TEMPERATURE AND THERMAL EXPANSIONS
2. EXPANSIONS OF LIQUIDS

POINTS TO REMBER

1. Linear and a real expansion have no significance for a liquid since shape of liquid depends on shape of vessel.
2. A liquid has two expansion i.e., real & apparent as liquid requires vessel for its existence.
   $\gamma_r = \gamma_a + 3\alpha_{container}$
3. The real expansion of a liquid does not depend upon the temperature of the container.
4. The apparent expansion of liquid depends ona) initial volume or liquid, b) rise in temperature) nature of liquid and d) nature of container.
5. The fraction of the volume of a glass flask that must be filled with mercury so that the volume of the empty space left may be the same at all temperatures is 1/7.
6. The density of a liquid usually decreases when heated. If d_1 and d_2 are the densities of a liquid at 0°C and t°C respectively, then

. $d_t = \frac{d_o}{1+\gamma_r t}$ and $\gamma_r = \frac{d_1 - d_2}{d_1 t_2 - d_2 t_1} / {}^o C$

7. $\gamma_{app} = \frac{Mass of\ the\ liquid\ expelled}{Mass of\ the\ remaining\ liquid \times Temp.diff.}$
8. When water at 0°C is heated, its volume decreases up to 4°C and from 4°C its volume increases with the increase of temperature. This peculiar behavior of water is called anomalous expansion of water.
9. Water has maximum density and minimum volume at 4°C.
10. Hope's apparatus is used to demonstrate that water has maximum density at 4°C.
11. Dilatometer is used to prove anomalous expansion of water.
12. Aquatic animals are surviving in cold countries due to the anomalous expansion of water.

LONG ANSWER QUESTIONS

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1. **Define the coefficient of real expansion and apparent expansion of a liquid. Deduce the relation between them. (Mar 2005, May 2005, Mar 2006, Mar 2008)**
   If same liquid is taken in two different containers made of different materials, is the apparent coefficient of expansion same in the two containers. Explain.

Ans. **Coefficient of apparent expansion of a liquid** (γ_a)**:** The coefficient of apparent of liquid is defined as the ratio of apparent increase in the volume to its original volume per 1ºC rise in temperature

$\gamma_a = \frac{v_2 - v_1}{v_1(t_2 - t_1)}$ Units: K^{-1} (or) $^oC^{-1}$

Coefficient of real expansion of liquid (γ_r)

The coefficient of real expansion of liquid is defined as the ratio of real increase in the volume to its original volume per 1ºC rise in temperature

$\gamma_a = \frac{v_2' - v_1'}{v_1'(t_2 - t_1)}$ Units: K^{-1} (or) $^oC^{-1}$

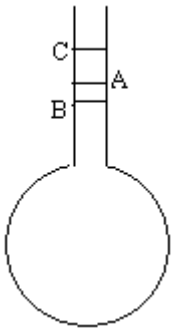
Relation between γ_a **and** γ_r

Consider a round bottomed glass flask with long neck which is filled with liquid of volume 'V' at a temperature t1ºC up to mark 'A'. When the liquid is heated to t_2ºC, glass and liquid expand simultaneously.

Considering the expansion of the flask alone the liquid falls from A to B. Hence 'AB' represents expansion of glass.

$$\gamma_g = \frac{AB \times a}{v(t_2 - t_1)} \Rightarrow AB = \frac{\gamma_g v(t_2 - t_1)}{a} ----(1)$$

Now considering the expansion of the liquid, its level increases from B to C. Hence BC represents real expansion of liquid.



$$\gamma_r = \frac{BC \times a}{v(t_2 - t_1)} \Rightarrow BC = \frac{\gamma_r v(t_2 - t_1)}{a} -----(2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But due to the simultaneous expansion of the liquid and the flask, the liquid level appears to increases from A to C. Hence AC represents the apparent expansion of liquid.

$$\gamma_a = \frac{AC \times a}{v(t_2 - t_1)} \Rightarrow AC = \frac{\gamma_a v(t_2 - t_1)}{a} ---(3)$$

From figure, BC = AB + AC

$$\frac{\gamma_r v(t_2 - t_1)}{a} = \frac{\gamma_g v(t_2 - t_1)}{a} + \frac{\gamma_a v(t_2 - t_1)}{a} \quad \Rightarrow \gamma_r = \gamma_a + \gamma_g \Rightarrow \gamma_r = \gamma_a + 3\alpha$$

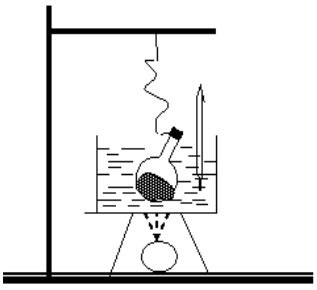
Apparent coefficient of expansion depends on the nature of the material of the container. Hence apparent coefficient of expansion is different in the two containers.

