TRASMISSION OF HEAT
CONDUCTION

POINTS TO REMEMBER

1. Transfer of heat takes place in three ways. These are conduction, convection and radiation.
2. It is the process of transfer of heat which takes place with out the movement of the particles of the medium from one region to another region.
3. Conduction of heat is possible in matter by transfer of energy among the molecules through collisions.
4. Conduction mainly occurs in solids.
5. Conduction is the slowest process.
6. Under steady state the temperature gradient (temperature difference per unit length) and rate of flow of heat through the rod are constant. $\frac{t_2 - t_1}{l}$ = Constant and $\frac{Q}{t}$ = constant
7. The amount of heat (Q) flowing through a rod is directly proportional to (1) the area of cross section (A) of the rod (2) temperature difference $(\theta_1 - \theta_2)$ between the ends of the rod (3) the time (t) of flow the heat through the rod and (4) is inversely proportional to the length (l) of the rod.
$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$
8. $\frac{l}{KA}$ = Thermal resistance (K-sec/J)
9. The process of transfer of heat from hotter part of the medium to the colder part of the medium by the bodily movement of the particles of the medium is called convection.
10. Convection of is of two types:
    1) **Natural convection** takes place in a still fluid. In natural convection heating is done from the bottom and cooling is done from the top.
    2) **Forced convection** which takes place in a stream of a fluid. In forced convection heat transfer may take place in any direction.
11. The rate of heat convection is proportional to 1) temperature difference between object and the fluid (2) area of contact $\frac{Q}{t} = hA(\theta_1 - \theta_2)$

SHORT ANSWER QUESTIONS

1. Explain thermal conductivity and coefficient of thermal conductivity.

Ans : Thermal conductivity

Definition: The ability to conduct heat in solids is called thermal conductivity

Explanation: When one end of a metal rod is heated, the atoms of that end gain heat energy and the amplitude of vibration of the atoms at the hot end increases. These atoms collide with the adjacent atoms by sharing their energy with them. Thus heat flows from higher temperature end to lower temperature end of the rod.

Coefficient of thermal conductivity (K) :

The amount of heat (Q) flowing through a rod is directly proportional to (1) the area of cross section (A) of the rod (2) temperature difference $(\theta_1 - \theta_2)$ between

the ends of the rod (3) the time (t) of flow the heat through the rod and (4) is inversely proportional to the length (l) of the rod. $Q = \frac{KA(\theta_1 - \theta_2)t}{l}$

where k is called the co-efficient of thermal conductivity of the rod.

Definition of K: $K = \frac{Q/t}{A(\theta_1 - \theta_2)/l}$ *If* $A = 1$ *and* $\frac{(\theta_1 - \theta_2)}{l}$ $, then K = \frac{Q}{t}$

It is defined as the quantity of heat flowing normally per second through unit area of the substance per unit temperature gradient.

2. Would loss of heat be reduced or increased by increasing the air gap between the double paned windows than the usual necessary gap. Explain (consider convection)?

Ans : When air gap between the double paned window is increased convection currents are set up are increased which transfer heat energy. Hence loss of heat is increased with the increase in the air gap. That means more heat flows through the window from high temperature to low temperature.

VERY SHORT ANSWER QUESTIONS

1. State the different modes of transmission of heat. Which of these modes require medium?

Ans : Transfer of heat takes place in three ways. These are conduction, convection and radiation. Out of modes conduction and convection require a medium.

2. Define coefficient of thermal conductivity and temperature gradient.?

Ans : Definition of K : $K = \frac{Q/t}{A(\theta_1 - \theta_2)/l}$ *If* $A = 1$ *and* $\frac{(\theta_1 - \theta_2)}{l}$ $, then K = \frac{Q}{t}$

It is defined as the quantity of heat flowing normally per second through unit area of the substance per unit temperature gradient.

Temperature gradient: Under steady state, the temperature difference per unit length of a metal rod is known as temperature gradient.

$\frac{t_2 - t_1}{l}$ = Constant

3. What is thermal resistance of a conductor? On what factors does it depend?

Ans : Thermal resistance = $\frac{l}{KA}$ (K-sec/J)

Thermal resistance of a conductor depends on the thermal conductivity of the material of the rod and its dimensions.

