FLUID MECHANICS
VISCOSITY

POINTS TO REMEMBER

1. **Viscosity:** It is the property of a fluid by virtue of which one layer of fluid opposes the relative motion of the other. This force of opposition is known as viscous force.

2. **Newton's formula:** The viscous force acting between two adjacent layers of the fluid is directly proportional to surface area of the layers in contact and the velocity gradient.

$$F \propto A$$
$$\mathrm{F} \propto \frac{-dv}{dx}$$
$$\Rightarrow F \propto A\frac{dv}{dx} \Rightarrow F = -\eta A\frac{dv}{dx}$$

The negative sign indicates that the viscous force is opposite to the direction of flow of fluid.

3. **Coefficient of viscosity (η):** It is defined as the tangential force per unit area of the layer of a fluid, required to maintain a unit velocity gradient.

S.I unit of coefficient of viscosity is Ns m^{-2} or Pa S
C.G.S unit is dyne sec cm^{-2} or poise
1 Pa S = 10 poise.

4. **Coefficient of Kinematics viscosity (μ):** It is defined as the ratio of coefficient of viscosity to density of the fluid. Its S.I unit is $m^2\ s^{-1}$,
Unit: stoke. **1 stoke** = $10^{-4}\ m^2\ s^{-1}$.

5. **Velocity gradient:** It is the change in velocity of the layers of the fluid per unit perpendicular distance.

Velocity gradient = $\frac{dv}{dx}$ **Unit**: s^{-1}

6. **Effect of temperature:** In case of liquids coefficient of viscosity decreases with increase in temperature as the cohesive forces decreases with increase in temperature.

In case of gases, coefficient of viscosity increases with increase in temperature, because the change in momentum of the molecules increases with increase in temperature.

7. **Effect of pressure:** For liquids the value of η increases with increase in pressure

For gases, value of η increases with increase in pressure at low pressure. But at high pressure η is independent of pressure

8. **Torricelli Theorem:** The speed of a liquid coming out of a hole at a depth 'h' below the surface of a liquid is same as that of a particle fallen through the height 'h' under gravity. This is called Torricelli Theorem. If a liquid is flowing from a hole at a depth h is below the free surface of the liquid.

Velocity of efflux (v) = $\sqrt{2gh}$

9. **Stokes formula**: The viscous force of high viscous liquids like castor oil, glycerin is given by $F = 6\pi\eta rv$
   Where r is the radius of the spherical body falling through a medium of coefficient of viscosity η.
   V_T is the velocity of the body.

10. Terminal velocity of a spherical body , $v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

    Where ρ = density of the body and σ = density of the fluid through which the body is falling.

LONG ANSWER QUESTIONS

1. Define coefficient of viscosity. (June 2005)

What is the terminal velocity for a body in motion through viscous medium?

Derive the expression for terminal velocity using Stoke's law.

A. **Coefficient of viscosity** (η):

If A =1 m^2 and Velocity gradient = $\frac{dv}{dx}$ =1 s^{-1}, then η = F

Co-efficient of viscosity is defined as the tangential viscous force acting per unit area per unit velocity gradient.

$$\mathbf{F = -\eta A}\frac{dv}{dx} \qquad \text{Or} \qquad \eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

CGS unit : dyne cm^{-2} (or) $gm\,cm^{-1}\,S^{-1}$ (or) poise.

Stoke's law:

It states that viscous force acting on the spherical body which is moving with a velocity (v) is directly proportional to

(i) Coefficient of viscosity of fluid η
(ii) Radius of the spherical body(r)
(iii) Velocity of the body (v)

$F \propto \eta rv$ Or $F = K\eta rv$, where K is proportionality constant K = 6π

$\therefore F = 6\pi\eta rv$

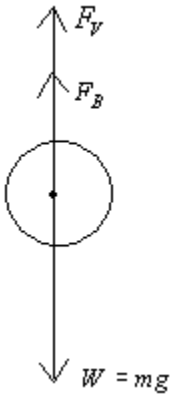
Terminal velocity:

Consider a spherical body of radius r, density ρ is falling through a fluid of density σ. The forces acting on the body are

1. Weight $W = mg = \frac{4}{3}\pi r^3\rho g$ acting in the downward direction
2. Viscous force $(F_v) = 6\pi\eta rv$ acting in upward direction.

