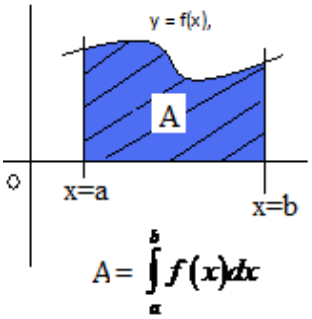
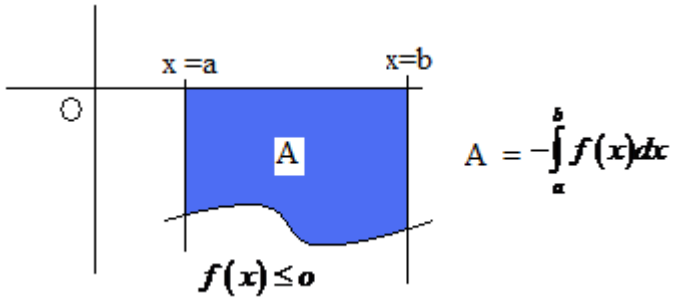
AREAS UNDER CURVES

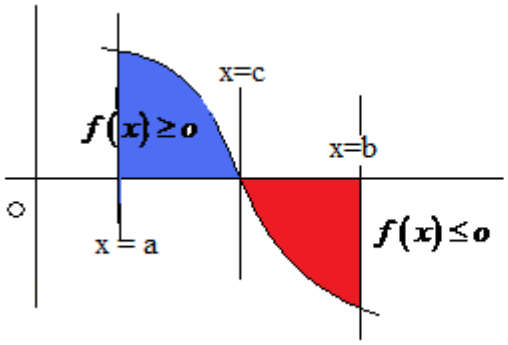
1. Let f be a continuous curve over [a,b]. If $f(x) \geq o$ **in [a,b], then the area of the region bounded by y = f(x), x-axis and the lines x=a and x=b is given by** $\int_a^b f(x)dx$ **. ?**



2. Let f be a continuous curve over [a,b]. If $f(x) \leq o$ **in [a,b], then the area of the region bounded by y = f(x), x-axis and the lines x=a and x=b is given by** $-\int_a^b f(x)dx$



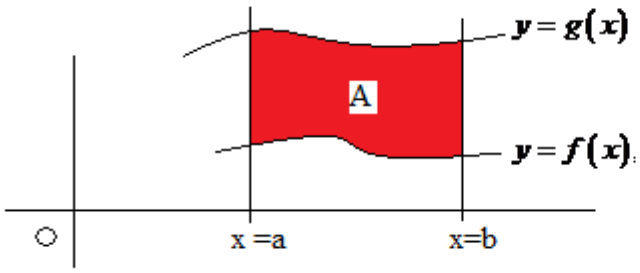
3. Let f be a continuous curve over [a,b]. If $f(x) \geq o$ **in [a, c] and** $f(x) \leq o$ **in [c , b] where a<c<b. Then the area of the region bounded by the curve y = f(x) , the x-axis and the lines x=a and x=b is given by** $\int_a^c f(x)dx - \int_c^b f(x)dx$ **.**



$$A = \int_a^c f(x)dx - \int_c^b f(x)dx$$

4. Let f(x) and g(x) be two continuous functions over [a, b]. Then the area of the region bounded by the curves $y = f(x), y = g(x)$ **and the lines x = a ,x=b is given by**

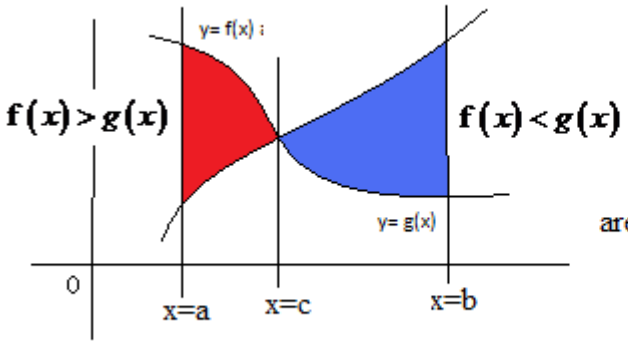
$$\left| \int_a^b f(x)dx - \int_a^b g(x)dx \right| .$$



$$A = \left| \int_a^b f(x)dx - \int_a^b g(x)dx \right|$$

5. Let f(x) and g(x) be two continuous functions over [a, b] and $c \in (a,b)$**. If** $f(x) > g(x)$ **in (a, c) and** $f(x) < g(x)$ **in (c, b) then the area of the region bounded by the curves y= f(x) and y= g(x) and the lines x=a, x=b is given by**

$$\left| \int_a^c (f(x) - g(x))dx \right| + \left| \int_c^b (g(x) - f(x))dx \right|$$

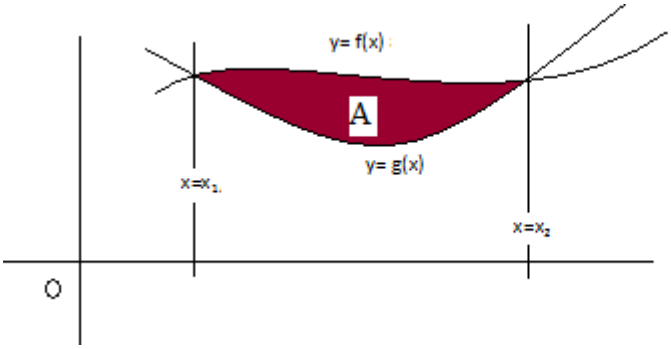


$$\text{area} = \left|\int_a^c (f(x)-g(x))dx\right| + \left|\int_c^b (g(x)-f(x))dx\right|$$

6. Let f(x) and g(x) be two continuous functions over [a, b] and these two curves are intersecting at

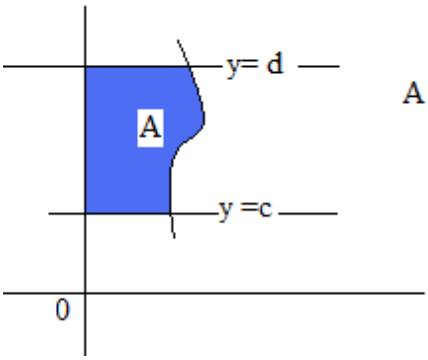
X =x_1 and x= x_2 where $x_1, x_2 \in (a,b)$ then the area of the region bounded by the curves y= f(x) and y= g(x) and the lines x=x_1, x=x_2 is given by

$$\left|\int_{x_1}^{x_2} (f(x)-g(x))dx\right|.$$



$$A = \left|\int_{x_1}^{x_2} (f(x)-g(x))dx\right|$$

Note: The area of the region bounded by x =g(y) where g is non negative continuous function in [c,d], the y axis and the lines y =c and y=d is given by $\int_c^d g(y)dy$.