4. State the units and dimensions of the co-efficient convection.?

Ans : The rate of heat convection is proportional to 1) temperature difference between object and the fluid (2) area of contact $\frac{Q}{t} = hA(\theta_1 - \theta_2)$

Where h is the convection co-efficient which depends on density, viscosity, specific heat thermal conductivity etc.

Units: Watt./ m^2 / K Dimensions : $h = ML^{-1}T^{-3}K^{-1}$

Exercise 1

1. **A room has a window fixed with a pane of area 1.2 m^2. The glass has thickness 2.2 mm. If the temperature outside the room is** 36^0C **and the temperature inside is** 26^0C **(a) calculate the heat flowing into the room every hour. (b) If the same single pane window is replaced by a double paned window with air gap of 0.50 cm between the two panes calculate the heat flowing into the room every hour. For glass** $K = 0.80 Wm^{-1}K^{-1}, K_{air} = 0.0234 Wm^{-1}K^{-1}$

Sol: (a) $\theta_2 - \theta_1 = 36^0C - 26^0C = 10^0C$

$d = 2.2mm = 2.2\times10^{-3}m \quad ; \quad A = 1.2m^2 \quad ; \quad K_{glass} = 0.80 Wm^{-1}K^{-1}$

But, $$Q = \frac{KA(\theta_2 - \theta_1)t}{d} = \frac{0.8\times1.2\times10\times3600}{2.2\times10^{-3}} = 1.57\times10^7 J$$

(b) For both the glass layers, thermal resistance.

$$R_g = \frac{d_g}{K_g A_g} = \frac{2.2\times10^{-3}}{0.8\times1.2} = 2.29\times10^{-3} K/W$$

For the air gap, $$R_a = \frac{d_a}{K_a A_a} = \frac{0.5\times10^{-2}}{0.0234\times1.2} = 178\times10^{-3} K/W$$

Total thermal resistance

$$R_r = R_g + R_a + R_g = (2\times2.29 + 178)10^{-3} = 182.6\times10^{-3} K/W$$

Now, $$Q = \frac{KA(\theta_2 - \theta_1)t}{d} = \frac{\theta \Delta t}{R_r} = \frac{10\times3600}{182.6\times10^{-3}} = 1.98\times10^5 J$$

2. **A runner moves along the road at in still air that is at a temperature of** 29.0^0C**. His surface area is** $1.4m^2$**, of which approximately 85% is exposed to the air. Find the rate of convective heat loss from his skin at a temperature** 35.0^0C **to the outside air? Coefficient of convection for dry air and bare skin at wind speed 22** $W/m^{2\ 0}C$

Sol: $\frac{Q}{t} = h\ A\ \Delta\theta$.

$h = 22 W/m^{2\ 0}C; \Delta\theta = 35.0^0C - 29.0^0C = 6.0^{\ 0}C$

A = 85% of the surface area of the number $1.4m^2 = 1.2m^2$

Upon substituting the values

$\frac{Q}{t} = 22\times1.2\times6 \quad 160W$

Exercise 2

1. **A copper bar 2m long has a circular cross-section of radius 1 cm. One end is kept at 100ºC and the other at 0ºC. It is assumed that negligible heat is lost through the surface. (**Coefficient of thermal conductivity of copper = 401 W/mk)

a) Find the thermal resistance of the bar **b) The temperature gradient** $\frac{d\theta}{dx}$

c) The temperature at 25cm from the hot end

Ans: Length of copper bar (l) = 2m

Radius of copper bar (r) = 1 cm = 10^{-2} m

$d\theta = \theta_2 - \theta_1 = 100^0 C$

a) Thermal resistance R = $\frac{l}{KA} = \frac{l}{K\pi r^2} = \frac{2}{(401)\times 3.14\times 10^{-4}}$ = 15.87 K / w

b) Temperature gradient = $\frac{d\theta}{l} = \frac{100}{2}$ = 50 K / m

c) Temperature gradient = $\frac{d\theta}{l}$ = Constant

$\therefore \frac{(100-0)}{200} = \frac{(100-\theta)}{25} \qquad \Rightarrow \theta = 87.5^0 C$