3. Buoyant force or weight of the fluid displaced

$F_B = = \frac{4}{3}\pi r^3 \sigma g$ acting in upward direction.



The net (or) resultant force acting on the body is

$F_R = W - F_v - F_B = W - (F_v + F_B)$

If the body attains a constant velocity called terminal velocity, then the net force acting on the body is zero.

$W - (F_v + F_B) = 0$ *or* $F_v = W - F_B$

$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$

$\therefore v_T = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{\eta}$

This is the expression for terminal velocity of a body falling through a viscous medium.

SHORT ANSWER TYPE QUESTIONS

1. For a steady flow of liquid, the viscous force tends to stop the flow why?

A: The property of a liquid by virtue of which it opposes the relative motion between the layers of the liquid is called viscosity (or) internal friction of the liquid. Considering the liquid as layers, in steady flow any layer of the liquid tries to decrease the velocity of upper layer due to frictional forces between molecules of adjacent layers. This force is called viscous drag. These opposing forces tend to decrease the velocity of the liquid layers. Due to viscous drag the relative velocity between two adjacent layers decreases so for a steady flow of liquid the viscous force tends to stop the flow.

2. The variation of viscosity in liquids and gases with temperature is opposite. Why? (June2010,May 2008,March 2007)

A. **Viscosity in liquids** is due to the cohesive forces between the molecules. When temperature increases, distance between the molecules will be increased and cohesive forces decrease due to which viscosity also decreases.

Viscosity in gases is mainly due to collision between the molecules in which exchange of momenta takes place. When temperature increases, velocity of gas molecules increases due to which more collisions and more exchange of momenta will be taken place. Hence, viscosity of gases increases with increase in temperature.

Hence when temperature increases, viscosity of liquids decreases where as viscosity of gases increases.

3. The terminal velocity is less if the co-efficient of viscosity is more for a given medium. Why?

A. According to stoke's law; the terminal velocity is given by $v_T = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{\eta}$

where r $\rightarrow$ radius of the spherical body

$\eta \rightarrow$ Co-efficient of viscosity

$g \rightarrow$ Acceleration due to gravity

$\rho \rightarrow$ Density of the falling spherical body

$\sigma \rightarrow$ Density of the fluid through which the spherical body is traveling

$$\therefore V_T \propto \frac{1}{\eta}$$

Thus if co-efficient of viscosity is more, then the terminal velocity is less for a given medium.

4. **Obtain the expression for Stokes law.**

A. **Stoke's law:**

It states that viscous force acting on the spherical body which is moving with a velocity (v) is directly proportional to

(iii) Coefficient of viscosity of fluid η

(iv) Radius of the spherical body(r)

(iii) Velocity of the body (v)

$F \propto \eta r v$ Or $F = K\eta r v$, where K is proportionality constant K = 6π

$\therefore F = 6\pi\eta r v$

VERY SHORT ANSWER QUESTIONS

1. Why is that a liquid set in rotation comes to rest after some time?

A: When a liquid is set in rotation, there exists a tangential force between the layers of the liquid called viscous drag (or) viscous force. This viscous drag or internal friction makes the liquid to come to rest.

2. When water and castor oil are set rotating which comes to rest earlier? Why

A: Caster oil. The viscous forces are more in caster oil.

3. When water flows through a pipe, which of the layers moves fastest and slowest?

A. The water layer which moves along the axis of the pipe is fastest. The water layer which is in contact with the pipe moves slowest.

4. Why is that we need a constant driving force for the maintenance of the flow of the oil through pipe lines in oil refineries.

Sol: Constant driving force is required to over come the fluid resistance offered by the pipes to flow of oil.