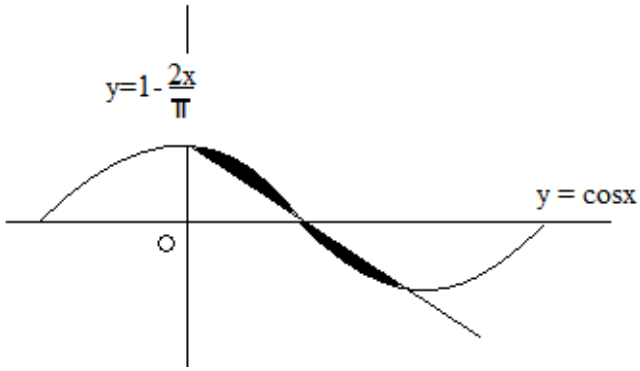
$A = \int_c^d g(y)dy$

EXERCISE – 10 (A)

1. Find the area of the region enclosed by the given area y = cos x , y = $1-\frac{2x}{\pi}$.

Sol: Equations of the given curves are y = cosx ……..(1) and $y=1-\frac{2x}{\pi}$------(2)

Solving (1) and (2) , $\cos x=1-\frac{2x}{\pi} \Rightarrow x=0,\frac{\pi}{2},\pi$



The curves are intersecting at $\Rightarrow x=0,\frac{\pi}{2},\pi$

And in $\left(0,\frac{\pi}{2}\right)$, (1) > (2) and in $\left(\frac{\pi}{2},\pi\right)$, (2) > (1)

Therefore required area = $\int_0^{\frac{\pi}{2}} (y \text{ of (1)-y of (2)})dx + \int_{\frac{\pi}{2}}^{\pi} (y \text{ of (2)-y of (1)})dx =$

$$\int_0^{\frac{\pi}{2}} \left(\cos x - 1 + \frac{2x}{\pi}\right)dx + \int_{\frac{\pi}{2}}^{\pi} \left(1 - \frac{2x}{\pi} - \cos x\right)dx$$

$$= \left(\sin x - x + \frac{x^2}{\pi}\right)_0^{\frac{\pi}{2}} + \left(x - \frac{x^2}{\pi} - \sin x\right)_{\frac{\pi}{2}}^{\pi} = 2 - \frac{\pi}{2}$$

2. y = cos x , y = sin2x , x = 0, $x = \frac{\pi}{2}$

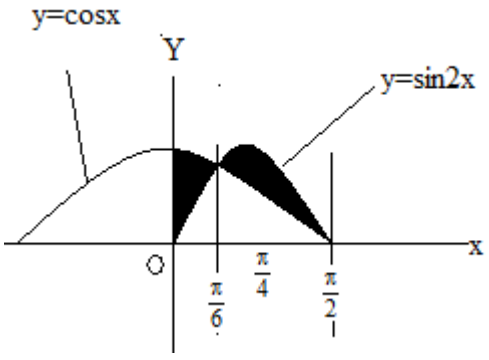
Sol: Given curves y = cos x ----(1) and y = sin 2x ----(2)

solving (1) and (2) , cosx = sin2x

cosx-2sinx cosx=0 $\Rightarrow$ cosx=0 and 1-2sinx =0

$\Rightarrow x = \frac{\pi}{2}, \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

Given curves are intersecting at x= $\frac{\pi}{2}, \frac{\pi}{6}$



Required area = $\int_0^{\frac{\pi}{6}} (\cos x - \sin 2x)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x)dx$

$$= \left(\sin x + \frac{\cos 2x}{2}\right)_0^{\frac{\pi}{6}} + \left(-\frac{\cos 2x}{2} - \sin x\right)_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$=\left[\left(\frac{1}{2}+\frac{1}{4}\right)-\left(0+\frac{1}{2}\right)\right]-\left[\left(\frac{1}{2}-1\right)-\left(-\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$=\frac{1}{2}+\frac{1}{4}-\frac{1}{2}-\frac{1}{2}+\frac{1}{4}+\frac{1}{2}=\frac{1}{2}\text{sq.units}$$

3. $\mathbf{y=x^3+3,\ y=0, x=-1, x=2}$

Sol: Given curve $\mathbf{y=x^3+3,\ y=0, x=-1, x=2}$

Given curve is continuous in [-1.2] and y>0.

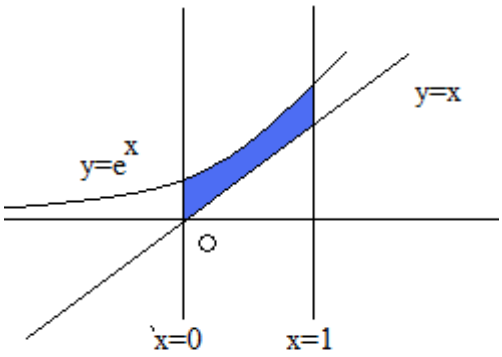
Area bounded by $\mathbf{y=x^3+3,\ x-axis, x=-1, x=2}$ is

$$\int_{-1}^{2} y\,dx=\int_{-1}^{2}\left(x^3+3\right)dx=\left(\frac{x^4}{4}+3x\right)_{-1}^{2}$$

$$=\left(\frac{2^2}{4}+3.2\right)-\left(\frac{(-1)^4}{4}+3(-1)\right)=12\frac{3}{4}\text{sq. units}$$

4. $\mathbf{y=e^x, y=x, x=0, x=1}$

Sol: Given curve is $\mathbf{y=e^x}$ and Lines are y =x, x=0 and x=1.