2. **Describe the necessary theory that specific gravity bottle method to determine the coefficient of apparent of expansion. (March2011,Mar 2004, Mar 2005, Mar 2006, May 2007, June 2008)**

Ans. **Procedure**:

Consider an empty, clean and dry specific gravity bottle whose mass 'm_1' is determined by using common balance. This bottle is completely filled with liquid at t_1°C and its mass m_2 is determined. The bottle along with the liquid is held in water bath and heated. As the temperature of the water bath increases, the liquid in the bottle expands and expels out from stopper. The expelled liquid is removed from time to time using a filter paper. Other wise the liquid from the bottle falls into the water bath and changes its temperature. The final temperature of the liquid t_2°C is noted when no more liquid comes out of the bottle. The bottle with the remaining liquid is removed from water bath and cooled to t_1°C and its mass 'm_3' is determined.

Theory:



Mass of empty specific gravity bottle = m_1

Mass of (Sp. gravity bottle + liquid) at $t_1\ ^0C = m_2$

Mass of liquid = $m_2 - m_1$

Volume of the liquid $(v_2) = \frac{m_2 - m_1}{\rho}$ Where ρ = density of liquid

Mass of (Sp. gravity bottle + remaining liquid) = m_3

Mass of remaining liquid = $m_3 - m_1$

Volume of remaining liquid $(v_1) = \frac{m_3 - m_1}{\rho}$

$\therefore$ Apparent expansion of liquid, $\gamma_a = \frac{v_2 - v_1}{v_1(t_2 - t_1)}$

$$\Rightarrow \gamma_a = \frac{\left(\frac{m_2 - m_1}{\rho}\right) - \left(\frac{m_3 - m_1}{\rho}\right)}{\frac{m_3 - m_1}{\rho}(t_2 - t_1)} \qquad \Rightarrow \quad \gamma_a = \frac{m_2 - m_3}{(m_3 - m_1)(t_2 - t_1)}$$

$$\gamma_a = \frac{\text{mass of liquid expelled}}{\text{mass of remaining liquid} \times \text{temp. difference}}$$

Thus the coefficient of apparent expansion (γ_a) of the given liquid can be determined.

SHORT ANSWER QUESTIONS:

1. Distinguish between absolute expansion and apparent expansion of a liquid. State also the relation between their coefficients (Mar 2004, Mar 2005, May 2005, Mar 2008)

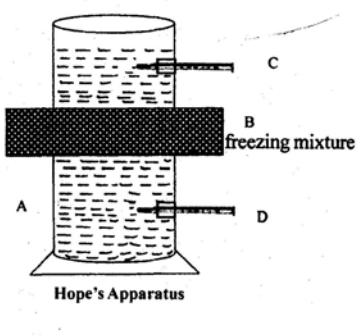
Ans.

Absolute expansion	**Apparent expansion**
1. The coefficient of apparent expansion of liquid is defined as the ratio of apparent increase in its volume to original volume per 1ºC rise in temperature.	1. The coefficient of real expansion of liquid is defined as the ratio of real increase in its volume to original volume per 1ºC rise in temperature.
2.Actual expansion of the liquid is the real expansion	2.Observed expansion of the liquid is the real expansion
3. This does not depend on the expansion in the vessel.	3.This depends on the expansion in the vessel.

Relation between γ_a and γ_r: $\gamma_r = \gamma_a + \gamma_g$ or $\gamma_r = \gamma_a + 3\alpha$

2. How can show that water has maximum density at 4^0C

Ans :



Hope's Apparatus

Hope's apparatus consists of a long tall metal cylinder A provided with two side thermometers C and D near the top and bottom as shown. The cylinder is filled with water. Its central part is

surrounded by a circular trough B which is filled with a freezing mixture of ice and salt. The freezing mixture can produce temperatures below 0ºC.

