TEMPERATURE AND THERMAL EXPANSIONS
1. EXPANSIONS OF SOLIDS

POINTS TO REMEMBER

1. The branch of physics that deals with the measurement of temperature is called thermometry.
2. Heat is a form of energy which has capacity to do the work.
3. Temperature is degree of hotness or coldness in a body which determines direction of flow of heat.
4. **Units of Heat: Joule**
   a) Standard calorie is amount of heat required to raise temperature of 1g of water from 14.5°C to 15.5°C (1 cal = 4.2J)
   b)1 B.Th.U = 252 cal
   c)1 Therm = 252×10^5 cal (It is the biggest unit of Heat)
5. **Units of Temperature**

	°C	F	R	K	Ra
LFP	0	32	0	273	492
UFP	100	212	80	373	692

6. $\frac{C}{100} = \frac{F-32}{212-32} = \frac{R-0}{80-0} = \frac{K-273}{373-273} = \frac{Ra-492}{672-492}$ Or $\frac{\Delta C}{100} = \frac{\Delta F}{180} = \frac{\Delta R}{80} = \frac{\Delta K}{100}$
7. Lower and upper fixed points of thermometer are determined by using Hypsometer.
8. Any property which changes with temperature is called thermometric property.

$$t = \left(\frac{X_t - X_0}{X_{100} - X_0}\right)100 \text{ and } t_2 = \left(\frac{X_2 - X_0}{X_1 - X_0}\right)t_1$$

9. To measure the temperature of a body, the maximum surface area of a thermometric bulb is considered. Since cylindrical surface has more surface area, bulbs are cylindrical.
10. Coefficient of linear expansion, $\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)} \Rightarrow L_2 = L_1[1 + \alpha\Delta t]$
11. Coefficient of areal expansion , $\beta = \frac{a_2 - a_1}{a_1(t_2 - t_1)} \Rightarrow a_2 = a_1[1 + \beta\Delta t]$
12. Coefficient of volume expansion , $\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} \Rightarrow V_2 = V_1[1 + \gamma\Delta t]$
13. $\beta = 2\alpha$ *and* $\gamma = 3\alpha$, *and* $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$
14. Lattice vibrations are purely harmonic, and then PE curve is a symmetric parabola indicating no thermal expansion the average inter-atomic distance remains same.
15. If the lattice vibrations are an-harmonic, PE curve is not a symmetric parabola indicating the thermal expansion. The average inter-atomic distance increases with increase of temperature.

LONG ANSWER QUESTIONS

1. **Mention different types of thermometers. Give a theoretical description of gas thermometers. Why are gas thermometers taken as primary standards?**

A. Thermometers are designed basing on the physical property of substance which changes with temperature.

1. Liquid and Gas thermometers are based on their respective thermal expansions.

2. Thermoelectric thermometers depend on the variation of thermo emf of a thermo couple with the temperature difference between the hot and cold junctions.

4. Resistance thermometers based on the variation of electric resistance of metals with temperature.

5. Radiation thermometers based on the measurement of the thermal radiation emitted by a body.

6. Vapour pressure thermometers which depend on the variation of vapour pressure with temperature.

1. Liquid Thermometers: Liquid thermometers are based on the thermal expansion of liquids such as mercury, alcohol etc. Mercury is used as a thermometric liquid because it has low specific heat, low thermal capacity, low vapour pressure, high thermal conductivity and more expansion. Since cylindrical surface has more surface area, thermometer bulbs are made cylindrical.

Mercury thermometers can not be used for measuring very low or very high temperatures because mercury freezes at - 38.8ºC and boils at 356ºC. This thermometer can be used to measure temperatures up to 300ºC. By filling nitrogen gas at high pressure above mercury column, it can be used up to 500ºC.

Alcohol thermometer can be used to measure low temperatures, as it exists in the liquid state upto – 112ºC and its coefficient of expansion is high.

2. Gas Thermometers:

A) **Constant Volume Thermometer** : –Let P_0 and P_t are the pressures of hydrogen gas filled in a thermometric bulb at 0°C and t°C respectively. The pressure coefficient β of the gas, is given by

$$\beta = \frac{P_t - P_0}{p_0 t} \quad \text{..... (1)}$$

If the pressure of the gas P_{100} at 100ºC, then the pressure coefficient can also be given as

$$\beta = \frac{P_{100} - P_0}{100.P_0} \text{ ..... (2)}$$

From equations (1) and (2)

$$\frac{P_t - P_0}{p_0 t} = \frac{P_{100} - P_0}{100.P_0} \quad \Rightarrow \quad t = \frac{P_t - P_0}{P_{100} - P_0} \times 100$$

B) **Constant Pressure Thermometer**: Calendar developed a constant pressure air thermometer by which temperatures upto 600ºC can be measured.

Constant volume hydrogen thermometers can be used for range of temperature from – 200ºC to 500ºC with platinum thermometric bulb. It is used upto 1100^0C with glazed porcelain bulb. For temperatures upto 1500ºC nitrogen is used as thermometric substance and for low temperatures below - 200ºC helium can be used.

3**. Resistance Thermometers**: The electrical resistance of a platinum wire increases gradually and uniformly with temperature over a wide range. If R_0, R_{100} and R_t are the resistances of a platinum wire at 0ºC, 100ºC and tºC respectively, then

$$\alpha = \frac{R_t - R_0}{R_0 t} \quad \text{and} \quad \alpha = \frac{R_{100} - R_0}{100 R_0}$$

Where α is the temperature coefficient of resistance which is defined as the increase in resistance per unit original resistance at 0°C per unit rise in temperature and t is the final temperature.

$$\therefore \frac{R_t - R_0}{R_0 t} = \frac{R_{100} - R_0}{100\, R_0} \quad Or \quad t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

Electrical resistance thermometers can be used over a wide range of – 272ºC to 1200ºC.

4. **Thermoelectric Thermometers**: When the junctions of two dissimilar metals are maintained at different temperatures an emf is generated. This effect is called thermoelectric effect or Seebeck effect. The thermo emf depens on the temperature difference between the hot and cold junctions of a thermocouple. It becomes maximum at a certain temperature of the hot junction called neutral temperature.

Thermocouples are used to measure temperatures in the range of -250ºC to 3000ºC.

5. **Radiation Thermometers**: Thermal radiation can be used to measure high temperatures since the radiation from a body depends only on its temperature. Such devices are called radiation pyrometers. Radiation thermometers are used to measure temperatures above 3000ºC.