Required area = $\int_0^1\left(e^x-x\right)dx=\left(e^x-\frac{x^2}{2}\right)_0^1$

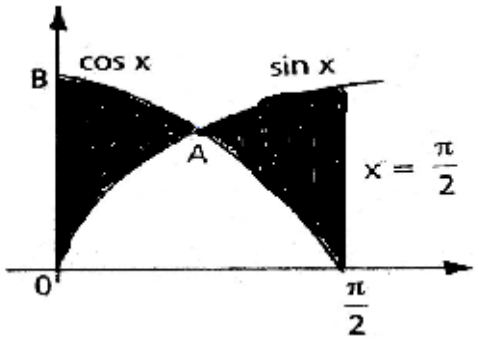
$$=\left(e-\frac{1}{2}\right)-(1-0)=e-\frac{3}{2}$$

5. y = sin x, y = cos x ; x = 0 , $x = \frac{\pi}{2}$

Sol. Given curves y = sin x----(1) and y = cos x -------(2)

from (1) and (2) , cosx = sinx $\Rightarrow x = \frac{\pi}{4}$

Therefore given curves intersecting at $x = \frac{\pi}{4}$ in the given interval.



In (0 , $\frac{\pi}{4}$) , Cos x > sin x and ($\frac{\pi}{4}$, $\frac{\pi}{2}$), Cos x < sin x , therefore

Required area = $\int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$

= $(\sin x + \cos x)_0^{\pi/4} - (\sin x + \cos x)_{\pi/4}^{\pi/2}$

= $\left(\sqrt{2}-1\right) + \left(\sqrt{2}-1\right) = 2\left(\sqrt{2}-1\right)$ sq.units .

6. $x = 4 - y^2$, x = 0. ?

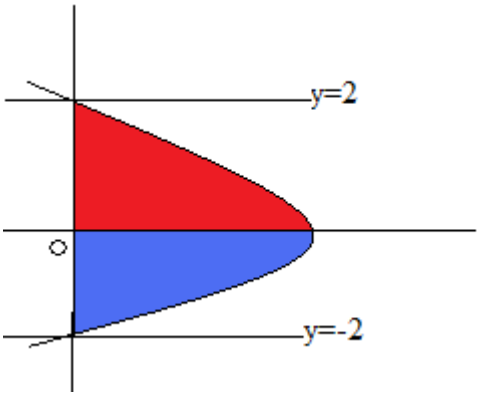
Sol: Given curve is $x = 4 - y^2$---(1)

Put y=0 then x=4.

The parabola $x = 4 - y^2$ meets the x – axis at A(4,0).

Require area = Region AQPA

Since the parabola is symmetrical about X – axis,



Required area = 2 Area of OAPO $= 2\int_0^2 (4-y^2)\,dy = 2\left(4y-\frac{y^3}{3}\right)_0^2$

$= 2\left(8-\frac{8}{3}\right) = 2.\frac{16}{3} = \frac{32}{3}$ sq.units

II.

1. $x = 2 - 5y - 3y^2$, $x = 0$.

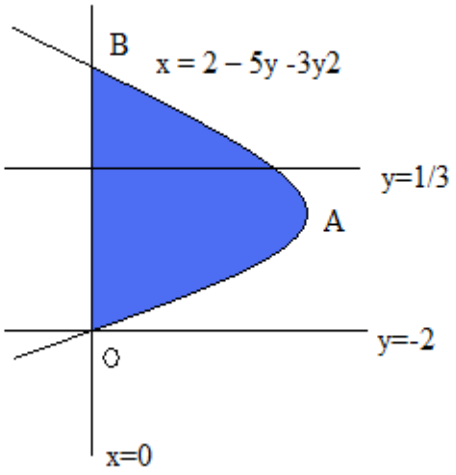
Sol:

Given curve $x = 2 - 5y - 3y^2$ and $x = 0$

Solving the equations, $2-5y - 3y^2 = 0$,

$\Rightarrow 3y^2 + 5y - 2 = 0$

$\Rightarrow (y + 2)(3y - 1) = 0 \Rightarrow y = -2$ or $\frac{1}{3}$



Required area =area OABO = $\int_{-2}^{\frac{1}{3}}\left(2-5y-3y^2\right)dy$

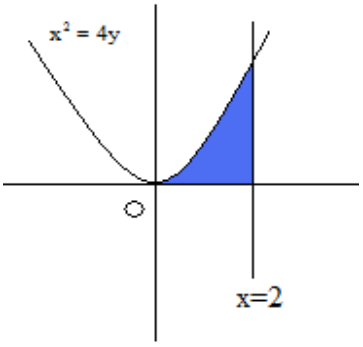
$= \left(2y-\frac{5}{2}y^2-y^3\right)_{-2}^{\frac{1}{3}} = \left(\frac{2}{3}-\frac{5}{2}.\frac{1}{9}-\frac{1}{27}\right)-\left(-4-\frac{5}{2}.4+8\right)$

$= \left(\frac{2}{3}-\frac{5}{8}-\frac{1}{27}\right)+6 = \frac{36-15-2+324}{54}=\frac{343}{54}$ sq. units

2. $x^2=4y,\ x=2\ , y=0$

Sol. Given curve $x^2=4y$,

And the lines are X=2 and y=0 i.e., x axis



Required curve = $\int_0^2 y\,dx=\int_0^2\frac{x^2}{4}dx=$

$\left(\frac{x^3}{12}\right)_0^2=\frac{8}{12}=\frac{2}{3}$ sq. units.

3. $y^2=3x, x=3$**.**

Given curve is $y^2=3x$ and the line is $x=3$

Required area = $\int_0^2 (2x - x^2)\,dx = \left(x^2 - \frac{x^3}{3} \right)_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$ sq. units

5. y = sin 2x , y = $\sqrt{3}$ sin x, x =0, $x = \frac{\pi}{6}$.

Sol; Given equations are y = sin 2x _____(1)

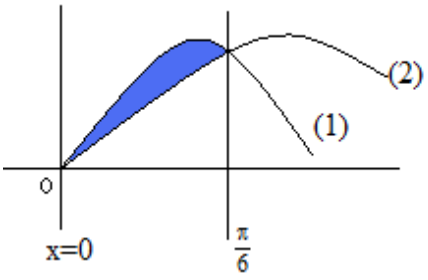
$y = \sqrt{3} \sin x$ _____(2)

Solving Sin2x = $\sqrt{3} \sin x$

$\Rightarrow 2\sin x.\cos x = \sqrt{3} \sin x \Rightarrow \text{Sin}x = 0$ or $2 \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow$ x = 0, $\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}$

Given curves are intersection at x = 0 and $x = \frac{\pi}{6}$



Required area = $\int_0^{\frac{\pi}{6}} (\sin 2x - \sqrt{3} \sin x)\,dx = \left(-\frac{\cos 2x}{2} + \sqrt{3} \cos x \right)_0^{\frac{\pi}{6}}$

$= \left(-\frac{1}{4} + \sqrt{3}.\frac{\sqrt{3}}{2} \right) - \left(-\frac{1}{2} + \sqrt{3} \right) = -\frac{1}{4} + \frac{3}{2} + \frac{1}{2} - \sqrt{3} = \frac{7}{4} - \sqrt{3}$ sq. units

6. $y = x^2, y = x^3$.