As the freezing mixture cools the water in the central region of the cylinder, the water becomes cooler and denser. This denser water sinks to the bottom and hence the temperature reading of the thermometer D gradually decreases to 4ºC. But the thermometer C shows almost a constant reading. When the temperature of the central layer falls below 4ºC the density of water decreases i.e., it becomes lighter. So, it rises upwards and cools the water above it. Now the temperature shown by C decreases while the temperature shown by D remains almost constant at the temperature of maximum density of water. The temperature of the water at the top falls rapidly to zero and ice begins to form first at the top of the water. The water at the bottom is still at the temperature of about 4ºC. Hence it can be concluded that the density of water is maximum at 4ºC.

3. In what way is the anomalous behavior of water advantageous to aquatic animals?

Ans. In cold countries, as atmospheric temperature decreases, the temperature of upper layers of the lakes, rivers etc., decreases and sink to the bottom. This goes on until the temperature of water reaches of 4ºC. When the temperature of water reaches of 4ºC, its density becomes maximum. It does not sink downwards and remains at the top. With further cooling the top layers gradually form ice at the surface. Ice and water are bad conductors of heat. So, the lower layers are protected against freezing by the layers of ice and cold water at 1ºC, 2ºC and 3ºC. Hence water at the bottom will be at 4ºC which enables the aquatic animals to live in the lake.

VERY SHORT QUESTIONS

1. Why do liquids have no linear and areal expansions?

Ans. Any liquid requires a container for its existence. Liquids do not have a specific size or shape. Hence the liquids do not have linear areal expansions.

2. What is the necessity to define two coefficient of volume expansion in case of a liquid?

Ans. A liquid is heated in a vessel. Hence if the liquid is heated, along with the liquid vessel also expands. Hence liquids have two coefficient expansions.

1) Co-efficient of real expansion and 2) Co-efficient of apparent expansion.

temperature of water (i) in contact with the lower surface of ice and (ii) at the bottom of the lake?

Ans. 1) -15ºC 2) 4ºC

4. **What is anomalous expansion of water? At what temperature has water maximum density**

(March 2010)

Ans. When water at 0°C is heated, its volume decreases up to 4°C and from 4°C its volume increases with the increase of temperature. This peculiar behavior of water is called anomalous expansion of water. Water has maximum density and minimum volume at 4°C.

Exercise 1

1. A glass bottle with a fine stem, fully immersed in melting ice $0^{o}C$ and completely filled with mercury, contains 300g of mercury. Calculate the amount of mercury that will

overflow if the bottle is kept for sufficiently long time in boiling water at 100^oC **.**

(Coefficient of apparent expansion of mercury is glass= $\frac{1}{6500}/^oC$ **)**

Sol: Mass of the mercury in the glass bottle, m_1=300g

Mass of the liquid expelled, $x=(m_1-m_2)$

Mass of the liquid remaining, $m_2=m_1-x=(300-x)$

Rise in temperature, $(t_2-t_1)=100^oC$ and $\gamma_a=\frac{1}{6500}/^oC$

$$\gamma_a=\frac{m_1-m_2}{m_2(t_2-t_1)} \Rightarrow \frac{1}{6500}=\frac{x}{(300-x)100} \Rightarrow x=4.545g$$

2. The mass of an empty specific gravity bottle is 20g. When it is filled with a liquid at 30^oC **, the mass of the bottle is 60g. When it is heated to** 80^oC **, 2.2g of liquid is expelled. Find the apparent expansion of the liquid.**

Sol: Mass of the empty specific gravity bottle $m_1=20g$

Mass of the specific gravity bottle and liquid at $30^oC, m_2=60g$

Mass expelled = 2.2g.