6. **Vapour Pressure Thermometers**: These depend on the fact that the saturation pressure of a pure liquid is a function of its temperature only. Such thermometers can be used below -268ºC to about -272ºC.

The advantages in using gas thermometers are

a. Gases expand more than the liquids for the same rise in temperature.
b. Gases have low thermal capacity and hence these are more sensitive for changes of temperature.
c. Gases expand uniformly over a wide range of temperatures.
d. Gases have low density.
e. Gases can be obtained of the same purity any where.

The volume and pressure coefficients of all permanent gases are nearly equal and hence the scale furnished by different gases is almost identical.

2. Define the coefficients of expansions of solids and deduce the relation between them, In what way if at all, do the coefficients of expansions depend on the unit of length used and the scale of temperature employed? (Mar 2007)

A. **Coefficient of Linear Expansion (α):** It is defined as the increase in the length of a solid per unit original length per unit rise in temperature.

Let l_1 and l_2 be the lengths of a solid at temperatures t_1 and t_2 respectively. Then the coefficient of a linear expansion is given by $\alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)}$ and $l_2 = l_1 [1+ \alpha\ (t_2 - t_1)]$

Coefficient of areal Expansion (β): It is defined as the increase in the area of a solid per unit original area per unit rise in temperature.

Let A_1 and A_2 be the areas of a solid at temperatures t_1 and t_2 respectively. Then the coefficient of areal expansion is given by $\beta = \frac{A_2 - A_1}{A_1(t_2 - t_1)}$ and $A_2 = A_1\left[1+\beta(t_2 - t_1)\right]$

Coefficient of Cubical Expansion (γ): It is defined as the increase in volume of a solid per unit original volume per unit rise in temperature.

Let V_1 and V_2 be the Volumes of a solid at temperatures t_1 and t_2 respectively. Then the coefficient of cubical expansion. $\gamma = \frac{v_2 - v_1}{v_1(t_2 - t_1)}$ and $V_2 = V_1 [1+ \gamma (t_2 - t_1)]$

The units of α, β and γ are $(^0 C)^{-1}$

Relation between α , β and γ :

Consider a cube of each side of unit length at temperature t_1ºC. Let the coefficient of linear expansion of the material of the cube is α . If The temperature of the cube is raised by 1ºC, the length of each side of the cube becomes $(1+ \alpha)$

a) **Relation between β and α**: Initial area of each face of the cube, A_1 = 1 Sq. unit. When the temperature is increase by 1ºC, final area of each face of the cube,

$A_2 = (1+\alpha)(1+\alpha) = (1+\alpha)^2 = 1+2\alpha+\alpha^2 = 1+2\alpha$ .Since α^2 is very small, it can be neglected.

But, $\beta = \frac{A_2 - A_1}{A_1(t_2 - t_1)} = \frac{(1+2\alpha]-1}{1(1)} = 2\alpha$

$\therefore \beta = 2\alpha$

(b) **Relation between γ and α**: Initial volume of the cube, V_1= 1 cubic unit, when the temperature is raised by 1ºC, final volume of the cube,

$V_2 = (1+\alpha)^3 = 1+3\alpha+3\alpha^2+\alpha^3 = (1+3\alpha)$ cubic unit. Since α is very small the terms α^2 and α^3 are neglected.

But, $\gamma = \frac{v_2 - v_1}{v_1(t_2 - t_1)} = \frac{(1+3\alpha)-1}{1} = 3\alpha$

$\therefore \gamma = 3\alpha$

Hence α: β: γ = 1:2:3

The expansion co-efficients do not depend on the units of length since they are the ratio of same physical quantities but depends on the scale of temperature.

- α in /°F = $\frac{5}{9}\alpha$ in /°C

  α in /°R = $\frac{5}{4}\alpha$ in /°C

  α In / K = α in /°C

3. Explain thermal expansion of solids on the basis of potential energy curve. How are anharmonic vibrations different from harmonic vibrations?

A. At any temperature, atoms in a solid vibrate about their mean position. When the temperature of the solid is increased the amplitude of vibration of the atoms increases which results in the expansion of the solid.

Potential Energy Curve: In a solid, atoms are close to each other. When the atoms come close to each other to a separation of the order of an angstrom, the distances between their nuclei and electron clouds change. Hence the inter atomic forces and potential energy between two atoms changes. The variation of inter atomic potential energy with inter atomic distance is shown in figure.

At large distances the potential Energy is negative indicating that the force between the atoms is attractive. As the distance between the atoms decreases the potential energy decreases till it reaches a minimum value. This is the equilibrium position. In this state of equilibrium, the atoms vibrate about the equilibrium position.

When the distance between the atoms is further decreased the force becomes repulsive and the potential energy increases. It then becomes positive. The potential energy curve is not symmetrical around the minimum since the potential energy decreases slowly with increasing distance whereas it increases much faster with decreasing distance.

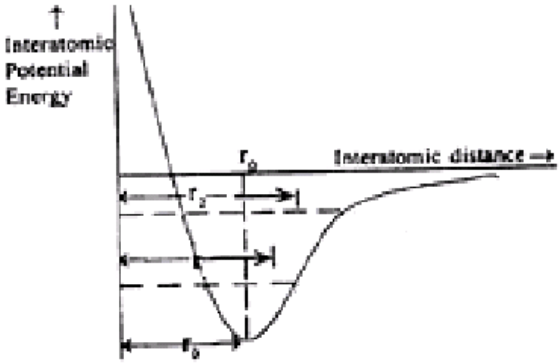
When the temperature is very low (T_0 = OK) , the atoms are at the equilibrium separation r_0.

When the temperature is increased to T_1 the energy increases. Corresponding to the increased energy there can be two possible values of separation between the atoms. If these values are taken as $\left(\frac{r_{min} + r_{max}}{2}\right)$ as shown in the mean position of vibration then becomes equal to $\gamma_{min} + \gamma_{max}$. This is not equal to due to the asymmetry of the potential energy curve. The mean position increases with temperature which causes thermal expansion of the solid.

An-harmonicity of Vibrations: If the vibrations are harmonic, the potential energy would be a parabolic function of interatomic distance. There would be no expansion even if the temperature increases because the potential energy curve would be symmetric about the equilibrium separation.

It shows that there would be no change in average interatomic distance as energy increases since is always equal to r_0 due to symmetry.