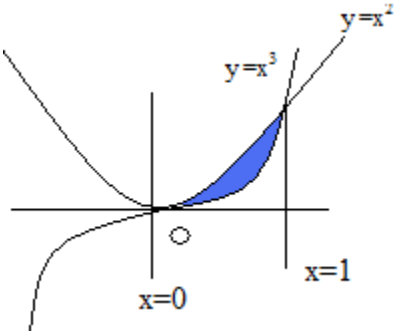
Sol: Given equations are $y = x^2$ _____(1)

$y = x^3$ _____(2)

From equation (1) and (2) $x^2 = x^3$

$\Rightarrow x^3 - x^2 = 0,\ x^2(x-1) = 0$

$\Rightarrow x = 0$ or 1



Given curves are intersection at x = 0 and x = 2

Required area = $\int_0^1 (x^2 - x^3)\,dx$

$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$ sq. units

7. $y = 4x - x^2,\ y = 5 - 2x$.

Sol:

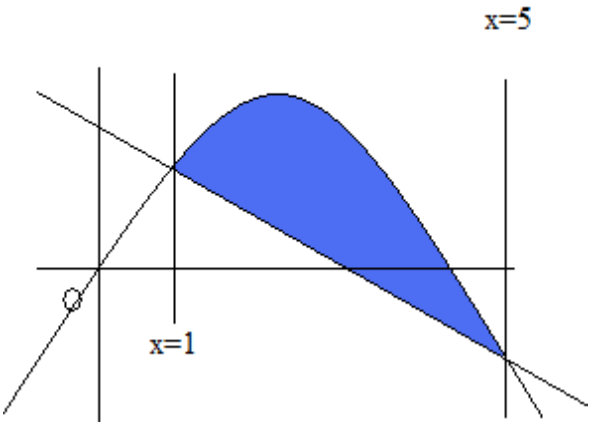
Given curves $y = 4x - x^2$ _____ (i)

$y = 5 - 2x$ _____ (ii)

Solving (i) and (ii) we get

$4x - x^2 = 5 - 2x,\ x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$, X = 1, 5



Required area $= \int_1^5 (\text{yof}(1) - \text{yof}(2))\,dx = \int_1^5 (4x - x^2 - 5 + 2x)\,dx$

$= \int_1^5 (6x - x^2 - 5)\,dx = \int_1^5 \left(3x^2 - \frac{x^3}{3} - 5x\right)_1^5$

$= \left(75 - \frac{125}{3} - 25\right) - \left(3 - \frac{1}{3} - 5\right) = 50 - \frac{125}{3} + 2 + \frac{1}{3}$

$= \frac{150 - 125 + 6 + 1}{3} = \frac{32}{3}\text{sq.units}$

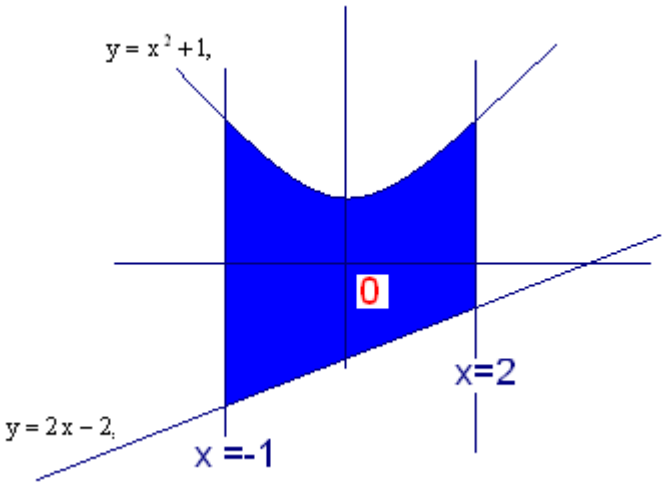
III.

1. $y = x^2 + 1, y = 2x - 2, x = -1, x = 2$ **.**

Sol: Equation of the curves are

$y = x^2 + 1$ _____ (1)

$y = 2x - 2$ _____(2)



Area between the given curves $= \int_{-1}^{2}\left(f(x)-g(x)\right)dx$

$= \int_{-1}^{2}\left[\left(x^2-1\right)-\left(2x-2\right)\right]dx = \int_{-1}^{2}\left(x^2-2x+3\right)dx$

$= \left(\frac{8}{3}-4+6\right)-\left(-\frac{1}{3}-1-3\right)$

$\frac{8}{3}+2+4+\frac{1}{3}=3+6=9\,\text{sq.units}.$

2. $y^2=4x,\ y^2=4(4-x)$

Sol: Equation of the curves are

$y^2 = 4x$ ____(1)

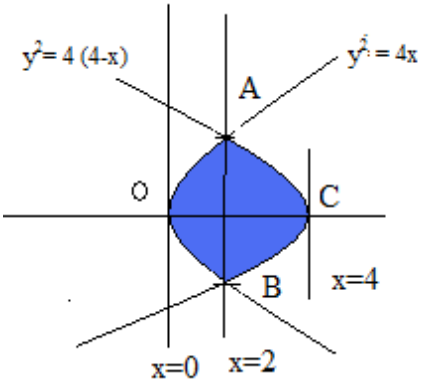
$y^2= 4\ (4-x)$ _____(2)

solving , we get

$4x= 4\ (4-x) \Rightarrow 2x = 4 \Rightarrow x = 2$

$y=0 \Rightarrow x=0$ and $x=4$

Given curves intersects at x=2 and those curves intersect the x axis at x=0 and x=4.



