moment $60 Am^2$

Sol: The magnetic induction on the equatorial line of a short bar magnet, $B_E = \frac{\mu_0}{4\pi} \frac{M}{d^3}$

$$\therefore B_E = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{60}{(20 \times 10^{-2})^3}$$

2. The magnetic moment of a bar magnetic is $45 Am^2$ **and its length is 10cm. Calculate the magnetic field induction at a point 10cm away from the centre of the magnet on the axial line.?**

Sol: Magnetic field induction on the axial line $B = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2}$

$$\therefore B = 10^{-7} \times \frac{2 \times 45 \times 10 \times 10^{-2}}{\left[(10 \times 10^{-2})^2 - (5 \times 10^{-2})^2\right]^2} = 16 \times 10^{-3} T$$

3. A bar magnet of magnetic moment $1.2\,Am^2$ is placed in the magnetic meridian with its south pole pointing the north. If a neutral point is found at a distance of 20cm from the centre of the magnet, what is the value of the horizontal component of the earth's magnetic field?

Sol: $$B_H = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 1.2}{(20 \times 10^{-2})^3} = 30 \times 10^{-6} T$$

4. Two short bar magnets having magnetic moments in the ratio 27 : 8 are placed on opposite arms of a deflection magnetometer. Find the distance of stronger magnet if the weaker magnet is at a distance of 10cm from the centre of the magnetometer for null deflection.?

Sol: $M_1 : M_2 = \mathbf{27} : \mathbf{8}$ and $d_1 = ?$ and $d_2 = 10cm = 0.1m$

$$\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3} \quad \Rightarrow \quad \frac{27}{8} = \frac{d_1^3}{(0.1)^3} \Rightarrow \quad d_1^3 = \frac{27 \times (0.1)^3}{8} = \frac{3^3 \times (0.1)^3}{2^3}$$

$$\therefore d_1 = \frac{3 \times 0.1}{2} = 0.15m = 15cm$$

5. The ratio of magnetic moments of two bar magnets is 4:5. If the deflection due to first bar magnet in the deflection magnetometer is 45^0, then what is the deflection due to second bar magnet?

Sol: $$\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2} \quad \text{and} \quad M_1 : M_2 = 4 : 5$$

$$\therefore \frac{4}{5} = \frac{Tan\ 45^0}{Tan\,\theta_2} = \frac{1}{Tan\,\theta_2} \Rightarrow Tan\,\theta_2 = \frac{5}{4} = 1.25 \Rightarrow \theta_2 = \tan^{-1}(1.25) = 51.34^0$$

<u>Unsolved Problem</u>

1. A short bar magnet produces magnetic fields of equal induction at two points one on the axial line and the other on the equatorial line. What is the ratio of their distances?

Asn: $B_a = B_e$

$$\Rightarrow \frac{\mu_0}{4\pi}\frac{2M}{d_1^3} = \frac{\mu_0}{4\pi}\frac{M}{d_2^3} \qquad \Rightarrow \left(\frac{d_1}{d_2}\right)^3 = 2 \qquad \Rightarrow \frac{d_1}{d_2} = 2^{1/3} \Rightarrow d_1 : d_2 = 2^{1/3} : 1$$

2. A bar magnet of magnetic moment $0.4Am^2$ is placed in the magnetic meridian with its north pole pointing north. A neutral point is obtained at a distance of 10cm from the centre of the magnet. If the length of the magnet is

also 10 cm what is the value of horizontal component of earth's field at the place?

Ans: $M = 0.4\,Am^2$; $2l = 10cm = 10^{-1}\,m \Rightarrow l = \frac{10^{-1}}{2}m$; d = 10cm = $10^{-1}m$

Since North Pole is pointing north, null point is obtained on equatorial line of the bar magnet.

Hence at null point, $B_H = \frac{\mu_0}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = \frac{10^{-7}\times 0.4}{\left(10^{-2}+\frac{10^{-2}}{4}\right)^{3/2}}$

$$\therefore B_H = 2.86\times 10^{-5}\,T\,.$$

ASSESS YOURSELF:

1. **What do you infer from the magnetic field lines around a bar magnet which are closed loops?**

Ans: Isolated poles do not exist.