If the vibrations are an-harmonic the potential Energy would be an asymmetric function of Separation between atoms. This curve is shown In As explained earlier, its asymmetric Nature gives rise to thermal expansion. Thus the thermal expansion of solids is due to an-harmonicity of vibrations.



An-armonic Vibrations

4. State the examples of thermal expansion of solids. State a difficulty that you come across due to thermal expansion of solids.

Ans. 1) While laying railway tracks, small gaps are left between adjacent rails to allow for free expansion without affecting the track during summer. Gap to be left (l) = . l t=expansion of each rail.

2) Platinum is used to seal inside glass because both have nearly equal coefficients of linear expansion

3) To fix iron tyre on to the wooden rim of a cart wheel, the iron tyre is heated so that it slips onto the wooden rim. On cooling the iron tyre contracts and grips the wheel firmly.

4) Invar steel (steel + nickel) has very low . So it is used in making pendulum clocks, balancing wheels and measuring tapes. (Composition of invar steel is 64% steel and 36% nickel).

5) Iron or steel is used for reinforcement in concrete because both have nearly equal coefficients of expansion.

6) Hot chimney cracks when a drop of water falls on it because of unequal contraction Thick glass tumbler cracks when hot liquid is poured into it because of unequal expansion.

7). Concrete roads are laid in sections and expansion channels are provided between them.

8). Measuring tapes are made of invar steel because of its least coefficient of linear expansion.

Difficulties:

1) Hot chimney cracks when a drop of water falls on it because of unequal contraction Thick glass tumbler cracks when hot liquid is poured into it because of unequal expansion.

2) Time period of a pendulum clock is directly proportional to square root of the length of the Pendulum. i.e. $T \alpha \sqrt{L}$

In winter due to less temperature, metals contract. Hence the length of the Pendulum decreases and its time period decreases. So clock goes fast.

In summer due to increase in temperature metals expend. Hence the length of the Pendulum increases and its time period increases. So the clock goes slow.

SHORT ANSWER QUESTIONS:

1. Define temperature and distinguish between heat and temperature.

Ans. **Temperature:** Temperature is the degree of hotness or coldness of a body. Temperature is the property of a body which determines the flow of heat. Temperature is a macroscopic property. Temperature is a scalar quantity. The heat always flows from a body at higher temperature to body at lower temperature.

Heat: Heat is a form of energy transferred between systems by virtue of temperature difference between them. The S.I unit of heat energy is Joules. C.G.S unit is Calorie. It is a scalar quantity.

At thermal equilibrium even though the bodies contain same amount of heat, they will be at the same temperature.

2. Explain how temperature is measured.

Ans. Temperature is measured by a device called thermometer. The property of the material which changes with temperature is called the thermometric property.

Thermometers are designed basing on the physical property of substance which changes with temperature.

1. Liquid and Gas thermometers are based on their respective thermal expansions.

2. Thermoelectric thermometers depend on the variation of thermo emf of a thermo couple with the temperature difference between the hot and cold junctions.

4. Resistance thermometers based on the variation of electric resistance of metals with temperature.

5. Radiation thermometers based on the measurement of the thermal radiation emitted by a body.

6. Vapour pressure thermometers which depend on the variation of vapour pressure with temperature.

Calibration of a thermometer: The temperature on any scale is measured by taking two fixed points.

i) **Ice point**: The temperature at which water is converted into ice is taken as lower fixed point (LFP).

ii) **Steam point:** The temperature at which pure water boils is taken as upper fixed point (UFP).

The distance between these two fixed points conveniently sub-divided into equal intervals.

3. Explain Celsius and Fahrenheit scales of temperature.

Ans. **Celsius scale:** In the centigrade (Celsius) scale of temperature, the lower fixed point is taken as 0ºC. The upper fixed point is taken as 100ºC. The difference between two fixed point values is 100ºC which is called as fundamental interval given by UFP-LFP i.e. 100º – 0º =100º. The interval between these two points is divided into 100 equal parts and each corresponds to 1ºC on the scale

Fahrenheit scale: In the Fahrenheit scale of temperature, the lower fixed point is taken as 32ºF. The upper fixed point is taken as 212ºF. The difference between two fixed points is 180ºF which is called as fundamental interval given by UFP-LFP i.e. 212ºF-32ºF=180ºF. The interval between these two points is divided into 180 equal parts and each division corresponds to 1ºF on the scale.

4. Obtain the relation between Celsius and Fahrenheit scales of temperature.

Ans. The temperature difference between the ice and steam points should be the same on Celsius and Fahrenheit scales.

100 ºC difference = 180 ºF difference Fahrenheit

Let C and F be the temperature readings of a body on the Celsius and Fahrenheit scales respectively.

$$\frac{C-0}{100}=\frac{F-32}{180} \Rightarrow \frac{C}{5}=\frac{F-32}{9} \Rightarrow C=\frac{5}{9}(F-32) \text{ or } F=\frac{9}{5}(C+32)$$

5. Give a theoretical description of liquid thermometers.

Ans. . **Liquid Thermometers**: Liquid thermometers are based on the thermal expansion of liquids such as mercury, alcohol etc. Mercury is used as a thermometric liquid because it has low specific heat, low thermal capacity, low vapour pressure, high thermal conductivity and more expansion. Since cylindrical surface has more surface area, thermometer bulbs are made cylindrical.

Mercury thermometers can not be used for measuring very low or very high temperatures because mercury freezes at - 38.8ºC and boils at 356ºC. This thermometer can be used to measure temperatures up to 300ºC. By filling nitrogen gas at high pressure above mercury column, it can be used up to 500ºC.

Alcohol thermometer can be used to measure low temperatures, as it exists in the liquid state upto – 112ºC and its coefficient of expansion is high.

6. Give a theoretical description of resistance thermometers. State the range in which these thermometers are used.

Ans. 3**. Resistance Thermometers**: The electrical resistance of a platinum wire increases gradually and uniformly with temperature over a wide range. If R_0, R_{100} and R_t are the resistances of a platinum wire at 0ºC, 100ºC and tºC respectively, then

$$\alpha=\frac{R_t-R_0}{R_0 t} \text{ and } \alpha=\frac{R_{100}-R_0}{100R_0}$$

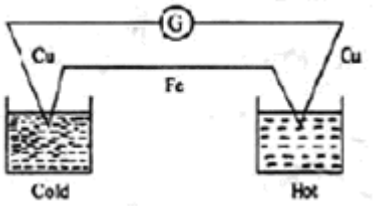
Where α is the temperature coefficient of resistance which is defined as the increase in resistance per unit original resistance at 0°C per unit rise in temperature and t is the final temperature.

$$\therefore \frac{R_t-R_0}{R_0 t}=\frac{R_{100}-R_0}{100\,R_0} \; Or \quad t=\frac{R_t-R_0}{R_{100}-R_0}\times 100$$

Electrical resistance thermometers can be used over a wide range of – 272ºC to 1200ºC.