Required area is symmetrical about X – axis,

$$\text{Area OACBO} = 2\left[\int_0^2 2\sqrt{x}\,dx + \int_2^4 2\sqrt{4-x}\,dx\right]$$

$$= 2\left(2\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 + 2\left\{\frac{(4-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right\}_2^4$$

$$= 2\left[\frac{4}{3}\left(2\sqrt{2}\right) - \frac{4}{3}\left(-2\sqrt{2}\right)\right] = 2\left(\frac{8\sqrt{2}}{3} + \frac{8\sqrt{2}}{3}\right)$$

$$= 2\left(\frac{16\sqrt{2}}{3}\right) = \frac{32\sqrt{2}}{3} \text{ sq. units}$$

3. $y = 2 - x^2, y = x^2$

Sol:

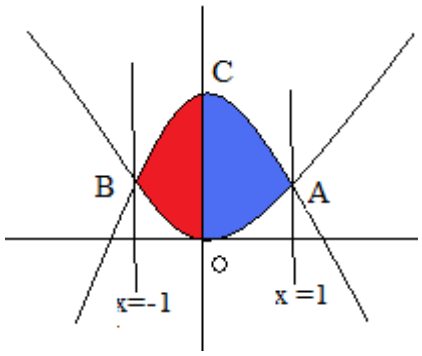
Given curves

$y = 2 - x^2$ _____(1)

$y = x^2$ _____(2)

FROM above equations,

$2 - x^2 = x^2, 2 = 2x^2$ or $x^2 = 1$



$x = \pm 1$

Area bounded by two curves is area ACBOA.

Since the curves are symmetric w.r.t. y axis,

Required area =2. Area OACO

$$2\times\int_{-1}^{1}(y\ of\ (1)-y\ of\ (2))dx = 2\int_{-1}^{1}\left(2-x^2-x^2\right)dx$$

$$=2\int_{-1}^{1}\left(2-2x^2\right)dx = 2\left(2x-\frac{2x^3}{3}\right)_{-1}^{1}$$

$$= \ 2\left[2-\frac{2}{3}\right]=\frac{8}{3}\text{ sq. units}.$$

4. Show that the area enclosed between the curve $y^2=12(x+3)$ and $y^2=20(5-x)$ is $64\sqrt{\frac{5}{3}}$.

Sol: Equation of the curves are $y^2=12(x+3)$ _____(1)

$y^2=20(5-x)$ _____(2)

Solving above equations, we get

12(x+3) = 20(5 – x)

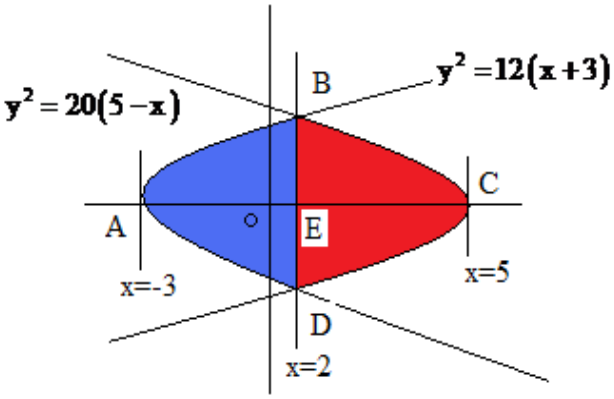
x + 9 = 25 – 5x, 8x = 16 , x = 2

Given curves are intersecting on x=2.

The points of intersection of the curves and the x axis are x=5 and x=-3.

$y^2=12(2+3)=60$

$y = \sqrt{60} = \pm\ 2\sqrt{15}$



The required area is symmetrical about X – axis

Required area =2x(AREA ABCOA) = 2.(AREA ABEA + AREA BECB)

$$= 2\left[\int_{-3}^{2} 2\sqrt{3}\sqrt{x+3}\,dx + \int_{2}^{5} 2\sqrt{5}\sqrt{5-x}\,dx\right]$$

$$= 4\sqrt{3}\left(\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right)_{-3}^{2} + 4\sqrt{5}\left(\frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right)_{2}^{3}$$

$$= \frac{8\sqrt{3}}{3}\left(\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right)_{-3}^{2} + 4\sqrt{5}\left(\frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right)_{2}^{5}$$

$$= \frac{8\sqrt{3}}{3}\left(5^{\frac{3}{2}} - 0\right) - \frac{8\sqrt{5}}{3}\left[0 - 3^{\frac{3}{2}}\right]$$

$$= \frac{8\sqrt{3}}{3}.5\sqrt{5} + \frac{8\sqrt{5}}{3}\left[0 - 3^{\frac{3}{2}}\right]$$

$$= \frac{40.\sqrt{15}}{3} + \frac{24\sqrt{15}}{3} = \frac{64}{3}\sqrt{15}\ \text{sq.units}$$

$= 64\sqrt{\frac{15}{9}}$ sq. units $= 64\sqrt{\frac{5}{3}}$ sq. units .

5. Find the area of the region $\left\{(x,y)/x^2-x-1\le y\le -1\right\}$

Sol. Let the curves be $y=x^2-x-1$ ------(1)

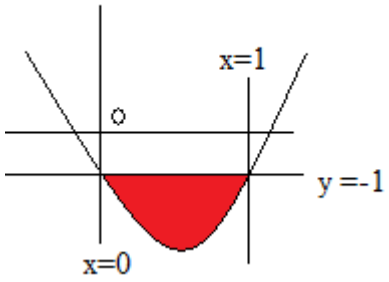
And $y=-1$ --------(2)

The curve $y=x^2-x-1$ represents a parabola with vertex $\left(\frac{1}{2},-\frac{5}{4}\right)$,which lies in 4th quadrant.

From (1) and (2),

$x^2-x-1=-1\Rightarrow x^2-x=0\Rightarrow x=0,x=1$

Given curves are intersecting at x=0 and x=1.



Required area = $\int_0^1 \left(y \text{ of } (1)-y \text{ of } (2)\right)dx$

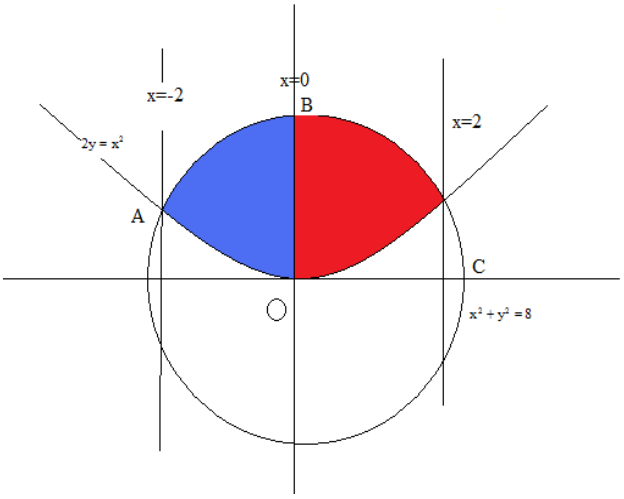
$A=\left|\int_0^1 (x^2-x-1)\,dx-\int_0^1 (-1)\,dx\right| = \left|\int_0^1 \left(\frac{x^3}{3}-\frac{x^2}{2}-x\right)-{}_0^1[-x]\right| = \frac{1}{6}$ sq.units

6. The circle $x^2 - y^2 = 8$ is divided into two parts by the parabola $2y = x^2$. Find the area of both the parts.

Sol: Equations of the curves are $x^2 + y^2 = 8$ _____ (1)

$2y = x^2$ _____(2)

Eliminating Y between equations (1) and (2)



Let $x^2 = t$, $4t + t^2 = 32$, $t^2 + 4t - 32 = 0$

$(t + 8)(t - 4) = 0 \Rightarrow t = -8$ or $t = 4$.

