EXERCISE – 8(F)

I. Evaluate the following integrals.

1. $\int e^x(1+x^2)dx$

Sol. $\int e^x(1+x^2)dx = \int e^x dx + \int x^2 e^x dx$

$= e^x + (x^2 \cdot e^x - 2\int x \cdot e^x dx)$

$= e^x + x^2 \cdot e^x - 2(x \cdot e^x - \int e^x dx)$

$= e^x + x^2 \cdot e^x - 2x \cdot e^x + 2e^x + C$

$= e^x(x^2 - 2x + 3) + C$

2. $\int x^2 e^{-3x} dx$

Sol. $\int x^2 e^{-3x} dx = \frac{x^2 e^{-3x}}{-3} + \frac{1}{3}\int e^{-3x} \cdot 2x\, dx$

$= -\frac{x^2 e^{-3x}}{3} + \frac{2}{3}\left(\frac{xe^{-3x}}{-3} + \frac{1}{3}\int e^{-3x} dx\right)$

$= -\frac{x^2 e^{-3x}}{3} - \frac{2}{9}x \cdot e^{-3x} - \frac{2}{27}e^{-3x} + C$

$= \frac{-e^{-3x}}{27}\left(9x^2 + 6x + 2\right) + C$

3. $\int x^3 e^{ax} dx$

Sol. $\int x^3 e^{ax} dx = \frac{x^3 e^{ax}}{a} - \frac{1}{a}\int e^{ax}(3x^2\, dx)$

$= \frac{x^3 e^{ax}}{a} - \frac{3}{a}\int x^2 e^{ax} dx$

$= \frac{x^3 e^{ax}}{a} - \frac{3}{a}\left(\frac{x^2 e^{ax}}{a} - \frac{1}{a}\int e^{ax} 2x\, dx\right)$

$= \frac{x^3 e^{ax}}{a} - \frac{3}{a^2} \cdot x^2 \cdot e^{ax} + \frac{6}{a^2}\int x \cdot e^{ax} dx$

$= \frac{x^3 e^{ax}}{a} - \frac{3x^2 e^{ax}}{a^2} + \frac{6}{a^2}\left(\frac{xe^{ax}}{a} - \frac{1}{a}\int e^{ax} \cdot dx\right)$

$$= \frac{x^3 e^{ax}}{a} - \frac{3x^2 e^{ax}}{a^2} + \frac{6x \cdot e^{ax}}{a^3} - \frac{6}{a^4} e^{ax} + C$$

$$= \frac{e^{ax}}{a^4}\left[a^3x^3 - 3a^2x^2 + 6ax - 6\right] + C$$

II.

1. Show that

$$\int x^n \cdot e^{-x} dx = -x^n e^{-x} + n \int x^{n-1} \cdot e^{-x} dx$$

Sol. $\int x^n \cdot e^{-x} dx = \frac{x^n e^{-x}}{(-1)} + \int e^{-x} \cdot n x^{n-1} dx$

$$= -x^n e^{-x} + n \int x^{n-1} \cdot e^{-x} dx$$

2. If $I_n = \int \cos^n x \, dx$ **, then show that** $I_n = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$ **.**

Sol. $I_n = \int \cos^n x \, dx = = \int \cos^{n-1} x \cos x \, dx$

$$= \cos^{n-1} x \sin x - \int \sin x (n-1) \cos^{n-2} x (-\sin x) dx$$

$$= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x (1 - \cos^2 x) dx$$

$$= \cos^{n-1} x \sin x + (n-1) I_{n-2} - (n-1) I_n$$

$$\therefore I_n (1 + n - 1) = \cos^{n-1} x \cdot \sin x + (n-1) I_n$$

$$I_n = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} I_{n-2}$$

III.