7. What are thermoelectric thermometers? What is the range in which thermoelectric thermometers are used?

Ans. **Thermoelectric Thermometers**: These works on the principle of Seebeck effect. When the junctions of two dissimilar metals are maintained at different temperatures, an emf is developed. This is called thermoelectric effect or See beck effect. The pair of metals used to produce the emf is called a thermocouple. The current developed is called thermoelectric current that can be detected by using a galvanometer G .



The thermo emf depends on the nature of pair of metals and the temperature difference between the junctions of a thermocouple. It reaches a maximum value at a certain temperature of the hot junction called neutral temperature.

Keeping the cold junction in melting ice and heating the hot junction to different temperatures determined by a constant volume hydrogen thermometer, the emf at each temperature is measured. A calibration curve is drawn plotting emf against temperature. Then the temperature of any body can be found from the graph by measuring the emf, keeping the body in contact with the hot junction.

Thermocouples are used to measure temperatures in the range of – 250ºC to 3000ºC

8. Derive the relation for the variation of density of a solid with temperature

Ans. When the temperature of a solid is increased, its volume increases without any changes in its mass and hence the density of the solid decreases. Consider a solid of mass m having volumes V_1 and V_2 and densities ρ_1 and ρ_2 at temperatures t_1 and t_2 respectively. Since mass of the remains same, volume and density are related as $\frac{\rho_1}{\rho_2}=\frac{V_2}{V_1}$

But, $V_2=V_1\left[1+\gamma\left(t_2-t_1\right)\right]$ Or $\frac{V_2}{V_1}=\left[1+\gamma\left(t_2-t_1\right)\right]$

$\therefore \frac{\rho_1}{\rho_2}=\left[1+\gamma\left(t_2-t_1\right)\right] \quad \Rightarrow \rho_1=\rho_2\left[1+\gamma\left(t_2-t_1\right)\right]$

9. Why a thick tumbler cracks when boiling water is poured in it?

Ans. When hot water is taken in a glass tumbler it breaks due to uneven expansion. When hot water is poured in a thick glass tumbler, the inner walls expand and the outer walls do not undergo expansion. Due to this uneven expansion in the walls of the tumbler, it cracks.

10. Two identical rectangular strips one of copper and the other of steel are riveted together to form a bimetallic strip. What will happen on heating?

Ans. Bimetallic Strip: A bimetallic strip consists of two strips of different metals with different coefficients of expansion riveted together. When it is heated, it bends such that the strip with the higher coefficient of expansion expending more than the other. Hence when copper – steel bimetallic strip is heated, then it bends so that copper is on convex side and steel on concave side. A bimetallic strip is used as (i) a thermostat to control the temperature of a body (ii) a fire alarm etc.

11. Pendulum clocks generally go fast in winter and slow in summer. Why? (May2009)

Ans. Time period of a pendulum clock is directly proportional to square root of the length of the Pendulum. i.e. T α $\sqrt{L}$
In winter due to less temperature, metals contract. Hence the length of the Pendulum decreases and its time period decreases. So clock goes fast.
In summer due to increase in temperature metals expend. Hence the length of the Pendulum increases and its time period increases. So the clock goes slow.

12. How does the diameter of the opening in the cast iron plate of a kitchen stove change, when the stove is heated?

Ans. The molecules at the edges of stove vibrate freely in outward direction when the stove is on. The inner molecules of stove vibrate towards the holes of the stove. But vibrations of the molecules at the edges of the stove are greater than that of the vibrations of the molecules at the inner sides of the stove. So that diameter of the opening in the cast iron plate of a kitchen stove increases.

13. The difference between the length of a certain brass rod and that of a steel rod is claimed to be constant at all temperature. Is this possible? It possible, state the condition under which it is possible.

Ans. Yes.
The condition under which the difference between the length of a certain brass rod and that of a steel rod is claimed to be constant at all temperature is $l_b\alpha_b = l_s\alpha_s$

Where l_b = length of brass rod
l_s = length of steel rod
α_b = co-efficient of linear expansion of brass rod
α_s = co-efficient of linear expansion of steel rod

14. A uniform pressure 'p' is exerted on all sides of a solid cube at temperature 0^0C. By what amount should that temperature of the cube be raised in order to bring its volume back to the value it had before the pressure was applied. The co-efficient of cubical expansion of the cube is 'γ' and the bulk modules of elasticity is 'K'

Ans. $K=\frac{pv}{\Delta v} \quad \Rightarrow \frac{\Delta v}{v}=\frac{p}{K} \quad \Rightarrow \frac{\Delta v}{v}=\gamma(t_2-t_1) \quad \Rightarrow \Delta t=\frac{p}{\gamma K}$

VERY SHORT QUESTIONS:

1. What is the basic principle of a thermometer?

Ans. A thermometer works as on the principle of expansion of liquids and gases.

2. On what law is the measurement of temperature based? State the law.

Ans. Zeroth law of thermodynamics.
If two bodies A and B are thermally in equilibrium with a third body C, then A and B are in thermal equilibrium with each other.

3. What are the lower and upper fixed points of Celsius scale?

Ans. The lower fixed point of centigrade scale is ice point and its value is 0ºC
The upper fixed point on a Celsius scale is steam point and its value is 100ºC

4. What are the lower and upper fixed points of Fahrenheit scale?

Ans. The lower fixed point is ice point and its value is 32ºF
The upper fixed point is steam point and its value is 212 ºF

5. Which thermometer is more sensitive – a mercury or a gas thermometer?

Ans. Gas Thermometer

6. Name the suitable thermometers to measure the following temperature:
- 80ºC, 60ºC, 250ºC, 780ºC, 2000ºC

Ans. Thermo Couple (or) thermoelectric thermometers

7. In what ways are the gas thermometers better than mercury thermometers? (March 2009)

Ans. The advantages in using gas thermometers are

a. Gases expand more than the liquids for the same rise in temperature.
b. Gases have low thermal capacity and hence these are more sensitive for changes of temperature.
c. Gases expand uniformly over a wide range of temperatures.