$x^2 = -8$ (not possible) $x^2 = 4 \Rightarrow x = \pm 2$

Given curves are intersecting at x=2 and x=-2.

$$\text{AREA OBCO} = \int_0^2 \sqrt{8 - x^2}\, dx - \int_0^2 \frac{x^2}{2}\, dx$$

$$= \left[\frac{1}{2} \times . \sqrt{8 - x^2} + \frac{8}{2} \sin^{-1} \frac{x}{2\sqrt{2}}\right]_0^2 - \left[\frac{x^3}{6}\right]_0^2$$

$$= \frac{1}{2}.2.2 + 4.\frac{\pi}{4} - \frac{8}{6} = \frac{2}{3} + \pi$$

As curve is symmetric about Y – axis, total area ABCOA= 2. OBCO

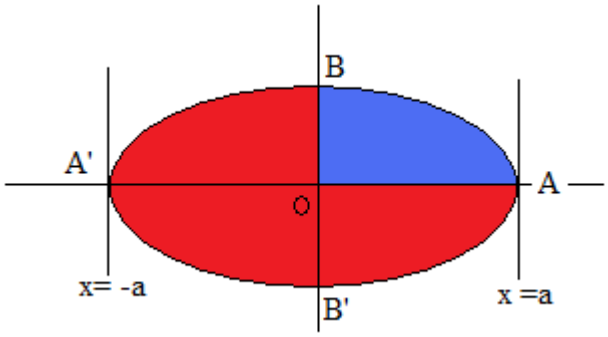
$$= 2\left(\frac{2}{3} + \pi\right) = \frac{4}{3} + 2\pi \text{ sq. units}.$$

AREA of the circle = $\pi r^2 = 8\pi$

Reaming part = $8\pi - \left(\frac{4}{3} + 2\pi\right) = \left(6\pi - \frac{4}{3}\right)$sq. units .

7. Show that the area of the region bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$(ellipse) is π ab . Also deduce the area of the circle $x^2 + y^2 = a^2$.

Sol:



Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow y = \frac{b}{a}\sqrt{a^2 - x^2}$

Put y=o, then x = a or x=-a i.e., given curve is intersecting x axis at x = a, x=-a.

Since the ellipse is symmetrical about X and Y axis, area of the ellipse is equal to 4 times area of the region bounded by the curve , x axis, and the lines x =0 and x=a . (4X area in 1st quadrant)

Area OABO = $\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\ dX$

$= \frac{b}{a}\left(\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)_0^a$

$= \frac{b}{a}\left(0 + \frac{a^2}{2}.\frac{\pi}{2} - ab\right) = \frac{\pi a^2}{4}.\frac{b}{a} = \frac{\pi}{4}ab$

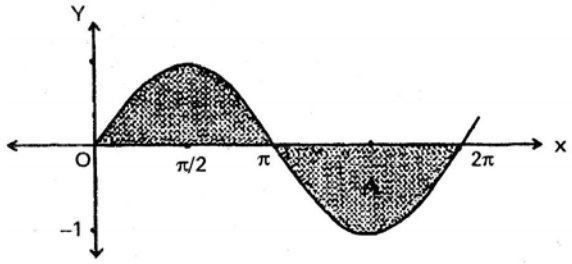
But area of the ellipse = 4X area of OABO = 4. $\frac{\pi}{4}ab = \pi ab$

Substituting b = a, we get the circle $x^2 + y^2 = a^2$

Area of the circle = $\pi a(a) = \pi a^2$ sq. units.

PROBLEMS FOR PRACTICE

1. Find the area under the curve f(x) = sin x in (0,2 π)



Sol:

f(x) = sin x ,

we know that in $[0,\pi], \sin x \geq 0$ and $[\pi, 2\pi], \sin x \leq 0$

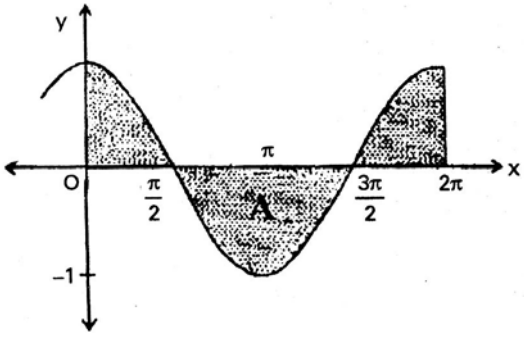
Required area $\int_1^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} (-\sin x)\,dx = (-\cos x)_0^{\pi} + [\cos x]_{\pi}^{2\pi}$

$= -\cos\pi + \cos 0 + \cos 2\pi - \cos\pi$

$= -(-1) + 1 + 1 - (-1) = 1 + 1 + 1 + 1 = 4$

2. Find the area under the curve f(x) = cos x in [0,2 π].

Sol: We know that cos $x \geq 0$ in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, \pi\right)$ and ≤ 0 in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



Required area = $\int_0^{\frac{\pi}{2}} \cos x\,dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos x)\,dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\,dx$

$= (\sin x)_0^{\pi/2} + (-\sin x)_{\pi/2}^{3\pi/2} + (\sin x)_{3\pi/2}^{2\pi}$

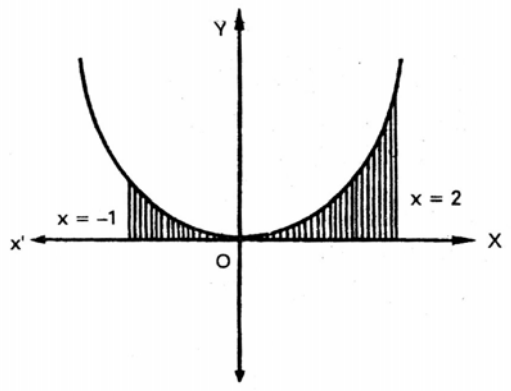
$= \sin\frac{\pi}{2} - \sin 0 - \sin\frac{3\pi}{2} + \sin\frac{\pi}{2} + \sin 2\pi - \sin\frac{3\pi}{2}$