1. Obtain reduction formula for $I_n = \int \cot^n x \, dx$ **, n being a positive integer, n ≥ 2 and deduce the value of** $\int \cot^4 x \, dx$ **.**

Sol. $I_n = \int \cot^n x \, dx = \int \cot^{n-2} x \cdot \cot^2 x \, dx$

$$= \int \cot^{n-2} x \cdot (\csc^2 x - 1) dx$$

$$= \int \cot^{n-2} x \cdot \csc^2 x dx - I_{n-2}$$

$$= -\frac{\cot^{n-1} x}{n-1} - I_{n-2}$$

$$n = 4 \Rightarrow I_4 = -\frac{\cot^3 x}{3} - I_2$$

$n = 2 \Rightarrow I_2 = -\cot x - I_0$ **where** $I_0 = \int dx = x$

$$I_2 = -\cot x - x$$

$$I_4 = -\frac{\cot^3 x}{3} - (-\cot x - x) + C \qquad = -\frac{\cot^3 x}{3} + \cot x + x + C$$

2. Obtain the reduction formula for $I_n = \int \csc^n x\,dx$, n being a positive integer, n ≥ 2 and deduce the value of $\int \cos ec^5 x\,dx$.

Sol. $I_n = \int \csc^n x\,dx = \int \csc^{n-2} x \cdot \csc^2 x\,dx$

$$= \csc^{n-2} x(-\cot x) + \int \cot x(n-2)\csc^{n-3} x(\cot x)dx$$

$$= -\csc^{n-2} x \cot x + (n-2)\int \csc^{n-2} x(\csc^2 x - 1)dx$$

$$= -\csc^{n-2} x \cot x + (n-2)I_{n-2} - (n-2)I_n$$

$$I_n(1 + n - 2) = -\csc^{n-2} x \cdot \cot x + (n-2)I_{n-2}$$

$$I_n = \frac{-\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

$$n = 5 \Rightarrow I_5 = -\frac{\csc^3 x \cdot \cot x}{4} + \frac{3}{4} I_3$$

$$I_3 = -\frac{\csc x \cdot \cot x}{2} + \frac{1}{2} I_1$$

$$I_1 = \int \csc x\,dx = \log\left|\tan\frac{x}{2}\right|$$

$$I_3 = -\frac{\csc x \cdot \cot x}{2} + \frac{1}{2}\log\left|\tan\frac{x}{2}\right|$$

$$I_5 = -\frac{\csc^3 x \cdot \cot x}{4} - \frac{3}{8}\csc x \cot x + \frac{3}{8}\log\left|\tan\frac{x}{2}\right| + C$$

3. If $I_{m,n} = \int \sin^m x \cos^n x\,dx$, then show that $I_{m,n} = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} I_{m-2}, n$ for a positive integer n and an integer m ≥ 2.

Sol. $I_{m,n} = \int \sin^m x \cos^n x\,dx$

$$= \int \sin^{m-1} x \cdot (\cos x)^n \sin x\,dx$$

$$= \int \sin^{m-1}(x)(\cos x)^n(-\sin x)dx$$

$$= -\left[\sin^{m-1}(x)\int(\cos x)^n(-\sin x)dx - \int\left\{\frac{d}{dx}\sin^{m-1}(x)\cdot\int\cos^n(x)(-\sin x)dx\right\}dx\right]$$

$$= -\left[\sin^{m-1}(x)\frac{\cos^{n+1}(x)}{n+1} - \int\left\{(m-1)\sin^{m-2}(x)\cos x\frac{\cos^{n+1}x}{n+1}\right\}dx \right]$$

$$= -\sin^{m-1}(x)\frac{\cos^{n+1}(x)}{n+1} + \frac{m-1}{n+1}\int\{\sin^{m-2}(x)\cos^{n}x\cos^{2}x\}dx$$

$$= -\frac{\sin^{m-1}(x)\cos^{n+1}(x)}{n+1} + \left(\frac{m-1}{n+1}\right)\int\{\sin^{m-2}(x)\cos^{n}x - \sin^{m}(x)\cos^{n}(x)\}dx$$

$$= -\frac{\sin^{m-1}(x)\cos^{n+1}(x)}{n+1} + \left(\frac{m-1}{n+1}\right)\left[\int\sin^{m-2}(x)\cos^{n}xdx - \int\sin^{m}(x)\cos^{n}(x)dx\right]$$