8. Do the values of coefficients of expansion differ, when the lengths are measured in centimeters or in meters?

Ans. No. The values of coefficients of expansion do not depend on the units of length.

9. Do the values of coefficients of expansion differ, when the temperatures are measured on Centigrade scale or on Fahrenheit scale?

Ans. Yes. $\frac{C}{5}=\frac{F-32}{9}$

10. State whether the following characteristics of a metal washer increase, decrease or remain the same when the washer is heated? (i) Internal diameter, (ii) volume, (iii) mass and (iv) density.

Ans. 1) Internal diameter increases.
2) Volume increases.
3) Mass remains constant.
4) Density decrease.

11. Can a substance contract on heating? Give an example.

Ans. Yes. Type metal, rubber, cast iron etc.

12. Iron tyres are heated red hot before fixing on wooden rim of a cart wheel. Why?

Ans. Before fixing on wooden rim of a cart wheel, the iron tyre is heated so that it can slip over the wooden ring due to expansion.

13. Why gap are left between rails on a railway track? (June2010,Mar 2009,Mar 2005)

Ans. A gap should be left between two successive rails to allow for linear expansion of the rails in summer,.

14. A brass disc snuggly fits in a hole in a steel plate. By what method, the disc is removed from the plate – by heating or cooling. Why? (March 2011)

A. A brass disc snuggly fits in a hole in a steel plate. To loosen the disc from the hole, the system should be cooled. Since brass contracts more on cooling ,it get separated from the plate.

SOLVED PROBLEMS:

1. Convert -10^0C into Fahrenheit scale.

Sol: C=-10^0C $\quad \frac{C}{5}=\frac{F-32}{9} \Rightarrow \frac{-10}{5}=\frac{F-32}{9} \Rightarrow F=14^0F$

2. Convert the normal temperature of a human body 98.4^0F into Celsius scale.

Sol: $F=98.4^0F$

$$C=\frac{9}{5}(F-32)=\frac{5}{9}(98.4-32)=36.9^0C$$

3. Show that -40^0 measure the same temperature on Celsius and Fahrenheit scales.

Sol. $Let\, C=F=x$

But, $\frac{C}{5}=\frac{F-32}{9} \quad \Rightarrow \frac{x}{5}=\frac{x-32}{9} \Rightarrow x=-40^o$

4. An aluminium rod of length 50cm is heated so that its temperature increases from $20^0C\, to\, 80^0C$. If the linear coefficient of expansion of expansion of aluminium is $24\times10^{-6}/^0C$, find the increase in the length of the aluminium rod.

Sol: $l_1=50cm; t_1=80^0C; t_2=20^oC; \alpha=24\times10^{-6}/^oC$

Coefficient of linear expansion, $\alpha=\frac{l_2-l_1}{l_1(t_2-t_1)}$

Increase in length of the rod, $(l_2-l_1)=\alpha l_1(t_2-t_1)$

$$=24\times10^{-6}\times50\times(80-20)=7.2\times10^{-2}cm$$

5. A brass rod and a copper rod have same difference in length at all temperatures. The difference in length is 10 cm. Find the length of the two rods at 0^0C. Coefficient of linear expansion of brass and copper are $19\times10^{-6}/^0C$ and $17\times10^{-6}/^0C$ respectively.

Sol. Let l_1 and l_1' be the lengths of the brass and copper rods at 0^0C respectively. At any temperature t^0C their lengths are $l_2$and l_2' respectively

$l_2=l_1(1+\alpha_1 t)$ And $l_2'=l_1'(1+\alpha_2 t)$

$(l_2'-l_2)=(l_1'-l_1)+(l_1'\alpha_2-l_1\alpha_1)t$

But the difference in lengths of the two rods is same at all temperatures i.e.,

$(l_2'-l_2)=(l_1'-l_1)$

$\therefore 0=(l_1'\alpha_2-l_1\alpha_1) \Rightarrow l_1\alpha_1=l_1'\alpha_2$

$\Rightarrow \frac{l_1}{l_1'}=\frac{17}{19}$ and $l_1'-l_1=10$

Solving, $l_1'=95cm$ *and* $l_1=85\ cm$

-efficient of linear expansion $=\frac{14\times10^{-6}}{1.8^oF}=7.778/^oF$

6. Express 200K on Farenheit scale.

Sol: $\frac{K-273.15}{373.15-273.15}=\frac{F-32}{212-32} \Rightarrow \frac{K-273.15}{100}=\frac{F-32}{180}$

$$\Rightarrow \frac{200-273.15}{100}=\frac{F-32}{180} \quad \Rightarrow F=-99.68K$$

7. The triple points of neon and carbon dioxide are 24.57K and 216.55 K respectively. Express these temperatures on the Celsius and Farenheit scales.

Sol: On Celsius scale: C = T-273.15

Triple point on neon = 24.57 - 273.15 = -248.58 ^{o}C

Triple point of carbon dioxide = 216.55 - 273.15 = -56.6. ^{o}C

On Farenheit scale :F= $\frac{9}{5}C+32$

Triple point of neon $=\frac{9}{5}(-56.6)+32=-415.26^{0}F$

Triple point of carbon dioxide $=\frac{9}{5}(-56.6)+32=-69.88^{0}F$

8. What is the temperature for which the readings on Kelvin and Fahrenheit scales are same? (June2010)

Sol: On the Kelvin and Fahrenheit scales

$$\frac{K-273.15}{100}=\frac{F-32}{180}$$

But, $K=F$

$$\frac{x-273.15}{100}=\frac{x-32}{180} \quad \Rightarrow x-273.15=\frac{5}{9}x-\frac{160}{9} \Rightarrow x=574.6^{0}$$

$$\therefore 574.6K=574.6^{o}F$$

9. A faulty thermometer has its fixed points marked as 5^{0} and 95^{0}. The temperature of a body as measured by the faulty thermometer is 59^{0}. Find the correct temperature of the body on Celsius scale.

Sol. On any scale of temperature

$$\frac{\text{Re}\,ading-LFP}{UFP-LFP}=cons\tan t \quad \Rightarrow \frac{x-5}{95-5}=\frac{C}{100} \Rightarrow \text{C}==60^{o}C$$

10 Two absolute scales A and B have triple points of water defined to be 200 A and 300B. What is the relation between $T_A and\, T_B$? Given, triple point of water = 273.15K.