Mass of the specific gravity bottle with the remaining liquid at 80^oC

$$m_3=60-2.2=57.8\text{g}.$$

$t_1=30^oC$ *and* $t_2=80^oC$

$$\gamma_a=\frac{(m_2-m_1)}{(m_3-m_1)(t_2-t_1)}=\frac{2.2}{37.8(50)}=1.164\times10^{-3}/^oC$$

3. The coefficient of real expansion of mercury is $0.00018/^oC$ **. Find the coefficient of apparent expansion of mercury in glass.** $(\alpha_g=9\times10^{-6}/^oC)$ (March 2011)

Sol: $\gamma_r=0.00018/^oC; \alpha_g=0.000009/^oC; \gamma_g=3\alpha_g=0.000027/^oC.$

$$\gamma_g=\gamma_r-\gamma_g=0.00018-0.000027=0.000153/^oC.$$

4. A block of quartz of mass 13.5g is weighed in aniline at 0^oC *and* t^oC **8.905 and 9.29g respectively. Calculate the co-efficient of cubical expansion of aniline, assuming the coefficient of cubical expansion of quartz to be zero.**

Sol. When a solid substance is immersed in a liquid at 0°C, the loss of wt. is w_0. Now liquid is heated to t°C, loss of wt. in substance is w_t. Then

Loss of weight = weight of liquid displaced.

$w_0=V_0d_0g$ And $w_t=V_td_tg$

$$\frac{w_t}{w_0}=\frac{V_td_t}{V_0d_0}=\frac{V_0(1+\gamma_0t)}{V_0(1+\gamma_Rt)} = (1+\gamma_st)(1+\gamma_Rt)$$

$$w_t=w_0\left[1+(\gamma_s-\gamma_R)t\right]$$

$W_0=(13.5-8.905)\times10^{-3}\times gN=4.595\times10^{-3}\times gN;\ t=0^oC;$

$W_t=(13.5-9.29)\times10^{-3}\times gN=4.21\times10^{-3}\times gN; t_2=100^oC; \gamma=0$

$$\gamma_l=9.166\times10^{-4}/^oC$$

5. How much mercury must be placed inside a glass flask, having an internal volume of 300cc., so that the volume of the remaining space inside the flask may be constant at all temperatures? (Co-efficient of real expansion of mercury= $0.00018/^oC$ **, co-efficient of linear expansion of glass is =** $0.00001/^oC$ **.**

Sol: For the volume of the empty space inside the vessel remains the same at all temperatures the increase in volume of the liquid is equal to the increase in volume of the vessel.

Increase in volume of the liquid = $V_1\gamma_1\Delta t$

Where V_1 the volume of the liquid is, γ_1 is its coefficient of real expansion and Δt is increase in temperature.

Increase in volume of the vessel = $V_2\gamma_2\Delta t$

Where V_2 is the internal volume of the vessel γ_2 is its coefficient of cubical expansion.

$V_1\gamma_1\Delta t = V_2\gamma_2\Delta t$ Or $V_1\gamma_1 = V_2\gamma_2$ Or $V_1\gamma_1 = V_2\times 3\alpha_2$

V_2=300c.c. $\alpha_2 = 0.00001/^oC, \gamma_1 = 0.00018/^oC$;

$$V_1 = \frac{V_2\times 3\alpha_2}{\gamma_1} = \frac{300\times 3\times 0.00001}{0.00018} = 50c.c$$

6. A barometer with brass scale, which is correct at 0^oC **, reads 75.000cm on a day when the air temperature is** 20^oC **. Calculate the correct reading at** 0^oC **. (Coefficient of real expansion of mercury=** $0.00018/^oC$ **and coefficient of linear expansion of brass =** $0.0000189/^oC$ **.**

Sol: Let H represent the observed height of the barometer at t^oC at H_o be the correct height at 0^oC. Let α be the linear expansion of the material of the scale and γ be the coefficient of real expansion of mercury.Let ρ_0 and ρ be the densities of mercury at and respectively.

Correcting for the expansion of the scale, the height will be

$$H(1+\alpha t)\rho g = H_0\rho_0 g$$

$$H(1+\alpha t)\rho = H_0\rho(1+\gamma t)\left(\because \rho_0 = \rho(1+\gamma t)\right)$$

$$H_0 = \frac{H(1+\alpha t)}{(1+\gamma t)} = H(1+\alpha t)(1-\gamma t)\left(\because (1-\gamma t)^{-1}\approx 1-\gamma t\right)$$

$$= H(1-(\gamma-\alpha)t)$$

Here, $H = 75.000cm, t = 20^oC, \gamma = 0.00018/^oC, \alpha = 0.0000189/^oC$

$$H = 75.000\left(1-(0.00018-0.0000189)20\right) = 74.758cm$$

Exercise 2

1. When a specific gravity bottle filled with mercury at 0ºC is heated to 100ºC, 5.190g of mercury overflows and 300g remains in the specific gravity bottle. Using glycerin in place of mercury, 1.439g of glycerin overflows and 27.333g remains. The coefficient of real expansion of mercury is 0.000183 /ºC. Determine (a) The coefficient of linear expansion of the material of the container,

(b) The coefficient of real expansion of glycerin.