$$= -\frac{\sin^{m-1}(x)\cos^{n+1}(x)}{n+1} + \left(\frac{m-1}{n+1}\right)I_{m-2,n} - \left(\frac{m-1}{n+1}\right)I_{m,n}$$

$$\therefore I_{m,n} + \left(\frac{m-1}{n+1}\right)I_{m,n} = -\frac{\sin^{m-1}(x)\cos^{n+1}(x)}{n+1} + \left(\frac{m-1}{n+1}\right)I_{m-2,n}$$

$$\Rightarrow \left(1+\frac{m-1}{n+1}\right)I_{m,n} = -\frac{\sin^{m-1}(x)\cos^{n+1}(x)}{n+1} + \left(\frac{m-1}{n+1}\right)I_{m-2,n}$$

$$\therefore \left(\frac{m+n}{n+1}\right)I_{m,n} = -\frac{\sin^{m-1}(x)\cos^{n+1}(x)}{n+1} + \left(\frac{m-1}{n+1}\right)I_{m-2,n}$$

$$\therefore I_{m,n} = \frac{1}{m+n}(\sin^{m-1}(x)\cos^{n+1}(x)) + \left(\frac{m-1}{n+1}\right)I_{m-2,n}$$

4. Evaluate $\int\sin^{5}x\cos^{4}xdx$ **.**

Sol. Reduction formula

$$I_{m,n}\frac{-\sin^{m-1}x\cdot\cos^{n+1}x}{m+n} + \frac{m-1}{m+n}I_{m-2,n}$$

$$I_{5,4} = -\frac{\sin^{4}x\cos^{5}x}{9} + \frac{4}{9}I_{3,4}$$

$$I_{3,4} = -\frac{\sin^{2}x\cos^{5}x}{7} + \frac{2}{7}I_{1,4}$$

$$I_{1,4} = \int\sin x\cos^{4}x\,dx = -\frac{\cos^{5}x}{5}$$

$$I_{3,4} = -\frac{\sin^{2}x\cos^{5}x}{7} - \frac{2}{35}\cos^{5}x$$

$$I_{5,4} = -\frac{\sin^{4}x\cos^{5}x}{9} + \frac{4}{9}\left(-\frac{\sin^{2}x\cos^{5}x}{7} - \frac{2}{35}\cos^{5}x\right) + C$$

$$= -\frac{\sin^4 x \cos^5 x}{9} - \frac{4}{63}\sin^2 x \cdot \cos^5 x - \frac{8}{315}\cos^5 x + C$$