Sol: 200A on absolute scale A corresponds to 273.16K on Kelvin scale.

Size of one degree on absolute scale A in terms of the size of the degree on Kelvin scale

$=\frac{273.16}{200}$

Temperature T_A on Kelvin scale $=\left(\frac{273.16}{200}\right)T_A$

Also, the value of temperature on Kelvin scale $=\left(\frac{273.16}{300}\right)T_B$

But, $T_A and\, T_B$ represent the same temperature.

$$\left(\frac{273.16}{300}\right)T_A=\left(\frac{273.16}{300}\right)T_B \quad \Rightarrow T_A=\frac{2}{3}T_B$$

.

11. **An iron rod of length 50cm is joined at an end to a copper rod of length 100cm at 20ºC. Find the length of the system at 100^oC and average coefficient of linear expansion of the system $\left(\alpha_{iron}=12\times10^{-6}/^oC \text{ and } \alpha_{copper}=17\times10^{-6}/^oC\right)$.**

Sol: Length of iron rod at 100^oC , $l_2=(50)\left(1+17\times10^{-6}(100-20)\right)=50.048cm$.

Length of copper rod at 100^oC, $l_2^1=(100)\left(1+17\times10^{-6}(100-20)\right)=100.136cm$

Length of the system at 100^oC = $l_2+l_2^1=150.184$ cm.

Length of the system at 20^oC = 50+100=150cm.

Increase in length of the system=150.184 - 150=0.184cm.

Rise in temperature of the system=100 - 20=80^oC .

Initial length of the system=150cm.

Average coefficient of linear expansion of the system $=\dfrac{0.184}{150\times80}=15.33\times10^{-6}/^oC$

12. **A brass scale which is correct at 20^0C gives the length of a steel rod as $51.6cm$ when both and scale are at 40^0C. Calculate (a) the true length of the rod at 40^0C and (b) the true length of the rod at 20^0C. Coefficients of linear expansion of brass and steel $19\times10^{-6}/^0C$ and $11\times10^{-6}/^0C$ are respectively**

Sol. a) Brass scale is correct at $t_1^{\,0}C$. The reading l_1 on it would be the correct length at $t_1^{\,0}C$.

At $t_2^{\,0}C$, the true length of the rod is $l_2=l_1\left(1+\alpha_s(t_2-t_1)\right)$.....(1)

$l_1=51.6cm, \alpha_s=19\times10^{-6}/^0C, t_1=20^0C, t_2=40^0C$

$\therefore$ True length of the rod = $51.6\left(1+19\times10-6(40-20)\right)=51.6196cm$.

(b)Let the true length of the rod at the temperature $t_1^{\,0}C$ of marking the scale be l_0

The length of the rod l_2 at $t_2^{\,0}C$ is given by, $l_2=l_0\left(1+\alpha_b(t_2-t_1)\right)$

But this length is equal to the true length of the rod given by Eq. (1).

$$l_0\left(1+\alpha_b(t_2-t_1)\right)=l_1\left(1+\alpha_s(t_2-t_1)\right)$$

$$l_0=\frac{\left(1+\alpha_s(t_2-t_1)\right)}{\left(1+\alpha_b(t_2-t_1)\right)}=l_1\left(1+\alpha_s(t_2-t_1)\right)\left(1+\alpha_b(t_2-t_1)\right)^{-1}$$

$$\therefore l_0=l_1\left(1+(\alpha_s-\alpha_b)(t_2-t_1)\right)\rightarrow(2)$$

$l_1=51.6cm, t_2=40^oC; t_1=20^oC, \alpha_s=19\times10^{-6}/^oC \text{ and } 11\times10^{-6}/^oC$

$l_o=51.6\left(1+(19-11)\times10^{-6}(40-20)\right)=51.6\left(1+160\times10^{-6}\right)=51.6083cm$

13. **A steel bar 1cm square and 50cm long at 30ºC fits into the space between two fixed supports. If the bar is now heated to 280ºC, What force will it exert against the supports? (α for steel = $11\times10^{-6}/^oC$ and Young's modulus for steel = $2\times10^{11}N/m^2$).**

Sol. Co-efficient of linear expansion $\alpha=\dfrac{l_2-l_1}{l_1(t_2-t_1)}\Rightarrow\dfrac{l_2-l_1}{l_1}=\alpha(t_2-t_1)$

$$\therefore Strain=\alpha(t_2-t_1)$$

Young's modulus, $Y = \dfrac{Stress}{Strain}$

Stress =Y (Strain) = $Y\alpha(t_2 - t_1)$

Force exerted on the supports=Stress X Area of cross section

$$= Y\alpha(t_2 - t_1)A = Y\alpha A(t_2 - t_1)$$

Now, $Y = 2\times10^{11} N/m^2;\ \alpha = 11\times10^{-6}/^o C;\ A = 1\times10^{-4} m^2;\ t_1 = 30^o C;\ t_2 = 280^o C$

The forced exerted on the supports $= 2\times10^{11}\times11\times10^{-6}\times10^{-4}\times250 = 55000N$

14. Find the increase in temperature of aluminium rod if its length is to be increased by1%. (α for aluminium= $25\times10^{-6}/^0 C$**.**

Sol: $\alpha = \dfrac{l_2 - l_1}{l_1(t_2 - t_1)} \Rightarrow \dfrac{l_2 - l_1}{l_1} = \alpha(t_2 - t_1)$

$\dfrac{1}{100} = 25\times10^{-6}(t_2 - t_1) \Rightarrow (t_2 - t_1) = \dfrac{1}{25\times10^{-4}} = 400^o C.$

15. A pendulum clock of an iron rod is connected to a small, heavy bob. If it shows correct time at $20^o C$**, how fast or slow will it go in one day at** $40^o C$**? (Coefficient of linear expansion for steel is** $12\times10^{-6}/^o C$**.**

Sol. $T = 2\pi\sqrt{\dfrac{l}{g}}$ If t is the temperature difference, then

$\dfrac{T_2}{T_1} = \sqrt{\dfrac{l_2}{l_1}} = \sqrt{\dfrac{l_1(1+\alpha t)}{l_1}}$ and $\dfrac{T_2}{T_1} = \sqrt{1+\alpha t} = 1 + \dfrac{1}{2}\alpha t$