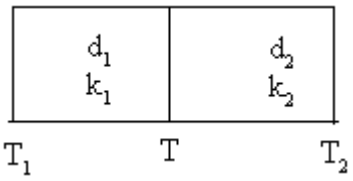
2. Two Metal slabs of thickness d_1 and d_2 and thermal conductivities k_1 and k_2 are in thermal contact with each other. The temperatures of their outer surfaces are T_1 and T_2 respectively. Assume $T_1 > T_2$. Find the temperature (T) of the surface of contacts ?

Ans: At steady state rate of flow of heat through the rod (Q/t) is constant.

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2$$

$$\frac{K_1 A(T_1 - T)}{d_1} = \frac{K_2 A(T - T_2)}{d_2}$$

$$\therefore T = \left(\frac{K_1 T_1 d_2 + K_2 T_2 d_1}{K_1 d_2 + K_2 d_1}\right)$$



3. Two slabs of thermal resistances R_1 and R_2 are placed in parallel, then show that the effective thermal resistance of the system is equal to $\left[\frac{R_1 R_2}{R_1 + R_2}\right]$

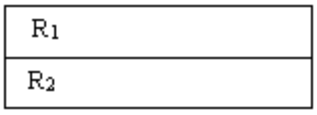
Ans :

Effective rate of heat flow through the parallel combination

$$\left(\frac{Q}{t}\right) = \left(\frac{Q_1}{t}\right) + \left(\frac{Q_2}{t}\right)$$

Or $\frac{\Delta\theta}{R} = \frac{\Delta\theta}{R_1} + \frac{\Delta\theta}{R_2} \Rightarrow \frac{(\theta_1 - \theta_2)}{R} = \frac{\theta_1 - \theta_2}{R_1} + \frac{\theta_1 - \theta_2}{R_2}$

$$\therefore R = \frac{R_1 R_2}{R_1 + R_2}$$

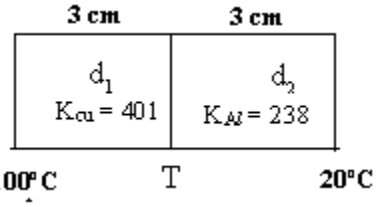


4. Two metal cubes with 3 cm edge of copper and aluminium are arranged in series. Find the (a) thermal resistance of each cube and the total thermal resistance of the system, (b) the temperature T at the surface of contact. The temperatures of outer surfaces of copper and aluminium are 100ºC and 20ºC respectively. (K_{cu} = 401 w/mk and K_{Al} = 238 w/mk)

Ans : (t) = 3cm = 3×10^{-2} m.

(a) Thermal resistance of copper cube i.e. $R_{cu} = \frac{L}{K_{cu}A} = \frac{L}{K_{cu} \times l^2}$

$R_{cu} = \frac{3x10^{-2}}{401x9x10^{-4}} \Rightarrow R_{cu} = 8.3\times10^{-2}\ kw^{-1}$



Thermal resistance of A*l* cube $R_{cu} = \frac{L}{K_{Al}A} = \frac{L}{K_{Al} \times l^2}$

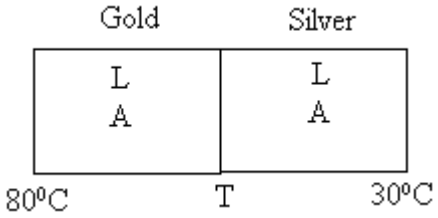
$R_{Al} = \frac{3x10^{-2}}{238x9x10^{-4}} \Rightarrow R_{Al} = 1.4 \times 10^{-1}\ kw^{-1}$

Total thermal resistance of the system when both are in series

$R = R_{cu} + R_{Al} = 8.3\times 10^{-2} + 1.4 \times 10^{-1} = 0.223\ kw^{-1}$

(b) Temperature of the interface $T = \frac{K_1T_1d_2 + K_2T_2d_1}{K_1d_2 + K_2d_1}$ =70.23 **°C**

5. A bar of gold is in thermal contact with a bar of silver of the same length and area. One end of the compound bar is maintained at 80.0 °C, and the opposite end is at 30.0^{0}C when steady state is reached. What is the temperature at the junction?