PROBLEMS FOR PRACTICE

1. Find $\int 2x^7 dx$ on R.

Ans. $\frac{x^8}{4} + C$

2. Evaluate $\int \cot^2 x\, dx$ on $I \subset R \setminus \{n\pi : n \in Z\}$.

Ans. –cot x – x + C

3. Evaluate $\int \frac{x^6 - 1}{1 + x^2} dx$ for $x \in R$.

Ans. $\frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1} x + C$

4. Find $\int (1-x)(4-3x)(3+2x)dx, x \in R$.

Ans. $\frac{3}{2}x^4 - \frac{5}{3}x^3 - \frac{13}{2}x^2 + 12x + C$

5. Evaluate $\int \left(x + \frac{1}{x}\right)^3 dx, x > 0$.

Ans. $\frac{x^4}{4} + \frac{3x^2}{2} + 3\log x - \frac{1}{2x^2} + C$

6. Find $\int \sqrt{1 + \sin 2x}\, dx$ on R.

Ans. cos x – sin x + C

7. Evaluate $\int \frac{2x^3 - 3x + 5}{2x^2} dx$ for x > 0 and check the result by differentiation.

Ans. $\frac{x^2}{2} - \frac{3}{2}\log x - \frac{5}{2x} + C$

8. Evaluate $\int \frac{x^5}{1+x^{12}}dx$ **on R.**

Ans. $\frac{1}{6}\tan^{-1}x^6 + C$

9. Evaluate $\int \cos^3 x \sin x\, dx$ **on R.**

Ans. $\frac{-\cos^4 x}{4} + C$

10. Find $\int\left(1-\frac{1}{x^2}\right)e^{\left(x+\frac{1}{x}\right)}dx$ **on I where I = (0, ∞)**

Ans. $e^{\left(x+\frac{1}{x}\right)} + C$

11. Evaluate $\int \frac{1}{\sqrt{\sin^{-1}x}\sqrt{1-x^2}}dx$ **on I = (0, 1).**

Ans. $2\sqrt{\sin^{-1}x} + C$

12. Evaluate $\int \frac{\sin^4 x}{\cos^6 x}dx, x \in I \subset R \setminus \left\{\frac{(2n+1)\pi}{2} : n \in Z\right\}$

Ans. $\frac{1}{5}\tan^5 x + C$

13. Evaluate $\int \sin^2 x\, dx$ **on R.**

Ans. $\frac{1}{2}\sin 2x + C$

14. Evaluate $\int \frac{1}{a\sin x + b\cos x}dx$ **where a, b ∈ and $a^2 + b^2 \neq 0$ on r.**

Sol. We can find real numbers r and θ such that

$a = \cos\theta$, $b = r\sin\theta$

Then $r = \sqrt{a^2+b^2}, \cos\theta = \frac{a}{r}$ and $\sin\theta = \frac{b}{r}$

$a\sin x + b\cos x = r\cdot\cos\theta\sin x + r\sin\theta\cos x$

$= r[\cos\theta\sin x + \sin\theta\cos x] = r\sin(x+\theta)$

$\int \frac{1}{a\sin x + b\cos x}dx = \frac{1}{r}\int \frac{1}{\sin(x+\theta)}dx$

$$= \frac{1}{r}(\csc(x+\theta))dx = \frac{1}{r}\log\left|\tan\frac{1}{2}(x+\theta)\right| + C$$

$$= \frac{1}{\sqrt{a^2+b^2}}\log\left|\tan\frac{1}{2}(x+\theta)\right| + C$$

For all $x \in$ I where I is an internal disjoint with $\{n\pi - \theta : n \in Z\}$.

15. Find $\int \frac{x^2}{\sqrt{x+5}}dx$ on (–5, ∞)

Ans. $\frac{2}{5}(x+5)^{5/2} - \frac{20}{3}(x+5)^{3/2} + 50\sqrt{x+5} + C$

16. Find $\int \frac{x}{\sqrt{1-x}}dx,\ x \in I = (0,1)$.

Ans. $\frac{2}{3}(1-x)^{3/2} - 2\sqrt{1-x} + C$

17. Evaluate $\int \frac{dx}{(x+5)\sqrt{x+4}}$ on (–4, ∞)

Ans. $2\tan^{-1}(\sqrt{x+4}) + C$

18. Evaluate $\int \frac{dx}{\sqrt{4-9x^2}}$ on $I = \left(-\frac{2}{3}, \frac{2}{3}\right)$

Ans. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$

19. Evaluate $\int \frac{1}{a^2-x^2}dx$ for $x \in I = (-a, a)$.

Ans. $\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

20. Evaluate $\int \frac{1}{1+4x^2}dx$ on R.

Ans. $\frac{1}{2}\tan^{-1}2x + C$

21. Find $\int \frac{1}{\sqrt{4-x^2}}dx$ on (–2, 2).

Ans. $\sin^{-1}\left(\frac{x}{2}\right) + C$

22. Evaluate $\int\sqrt{4x^2+9}\,dx$ **on R.**

Ans. $\frac{x\sqrt{4x^2+9}}{2}+\frac{9}{4}\sinh^{-1}\left(\frac{2x}{3}\right)+C$

23. Evaluate $\int\sqrt{9x^2-25}\,dx$ on $\left(\frac{5}{3},\infty\right)$.

Ans. $\frac{x\sqrt{9x^2-25}}{2}-\frac{25}{6}\cosh^{-1}\left(\frac{3x}{5}\right)+C$