$T_2 = T_1 + \dfrac{1}{2}\alpha t T_1 \quad \Rightarrow \quad T_2 - T_1 = \dfrac{1}{2}\alpha t T_1$

$\alpha = 12\times10^{-6}/^o C;\ t_1 = 20^o C, t_2 = 40^o C.$

Loss of time per day $= \dfrac{\alpha t}{2}\times86400 = 43200\times12\times10^{-6}(40-20) = 10.368s$

16. Two equal, straight strips of two different metals are fastened together, parallel to each other, a small fixed distance 'd' apart to form a bimetallic strip. Find the radius of curvature of the bimetallic strip when it is heated from $0^o C$ **to** $t^o C$**.**

Sol: A bimetallic strip containing two metal strips A and B is heated from $0^o C$ **to** $t^o C$**.**

If l_0 is the length of the metal strips at $0^o C$ and l_1, l_2 are the lengths of strips A, B respectively at $t^o C$, then

$l_1 = l_0(1+\alpha_1 t)$ *and* $l_2 = l_0(1+\alpha_2 t) \rightarrow (1)$

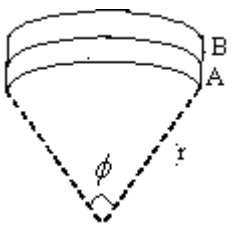
Where α_1 and α_2 are the co-efficient of linear expansion of strips A and B respectively.

If ϕ is the angle subtended by the bent strips at their common centre of curvature O and are $r_1 r_2$ the radii of curvature of strips A, B respectively then

$l_1 = r_1\phi$ *and* $l_2 = r_2\phi \rightarrow (2)$

Substituting l_1 and l_2 from equation (i)

$l_0(1+\alpha_1 t) = r_1\phi$ *and* $l_0(1+\alpha_2 t) = r_2\phi$



$$\phi = \frac{l_0(\alpha_1 - \alpha_2)t}{(r_1 - r_2)} = \frac{l_0(\alpha_1 - \alpha_2)t}{d} \qquad (\because r_1 - r_2 = d)$$

$Adding\ l_o(1+\alpha_1 t) = r_1\phi\ and\ l_o(1+\alpha_2 t) = r_2\phi$

$$l_o(1+\alpha_1 t+1+\alpha_2 t) = (r_1 + r_2)\phi$$

$$l_o(2+(\alpha_1+\alpha_2)t) = (r_1+r_2)\frac{l_0(\alpha_1-\alpha_2)t}{d}$$

$$2 = (r_1+r_2)\frac{(\alpha_1-\alpha_2)t}{d}(\because (\alpha_1+\alpha_2)t\ is\ neglected\ )$$

$$\frac{r_1+r_2}{2} = \frac{d}{(\alpha_1-\alpha_2)t}$$

Average radius of curvature of the strip = $r = \frac{r_1+r_2}{2}$

$$\therefore r = \frac{d}{(\alpha_1-\alpha_2)t}.$$

Exercise 1

1. Express O K on Fahrenheit scale. (March - 2009)

Sol: $\frac{K-273.15}{100} = \frac{F-32}{180} \Rightarrow \frac{0-273.15}{100} = \frac{F-32}{100} \Rightarrow F = -459.67^0 F$

2. On Reaumer scale of temperature the melting point of ice and the boiling point of water are taken as 0ºR and 80ºR respectively. The freezing point and boiling point of mercury on Celsius scale -39ºC and 367ºC respectively. Express these temperatures on reaumer scale.

Sol: $\frac{C}{100} = \frac{R}{80}$

i) $\frac{-39}{100} = \frac{R}{80}$ $-39\times 8 = R\times 100$ = R = -31.2°C ii) $\frac{367}{100} = \frac{R}{80} \Rightarrow$ R = 293.6°

3. A faulty thermometer has its fixed points marked as 3º and 102º. The temperature of a body as measured by the faculty thermometer is 80^0. Find the correct temperature of the body on Celsius scale.

Sol: $\frac{x-L.F.P}{U.F.P-L.F.P} = \frac{C-O}{100} \Rightarrow \frac{x-3}{102-3} = \frac{C}{100} \Rightarrow C = 77.8^0 C$

4. At what temperature on the Fahrenheit scales will the reading be double of the reading on the Celsius scale?

Sol: $C = x$ then F=2x

But, $C = \frac{5}{9}(F-32) \Rightarrow X = 160^0 C$

$F = 2x$ $F = 2\times 160^0 F = 320^0 F$

5. An accurate Celsius thermometer and a Fahrenheit thermometer register 60º and 141º respectively when places in the same constant temperature enclosure. What is the error in the Fahrenheit thermometer?

Sol: $\frac{C}{100}=\frac{F-32}{180} \Rightarrow \frac{60}{100}=\frac{F-32}{180} \Rightarrow F=140^0$

Error in $F^0 = 141-140 = 1^0 F$

6. Find the ratio of the lengths of an iron rod and an aluminium rod for which the difference in the lengths is independent of temperature. Coefficient of linear expansion of iron and aluminium are 12×10^{-6}/ºC and 24×10^{-6}/ºC respectively.

Sol: $\alpha_1 = 12\times10^{-6}/^0 C$; $\alpha_2 = 24\times10^{-6}/^0 C$

$L_1\alpha_1 = L_2\alpha_2 \Rightarrow \frac{L_1}{L_2}=\frac{\alpha_2}{\alpha_1} \Rightarrow L_1 : L_2 = 2:1$

7. A piece of copper wire has a length of 2 m at 10ºC. Find its length at 100ºC. Coefficient of linear expansion of copper = 17×10^{-6}/ºC?

Sol: $l_2 = l_1\left[1+\alpha(t_2-t_1)\right] = 2\left[1+17\times10^{-6}\times90\right]$

$\therefore L_2 = 2.00306\ m$

8. A piece of steel has a length of 30 cm at 15ºC. At 90º its length increases by 0.027 cm. Find its coefficient of linear expansion.

Sol: $\alpha=\frac{l_2-l_1}{l_1(t_2-t_1)}=\frac{0.027}{30(75)}$

$\therefore \alpha = 12\times10^{-6}/^0 C$

9. The density of lead at 0ºC is 11.34 g/cm^3. What is the density of 100ºC, given that the coefficient of liner expansion of lead = 28×10^{-6}/ºC?