$= 1 - 0 - (-1) + 1 + 0 - (-1)$

$= 1 + 1 + 1 + 1 = 4.$

3. Find the area bounded by the parabola $y = x^2$, the X–axis and the lines x =-1, x =2

Sol:



Required area = $\int_{-1}^{2} x^2 dx = \left(\frac{x^3}{3}\right)_{-1}^{2} = \frac{1}{3}\left(2^3 - (-1)^3\right) = \frac{1}{3}(8+1) = \frac{9}{3} = 3$

4. Find the area cut off between the line y=0 and the parabola $y = x^2 - 4x + 3$.

Sol:

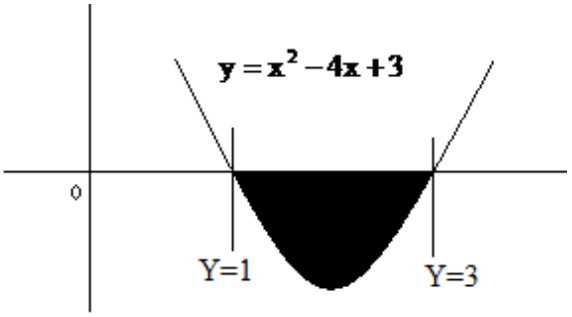
Equation of the parabola is $y = x^2 - 4x + 3$

Equation of the line is y = 0

$x^2 - 4x + 3 = 0$, $(x-1)(x-3) = 0$, x = 1,3

The curve takes negative values for the values of x between 1 and

3.



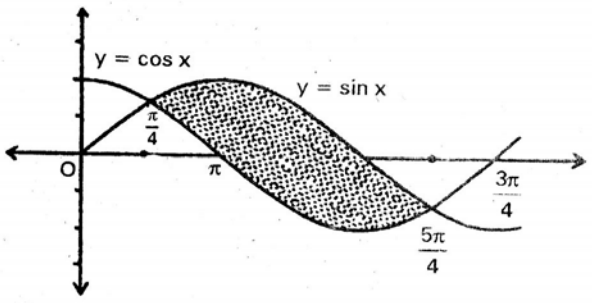
Required area $= \int_1^3 -\left(x^2-4x+3\right)dx$

$$= \int_1^3 \left(-x^2+4x-3\right)dx = \left(-\frac{x^3}{3}+2x^2-3x\right)_1^3$$

$$= \left(-9+18-9\right)-\left(-\frac{1}{3}+2-3\right) = \frac{10}{2}-2 = \frac{4}{3}$$

5. Find the area bounded by y = sin x and y = cos x between any two consecutive points of intersection.

Sol:



Two consecutive points of intersection are $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$

AND $\sin x \geq \cos x$ for all $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

Required area $= \int_{\pi/4}^{5\pi/4} \left(\sin x-\cos x\right)dx = \left(-\cos x-\sin x\right)_{\pi/4}^{5\pi/4}$

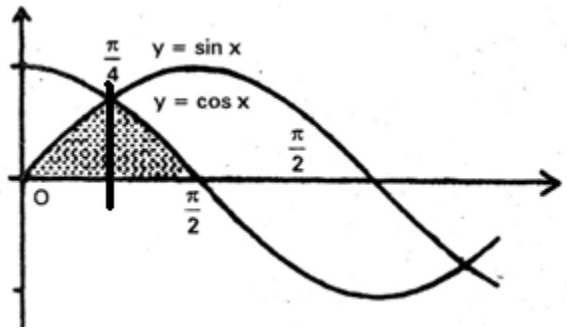
$$= \left(-\cos\frac{5\pi}{4}-\sin\frac{5\pi}{4}\right)+\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} = 4\frac{1}{\sqrt{2}} = 2\sqrt{2}$$

6. Find the area of one of the curvilinear triangles bounded by y = sin x , y = cos x and X – axis.

Sol: Given curves y = sinx and y = cos x.

Solving these curves sinx= cosx $\Rightarrow x=\frac{\pi}{4}$

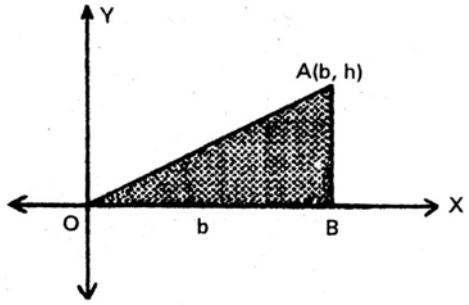


In $\left(0,\frac{\pi}{4}\right)\cos x \geq \sin x$ and $\left(\frac{\pi}{4},\frac{\pi}{2}\right), \cos x \leq \sin x$.

Required area = $\int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx = \left(-\cos x\right)_0^{\pi/4} + \left(\sin x\right)_{\pi/4}^{\pi/2}$

$$= -\cos\frac{\pi}{4}+\cos 0+\sin\frac{\pi}{2}-\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}+1+1-\frac{1}{\sqrt{2}} = 2\left(1-\frac{1}{\sqrt{2}}\right) = 2-\sqrt{2}$$

7. Find the area of the right angled triangle with base b and altitude h, using the fundamental theorem of integral calculus.



Sol:

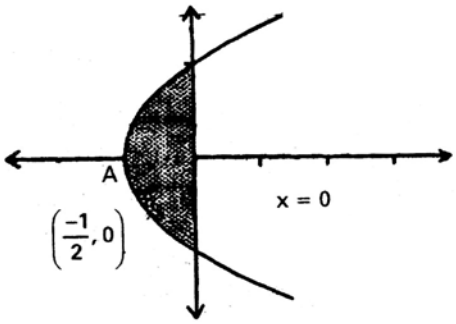
OAB is a right angled triangle and $\angle B = 90°$. Let 'O' as the origin and OB as positive X – axis

If OB = b and AB = h, then co – ordinates of A are (b, h)

Equation of OA is $y = \frac{h}{b}x$

Area of the triangle OAB = $\int_0^b \frac{h}{b}\,x\,dx = \frac{h}{b}\left(\frac{x^2}{2}\right)_0^b = \frac{h}{b}\,\frac{b^2}{2} = \frac{1}{2}\,bh$.