24. Evaluate $\int\sqrt{16-25x^2}dx$ on $\left(\frac{-4}{5},\frac{4}{5}\right)$.

Ans. $\frac{x\sqrt{16-25x^2}}{2}+\frac{8}{5}\sin^{-1}\left(\frac{5x}{4}\right)+C$

25. Evaluate $\int x\sin^{-1}x\,dx$ **on (–1, 1).**

Ans. $\frac{x^2\sin^{-1}x}{2}-\frac{1}{4}(\sin^{-1}x-x\sqrt{1-x^2})+C$

26. Evaluate $\int x^2\cos x\,dx$

Ans. $(x^2-2)\sin x+2x\cos x+C$

27. Evaluate $\int e^x\sin x\,dx$ **on R.**

Ans. $\frac{1}{2}e^x(\sin x-\cos x)+C$

28. Find $\int e^{ax}\cos(bx+c)dx$ **on R where a, b, c are real numbers and b ≠ 0.**

Sol. Let A = $\int e^{ax}\cos(bx+c)dx$

Then from the formula for integration by parts

$$A=e^{ax}\left[\frac{\sin(bx+c)}{b}\right]-\int ae^{ax}\left[\frac{\sin(bx+c)}{b}\right]dx$$

$$=\frac{1}{b}e^{ax}\sin(bx+c)-\frac{a}{b}\int e^{ax}\sin(bx+c)dx$$

$$=\frac{1}{b}e^{ax}\sin(bx+c)-\frac{a}{b}\left[e^{ax}\left\{\frac{-\cos(bx+c)}{b}\right\}-\int ae^{ax}\left\{-\frac{\cos(bx+c)}{b}\right\}dx\right]+C_1$$

$$=\frac{1}{b}e^{ax}\sin(bx+c)+\frac{a}{b^2}e^{ax}\cos(bx+c)s-\frac{a^2}{b^2}A+C_2$$

$$\left(1+\frac{a^2}{b^2}\right)A=\frac{a}{b^2}e^{ax}\cos(bx+c)+\frac{1}{b}e^{ax}\sin(bx+c)+C_2$$

$(a^2+b^2)A = ae^{ax}\cos(bx+c) + be^{ax}\sin(bx+c) + C_3$

Hence $A = \frac{e^{ax}}{a^2+b^2}[a\cos(bx+c) + b\sin(bx+c)] + K$

where $k = \frac{c_3}{a^2+b^2}$ **a constant**

By taking c = 0, we get

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx] + K$$

29. Evaluate $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$ **on (–1, 1).**

Sol. Put x = cos θ, θ ∈ (0, π)dx = –sin θ dθ

$$\frac{1-x}{1+x} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{2\sin^2\theta/2}{2\cos^2\theta/2} = \tan^2\frac{\theta}{2}$$

$$\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx = \int \tan^{-1}\sqrt{\tan^2\frac{\theta}{2}}(-\sin\theta)d\theta$$

$$= -\int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(\sin\theta)d\theta$$

$$= -\frac{1}{2}\int \theta\cdot\sin\theta\,d\theta$$

$$= -\frac{1}{2}\left[\theta(-\cos\theta) - \int(-\cos\theta)d\theta\right] + C$$

$$= \frac{1}{2}(\theta\cos\theta - \sin\theta) + C$$

$$= \frac{1}{2}\left(x\cos^{-1}x - \sqrt{1-x^2}\right) + C$$

30. Evaluate $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ **on I ⊂ R \ {2nπ : n ∈ Z}.**

Sol. $\frac{1-\sin x}{1-\cos x} = \frac{1-\sin x}{2\sin^2 x/2}$

$$= \frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} = \frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}$$

$$= \frac{1}{2}\csc^2\frac{x}{2} - \cot\frac{x}{2}$$

$$\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx = \int e^x\left(\frac{1}{2}\csc^2\frac{x}{2}-\cot\frac{x}{2}\right)dx$$