Sol: $d_t = d_0\left[1-\gamma(\Delta t)\right]$ and $\gamma = 3\alpha$

$d_t = d_0\left[1-\gamma(t_2-t_1)\right] = 11.34\left[1-3\times28\times10^{-6}\times100\right]$

$d_{100} = 11.24\ g/cm^3$

10. A steel scale is correct at 0ºC. The length of a brass tube measured by it at 40ºC is 4.5m. What is the length of the tube at 0ºC? (Coefficient of linear expansion of steel and brass are 11×10^{-6}/ºC and 19×10^{-6}/ºC respectively).

Sol: $H_0 = H_t\left[1-(\alpha_2-\alpha_1)t\right] = 4.5\left[1-\left(19\times10^{-6}-11\times10^{-6}\right)\times40\right]$

$H_0 = 4.49\ m$

11. A brass meter bar is correct at 20ºC. The length of a rod as measured by it at 35ºC is 50 cm. What is the true length of the rod at 35ºC if the coefficient of linear expansion of brass is 19 $\times10^{-6}$/ºC?

Sol: The brass meter is correct at t_1^0C the reading l_1 on it would be the correct length at t_1^0C at a t_2^0C the length the true length of this rod is, $l_2 = l_1\left[1+\alpha\ (t_2 - t_1)\right]$

l_1 = 50 cm; $\alpha = 19\times10^{-6}/^0 C$

t_1 = 20ºC; t_2 = 35ºC

The true length of the rod = $50\ \left[1\ +\ 19\ \times10^{-6}(35-20)\right]$ = 50.10425 cm

12. Two thin metal strips, one of brass and the other of iron are fastened together parallel to each other, a distance 2 mm apart to form a bimetallic strip. If the strips are of equal

length at 0ºC, calculate the radius of the arc formed by the bimetallic strip when heated to 80ºC. (Coefficient of linear expansion of brass = 19 ×10^{-6}/ºC and iron = 12 ×10^{-6}/ºC)

Sol: $r = \dfrac{d}{(\alpha_1 - \alpha_2)\Delta t} = r = \dfrac{2\times10^{-3}}{7\times10^{-6}\times80} = r = \dfrac{2}{56}\times10^{2} = r = 3.57\,m$

13. An iron ring 1.20m in diameter is to be put on a wooden cart wheel. The diameter of this wheel is 1.0 cm greater than of the ring. By how mush must the temperature of the ring be raised before it will go on it? (Coefficient of linear expansion of iron = 12 ×10^{-6}/ºC).

Sol: $\Delta l = l.\alpha.\Delta t$

$1\times10^{-2} = 1.2\times12\times10^{-6}\times\Delta t$

$\Delta t = \dfrac{1\times10^{-2}}{1.2\times12\times10^{-6}} \Rightarrow \Delta t = 694.4^{0}C$

14. A steel bar 0.5cm square and 25 cm long at 30ºC fits into the space between two fixed supports. If the bar is now heated to 230ºC, what force will it exert against the supports? (Coefficient of linear expansion for steel = 11×10^{-6}/ºC and Young's modulus for steel = 2×10^{11} N/m^2).

Sol:

Force $= Y\alpha(t_2 - t_1)A = Y\alpha A(t_2 - t_1)$

$Y = 2\times10^{11}\,N/m^2$; $\alpha = 11\times10^{-6}/^{0}C$; $A = 0.5\times0.5\times10^{-4}\,m^2$;

$t_1 = 30^{0}C; t_2 = 230^{0}C$

$\therefore$ Force of excreted on the supports $= 2\times10^{11}\times11\times10^{-6}\times0.25\times10^{-4}\times200$

$= 11000N$

15. A clock, with a brass pendulum, keeps correct time at 20ºC, but loses 8.212 s per day, when the temperature rises to 30ºC. Calculate the coefficient of linear expansion of brass.

Sol: Loss of time for day $= \dfrac{1}{2}\times\alpha(t_2 - t_1)\times86400$

$8.212 = \dfrac{1}{2}\times\alpha\times10\times86.400$

$\therefore \alpha = 19\times10^{-6}/^{0}C$

ASSESS YOURSELF

1. **Why is the bulb of a thermometer cylindrical?**

Ans: A cylinder has maximum surface area for a given volume.

2. **Mercury is used as thermoelectric liquid to all liquids, why?**

Ans: Mercury is used as a thermometric liquid because it has low specific heat, low thermal capacity, low vapour pressure, high thermal conductivity and more expansion.

3. **Can we find the correct temperature of a body on Celsius scale using a faulty thermometer?**

Ans: The correct temperature of a body on Celsius scale using a faulty thermometer can be found, if we know the upper and lower fixed points of the thermometer.

4. **The melting point of ice and the boiling point of water are taken as standard fixed points, is it correct?**

Ans: No, the melting point of ice and boiling point of water change if the pressure changes and impurities are dissolved in water.

5. **A metal disc has a hole in it. What happens to the size of the hole, when the disc is heated?**

Ans: The size of the hole increases.

6. **Can we apply the relations $\beta = 2\alpha$ and $\gamma = 3\alpha$ for anisotropic solids.**

Ans: No, these relations are applicable only for isotropic solids. For a anisotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ where α_1, α_2 and α_3 are co-efficient of linear expansions along three mutually perpendicular axes.

7. **The length of an iron rod is found to be smaller on a hot summer day than a cold winter day as measured by the Aluminium scale. Is it correct to conclude that iron shrinks on heating $\alpha\, of\ \ iron = 12\times10^{-6}\ /^{o}C$ and α of $Alu\min ium = 24\times10^{-6}\ /^{o}C$**

Ans: No. During winter iron contracts less than aluminium. Hence iron rod appears to be longer during winter.

8. **A thick tumbler and thin tumbler of glass are filled with hot water. The thick tumbler cracks but not the thin tumbler, why?**

Ans: The thick tumbler cracks due to uneven expansion of its inner and outer surfaces. Whereas thin tumbler does not crack due to uniform expansion of inner and outer surfaces.

9. **A bimetallic strip made of brass and steel strips shown in the figure be low is heated. What shape does it acquire?**

(α of brass $= 19\times10^{-6}\ /^{o}C$ and α of steel $= 11\times10^{-6}\ /^{o}C$)



Ans: It bends with brass on convex side as α of brass is greater than of steel.

10. **The bimetallic strip of in the above question is cooled. What shape does it acquire?**

Ans: It bends with brass on concave side.