Sol : $\gamma_a = \frac{5.190}{300\times 100} = \frac{5.190}{3}\times 10^{-4}$

$\gamma_a = 1.73\times 10^{-4}$,

But, $\gamma_r = \gamma_a + 3\alpha \Rightarrow 0.000183 = 0.000173 + 3\alpha$

$\therefore \alpha = 3.3\times 10^{-6}/^0C$

$$\gamma_a = \frac{1.439}{27.33\times100} = \frac{1.439}{2733} \quad \Rightarrow \quad \gamma_a = 0.0005271$$

$$\gamma_r = \gamma_a + \gamma_g = 0.000527 + 9.9\times10^{-6}$$

$$\therefore \gamma_r = 5.21\times10^{-4}\ /^0 C$$

2. A glass vessel which holds exactly 1000g of mercury at 15ºC is put into boiling water at 100ºC. What mass of mercury is expelled? (Coefficient of mercury is 0.00018 / ºC.; coefficient of linear expansion of glass is 0.00001/ ºC).

Sol: $\gamma_r = \gamma_a + \gamma_g$

$$0.00018 = \gamma_a + 3\times0.00001 \quad \Rightarrow \quad \gamma_a = 0.00015\ /^0 C$$

$$\gamma_a = \frac{x}{(1000-x)\times85} \quad \Rightarrow \quad x = 12.589g$$

3. A barometer with a brass scale correct at 0ºC reads 70cm of mercury on a day when the air temperature is 40ºC. Calculate the correct reading at 0ºC. (Coefficient of real expansion of mercury is 0.00018/ºC and the coefficient of linear expansion of brass is 0.000018/ ºC).

Sol: $\Delta x = R_1.\Delta\alpha.\Delta t$, $\Delta\alpha = 0.00018 - 0.000018 = 162\times10^{-6}$

$\Delta x = 0.4536$

Correct reading = 70 - 0.4536 = 69.55 cm

4. A specific gravity bottle is completely filled by 271.92g of mercury at 0ºC. Calculate the mass of the mercury which would fill it at 100ºC. (Coefficient of linear expansion of the material of the specific gravity bottle = 8.0×10^{-6}/ºC and coefficient of real expansion of mercury = 0.000182/ºC).

Sol: $\gamma_\gamma = \gamma_a + \gamma_g$

$$0.000182 = \gamma_a + 3\alpha$$

$$\therefore \gamma_a = 0.000182 - 0.000024 \quad \Rightarrow \quad \gamma_a = 0.000158$$

$$\gamma_a = \frac{271.92-x}{x\times100} \Rightarrow 0.000158 = \frac{271.92-x}{100x} \Rightarrow \quad x = 267.6\,gm$$

ASSESS YOURSELF

1. Is the real co-efficient of expansion of water between 0^oC and 4^oC positive or negative?

Ans: Negative because the volume of water decreases from 0^oC and 4^oC.

2. **Is it possible to have apparent expansion equal to real expansion?**

Ans: Yes, if the expansion of the container is compensated by introducing a certain amount of mercury whose expansion is just equal to that of the container used.

3. **A flask with a capillary tube is filled with water at room temperature when it is kept in boiling water for sufficiently long time water over flows. Does the amount of overflow of water depend on atmospheric pressure?**

Ans: The boiling point depends on atmospheric pressure and the expansion depends on the difference between the boiling point and the initial temperature.

4. **In cold countries, water pipes are laid much below the ground. Why?**

Ans: If the pipes are laid over the ground or just below the ground the pipes break when the atmospheric temperature falls from 4^oC *and* 0^oC due to anomalous expansion.

5. **Water is filled in a vessel at a certain temperature. When the vessel is heated or cooled water overflows. What is the initial temperature of the water?**

Ans: Water at 4^oC expands when it is heated or cooled due to anomalous expansion.

6. **Water at 0^oC is filled in a vessel. On heating up to 4^oC does the water overflow?**

Ans: No.