$$= \int e^x\left[f(x)+f'(x)\right]dx \text{ where } f(x) = -\cot\frac{x}{2}$$

$$= e^x f(x)+C = -e^x\cot\frac{x}{2}+C$$

31. Evaluate $\int\tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$ **on I $\subset$ R \ (–1, 1).**

Sol. Let x = tan θ $\Rightarrow$ dx = sec² θ dθ

$$\frac{2x}{1-x^2} = \frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta) = 2\theta+n\pi$$

where n = 0 if |x| < 1

= –1 if x > 1

= 1 if x < –1

We have $d\theta = \frac{1}{1+x^2}dx$ **and**

$1+x^2 = 1+\tan^2\theta = \sec^2\theta$

$$\therefore \int\tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int\left(\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)(1+x^2)\frac{1}{1+x^2}dx$$

$$= \int(2\theta+n\pi)\int\sec^2\theta d\theta$$

$$= 2\int\theta\sec^2\theta d\theta + n\pi\int\sec^2\theta d\theta + c$$

$$= 2\left(\theta\tan\theta - \int\tan\theta d\theta\right) + n\pi\tan\theta + c$$

$$= 2(\theta\tan\theta + \log|\cos\theta| + n\pi\tan\theta + c$$

$$= (2\theta+n\pi)\tan\theta + 2\log\cos\theta + c$$

$$= (2\theta+n\pi)\tan\theta + \log\cos^2\theta + c$$

$$= (2\theta+n\pi)\tan\theta + \log\sec^2\theta + c$$

$$= x\tan^{-1}\left(\frac{2x}{1-x^2}\right) - \log(1+x^2) + c$$

32. Find $\int x^2 \frac{\exp(m\sin^{-1}x)}{\sqrt{1-x^2}}dx$ **on (–1, 1) where m is a real number. (Here for Y ∈ R, exp.(y) stands for e^y).**

Sol. Let t = $\sin^{-1}$ x, then

$x = \sin t, dt = \frac{1}{\sqrt{1-x^2}}dx$ **, for x ∈ (–1, 1)**

Hence $\int x^2 \frac{\exp(m\sin^{-1}x)}{\sqrt{1-x^2}}dx = \int e^{mt}\sin^2 t dt$

$= \int e^{mt}\left(\frac{1-\cos 2t}{2}\right)dt$

$= \frac{1}{2}\int e^{mt}dt - \frac{1}{2}\int e^{mt}\cdot\cos 2t\,dt + c$ **…(1)**

Case (i) : m = 0

From (1) $\int x^2 \frac{\exp(m\sin^{-1}x)}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int dt - \frac{1}{2}\int \cos 2t\,dt + C$

$= \frac{t}{2} - \frac{\sin 2t}{4} + C$

$= \frac{\sin^{-1}x}{2} - \frac{1}{4}\sin(2\sin^{-1}x) + C$

Case (ii) : m ≠ 0

From (1) $\int x^2 \frac{\exp(m\sin^{-1}x)}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\frac{e^{mt}}{m} - \frac{1}{2}\frac{e^{mt}}{m^2+4}(m\cos 2t + 2\sin 2t) + C_1$

$= \frac{e^{mt}}{2}\left(\frac{1}{m} - \frac{1}{m^2+4}(m\cos 2t + 2\sin 2t)\right) + C_1$

$= \frac{e^{m\sin^{-1}x}}{2}\left(\frac{1}{m} - \frac{1}{m^2+4}(m\cos(2\sin^{-1}x) + 2\sin(2\sin^{-1}x)\right) + C_1$

33. Evaluate $\frac{dx}{4x^2-4x-7}$

Ans. $\frac{1}{8\sqrt{2}}\log\left|\frac{2x-1-2\sqrt{2}}{2x-1+2\sqrt{2}}\right| + C$

34. Find $\int \frac{dx}{5-2x^2+4x}$.

Ans. $\frac{1}{2\sqrt{14}} \log \left| \frac{x-1+\sqrt{\frac{7}{2}}}{x-1-\sqrt{\frac{7}{2}}} \right| + C$