Ans : K_{Gold} =318 w/mk ; and K_{Silver} = 429 w/mk

Temperature of the interface $T = \frac{K_1T_1d_2 + K_2T_2d_1}{K_1d_2 + K_2d_1}$

= 51.3°C

6. A copper bar of thermal conductivity 401 W / (mK has one end at 104°C and the other end at 24°C. The length of the bar is 0.10 m and the cross-sectional area is 1.0 × 10^{-6} m^2. (a) What is the rate of heat conduction P along the bar? (b) What is the temperature gradient in the bar? (c) If two such bars were placed in series (end to end) between the same temperature baths what would P be? (d) If two such bars were placed in parallel (side by side) with the ends in the same temperature baths what would P be? (e) In the series case, what is the temperature at the junction where the bars meet?

Ans: K = 401 w/mk ; L = 0.1 m ; A = 1 × 10^{-6} m^2 ; $\Delta\theta$ = 104 - 24 = 80°C

(a) Rate of heat conduction (P) = $\left(\frac{Q}{t}\right) = \frac{\Delta\theta}{R}$

Where R = thermal resistance of Cu = $\frac{L}{K_{cu}A} = \frac{0.1}{401x10^{-6}} = 249.3$

$\therefore\ P = \frac{\Delta\theta}{R} = \frac{80}{249.3} = 0.32\,w$

(a) Temperature gradient $\frac{\Delta\theta}{L} = \frac{80}{0.1} = 800\,k/m$

(c) If two such bars placed in series then rate of heat conduction (P) = $\frac{\Delta\theta}{R_{series}}$

$R_{series} = R_1 + R_2 = 2R$

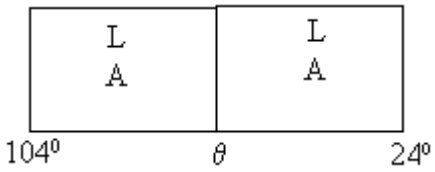
(P) = $\frac{\Delta\theta}{2R} = \frac{1}{2}\left(\frac{\Delta\theta}{R}\right) = \frac{1}{2}(0.32) = 0.16$ w

(d) If two such bars placed in parallel, then rate of heat conduction (P) = $\frac{\Delta\theta}{R_{Parallel}}$

$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(R)(R)}{R + R} = \frac{R}{2}$$

$$P = \frac{\Delta\theta}{R/2} = 2\left(\frac{\Delta\theta}{R}\right) = 2(0.32) = 0.64\,w$$

(e) Both in series temperature at the junction where the bars meet is



Temperature of the interface $T = \frac{K_1 T_1 d_2 + K_2 T_2 d_1}{K_1 d_2 + K_2 d_1} = 64^0 C$

ASSESS YOURSELF:

1. **Why copper is used at the bottom of cooking utensils and why they are provided with wooden handles?**

Ans: Copper is a good conductor of heat. Hence copper is used at the bottom of cooking utensils. Wood has low value of the coefficient thermal conductivity K=0.13, hence a bad conductor of heat to provide it as handles.

2. **Rub one of your palms on a metal surface for some time say 30-40s. Place the other palm on a unrubbed portion of the surface and then rubbed portion of metal surface. The rubbed will fee warmer. Now repeat the same on a wooden surface. You notice that the difference in temperature between the rubbed and un-rubbed portion of the wooden surface is larger than that of metal surface why?**

Ans: Metals are good conductors of heat. Heat generated due to rubbing of palm spreads rapidly from rubbed regions to other regions of the metal surface. Wood on the other hand is a bad conductor. The heat generated at the rubbed region cannot move rapidly to other areas of the wooden surface. Hence rubbed and unrobed regions exhibit a distinct temperature difference.

3. **Which type of cooling is involved in cooling the air in the air coolers?**

Ans: Forced convection