8. Find the area bounded between the curves $y^2 - 1 = 2x$ **and** $x = 0$



Sol:

Given curve $y^2 - 1 = 2x$.

Put x =0, then y = 1 and y=-1

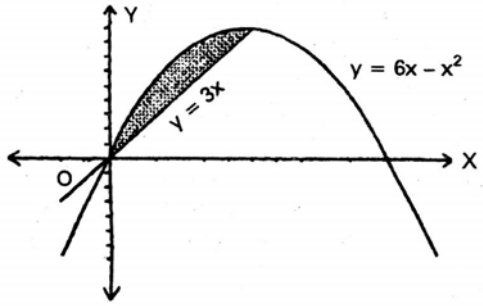
The curve meets y – axis at y = 1 andy =-1.

The curve is symmetrical about X – axis Required area $= \int_{-1}^{1}(-x)\,dy = \int_{-1}^{1} -\left(\frac{y^2-1}{2}\right)dy$

$$= \int_0^1 -\left(y^2 - 1\right) dy = \left(-\frac{y^3}{3} + y \right)_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$$

9. Find the area enclosed by the curves y = 3 and y = 6x- x^2.

Sol:



Given curve is y = 6x- x^2 and the line is y =3x.

Solving these equations $3x = 6x - x^2 \Rightarrow x = 0$ or 3

Given curves are intersecting at x = 0 or 3

Required area = $\int_0^3 \left(6x - x^2 - 3x\right) dx = \int_0^3 \left(3x - x^2\right) dx = \left(\frac{3x^2}{2} - \frac{x^3}{3} \right)_0^3$

$= \frac{27}{2} - \frac{27}{3} = \frac{27}{6} = \frac{9}{2}$.

10. Find the area enclosed between $y = x^2 - 5x$ and y = 4-2x.

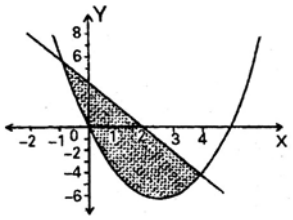
Sol: Given curves are

$y = x^2 - 5x$ …….(1)

y = 4-2x …….(2)

solving, $x^2 - 5x = 4 - 2x$, $x^2 - 5x = 4 - 2x$

$x^2 - 3x - 4 = 0 \Rightarrow (x+1)(x-4) = 0 \Rightarrow x = -1, 4$



Given curves are intersecting at x =-1,4 .

Required area $\int_{-1}^{4}\left[(4-2x)-\left(x^2-5x\right)\right]dx$

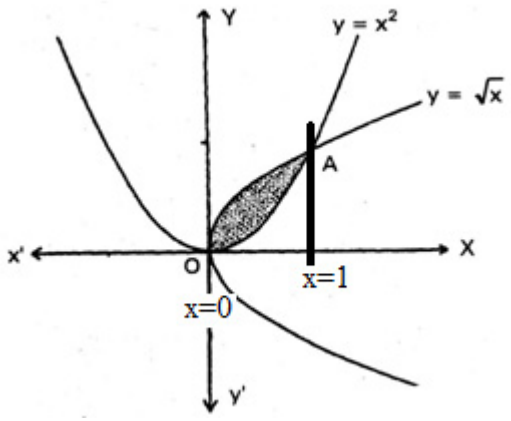
$= \int_{-1}^{4}\left(4+3x-x^2\right)dx = \left(4x+\frac{3}{2}x^2-\frac{x^3}{3}\right)_{-1}^{4}$

$= \left(16+\frac{3}{2}16-\frac{64}{3}\right)-\left(-4+\frac{3}{2}+\frac{1}{3}\right) = 16+24-\frac{64}{3}+4-\frac{3}{2}-\frac{1}{3}$

$= 44-\frac{64}{3}-\frac{3}{2}-\frac{1}{3} = \frac{264-128-9-2}{6} = \frac{125}{6}$

11. Find the area bounded between the curves $y = x^2,\ y = \sqrt{x}$ **.**

Sol:



Given curves are $y = \sqrt{x}$ ……(1) and $y = x^2$ ……(2)

Solving , $\sqrt{x} = x^2 \Rightarrow x^4 = x$

$x\left(x^3-1\right) = 0$, x=0 or x =1

$\therefore$ The curves intersect at x=0 and x =1.

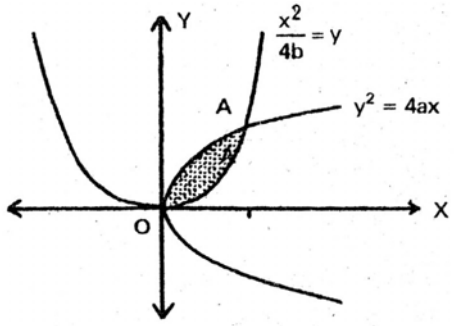
Required area = $= \int_0^1 \left(\sqrt{x} - x^2\right) dx = \left(\frac{2}{3} \times \sqrt{x} - \frac{x^3}{3}\right)_0^1 - \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

12. Find the area bounded between the curves $y^2 - 4ax,\ x^2 = 4by\left(a > 0, b > 0\right)$**.**

Sol: Given curves are $y^2 = 4ax$ …….(1) and $x^2 = 4by$ ……..(2)

From (2), $y = \frac{x^2}{4b}$ Substituting in (1), we get $\left(\frac{x^2}{4b}\right)^2 = 4ax$

$x^4 = \left(16b^2\right) | 4ax |$



$x\left[x^3 - 64b^2a\right] = 0$

X = 0, x = 4 $\left(b^2a\right)^{1/3}$

Therefore given curves are intersecting at X = 0, x = 4 $\left(b^2a\right)^{1/3}$

Required Area = $\int_0^{4\left(b^2a\right)^{1/3}} \left[\sqrt{4ax} - \frac{x^2}{4b}\right] dx$

$= {}_0^{4\left(b^2a\right)^{1/3}} \left[\left(4a\right)^{1/2} x^{3/2} . \frac{2}{3} - \frac{x^3}{12b}\right]$

$= \left[\left(4a\right)^{1/2} 8\left(b^2a\right)^{\frac{1}{3}\cdot\frac{3}{2}} \frac{2}{3} - \frac{4^3\left(b^2a\right)^{3.\frac{1}{3}}}{12b}\right]$

$$= \left[2ab\frac{16}{3} - \frac{64.b^2a}{12b} \right] = ab\left(\frac{32}{3} - \frac{16}{3} \right)$$

$$= \frac{16}{3} \text{ ab sq.units}$$