35. Evaluate $\int \frac{dx}{x^2+x+1}$

Ans. $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

36. Evaluate $\int \frac{dx}{\sqrt{x^2+2x+10}}$

Ans. $\sinh^{-1}\left(\frac{x+1}{3}\right) + C$

37. Evaluate $\int \frac{dx}{\sqrt{1+x-x^2}}$

Ans. $\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) + C$

38. Evaluate $\int \sqrt{3+8x-3x^2}\,dx$

Ans. $\frac{(3x-4)\sqrt{3+8x-3x^2}}{6} + \frac{25}{6\sqrt{3}} \sin^{-1}\left(\frac{3x-4}{5}\right) + C$

39. Evaluate $\int \frac{x+1}{x^2+3x+12}\,dx$

Ans. $\frac{1}{2}\log|x^2+3x+12| - \frac{1}{\sqrt{39}} \tan^{-1}\left(\frac{2x+3}{\sqrt{39}}\right) + C$

40. Evaluate $\int (3x-2)\sqrt{2x^2-x+1}\,dx$

Ans. $\sqrt{\left(x-\frac{1}{4}\right)^2+\frac{7}{16}} - \frac{35}{64\sqrt{2}} \sinh^{-1}\left(\frac{4x-1}{\sqrt{7}}\right) + C$

41. Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}}dx$

Ans. $2\sqrt{x^2-2x+10}+7\sinh^{-1}\left(\frac{x-1}{3}\right)+C$

42. Evaluate $\int \frac{dx}{5+4\cos x}$

Ans. $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+C$

43. Find $\int \frac{dx}{3\cos x+4\sin x+6}$

Ans. $\frac{2}{\sqrt{11}}\tan^{-1}\left(\frac{3\tan(x/2)+4}{\sqrt{11}}\right)+C$

44. Evaluate $\int \frac{\cos x+3\sin x+7}{\cos x+\sin x+1}dx$

Ans. $-\log|\cos x+\sin x+1|+2x+5\log\left|1+\tan\frac{x}{2}\right|+C$

45. Find $\int \frac{x^3-2x+3}{x^2+x-2}dx$

Ans. $\frac{(x-1)^2}{2}+\log|c(x+2)^{1/3}(x-1)^{2/3}+C$

46. Find $\frac{dx}{x^2-81}$

Ans. $\frac{1}{18}\log\left|\frac{x-9}{x+9}\right|+C$

47. Find $\int \frac{2x^2-5x+1}{x^2(x^2-1)}dx$

Ans. $\frac{1}{x}+\log\left|\frac{x^5}{(x^2-1)(x+1)^3}\right|+C$

48. Find $\int \frac{3x-5}{x(x^2+2x+4)}dx$

Ans. $-\frac{5}{4}\log|x|+\frac{5}{8}\log|x^2+2x+4|+\frac{17}{4\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)+C$

49. $\int \frac{2x+1}{x(x^2+4)^2}\,dx$

Ans. $\frac{1}{16}\log|x| - \frac{1}{32}\log(x^2+4) + \frac{1}{8(x^2+4)} + \frac{1}{8}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{4}\left(\frac{x}{4+x^2}\right) + C$

50. Evaluate $x^3 \cdot e^{5x} dx$ **.**

Ans. $\frac{x^3 \cdot e^{5x}}{5} - \frac{3}{25}x^2 \cdot e^{5x} + \frac{6}{125}x \cdot e^{5x} - \frac{6}{625}e^{5x} + C$

51. Evaluate $\int \sin^4 x dx$

Ans. $-\frac{\sin^3 x \cdot \cos x}{4} - \frac{3}{8}\sin x \cdot \cos x + \frac{3}{8} + C$

52. $\int \sec^5 x\,dx$

Ans. $\frac{\sec^3 x \cdot \tan x}{4} + \frac{3}{8}\sec x \tan x + \frac{3}{8}\log|\sec x + \tan x| + C$