5. Why the machine parts are jammed in winter?

Sol: In winter as the temperature is low, the viscosity of the engine oil and lubricant increases. Hence the machine parts are jammed in winter.

6. Hotter liquids flows faster then cold ones. Explain.

Sol: As temperature increases viscosity decreases and hence hotter liquids flows faster than colder ones.

7. What is SI unit of viscosity? What is its CGS unit?

Sol: SI unit: NSm^{-2} (or) $Kgm^{-1}S^{-1}$ (or) Decapoise

CGS unit: dyne cm^{-2} (or) $gm\,cm^{-1}S^{-1}$ (or) poise

8. Show that 10 poise = $1 Nm^{-2} S$.

Sol: Deca poise = $1 NSm^{-2} = 10^5\, dy \times S \times (100\, cm)^2$

$1\,\text{NSm}^{-2} = 10\,\text{paise}$

9. What are units and dimensions of the coefficient of viscosity?

Sol: SI unit : Nsm^{-2} (or) $\text{Kg m}^{-1}s^{-1}$ (or) Deca paise

Dimension : $[ML^{-1}T^{-1}]$

10. What is velocity gradient? State its unit.

Sol: The change in velocity between the layers of a liquid per unit distance perpendicular to the flow of the liquid is called velocity gradient.

Unit : Sec^{-1}

11. According to Newton, how is the viscous force in a fluid dependent on various factors?

Sol: According to Newton, the viscous force (F) is directly proportional to (1) surface area (A) of the layer and (2) velocity gradient $\left(\frac{dv}{dx}\right)$

$F \propto A$

$\propto \frac{dv}{dx}$

$F = -\eta A \frac{du}{dx}$

Where η is called the coefficient of viscosity. The negative sign indicates that viscous force is opposite to the direction of flow.

12. Terminal velocity is more if surface area of the body is more. True or false. Give reason?

Sol: Terminal velocity $(u_T) = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - \sigma)$

$N_T \propto r^2 \propto$ Surface area

As surface area increases, terminal velocity also increases.

EXERCISE 1

1. Rain drops falling through air acquire terminal velocity. If a rain drop of a given radius is found to fall on the ground at a speed of 20 cm/s, what would be the speed of a rain drop of double radius?

Sol: $V_t \propto r^2 \Rightarrow \frac{V_2}{V_1} = \frac{r_2^2}{r_1^2} \Rightarrow V_2 = 80 cm/s.$

2. A hose shoots water straight up for a distance of 2.5 m. The end opening on the hose has an area of 0.75cm^2. What is the speed of the water as it leaves the hose? How much water will comes out in one minute?

Sol: $v = \sqrt{2gh}$

$v = \sqrt{2 \times 9.8 \times 2.5} = 7 m/s$

The rate of flow of water in 1 min = Av =

$$(0.75)\times 700\times 60 = 0.75\times 42\times 10^3 = 31.5 lt$$

EXERCISE 2

1. Water in a storage tank stands 2.5 m above the level of a valve in the side of the tank with what speed the water will rush out, neglecting friction?

A. h = 2.5 m v = ?

$$v = \sqrt{2gh} = \sqrt{2\times 9.8\times 2.5} = \sqrt{49} = 7 m/s$$

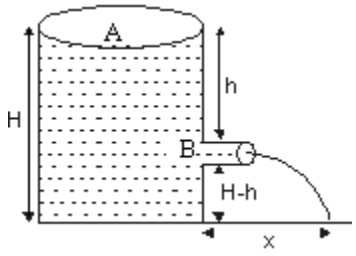
2. A tank is filled with water to a height H. There is a small hole at a depth h below water surface in the wall of the tank. At what distance from the foot of the wall, the water coming out of hole strikes the floor?

A. As the vertical velocity of liquid at the orifice is zero and it is at a height (H – h) from the base, the time taken by the liquid to reach the base level.

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$x = vt = \sqrt{2gh}\times\sqrt{2(H-h)/\mathrm{g}} \qquad (\therefore V = \sqrt{2gh})$$

$$x = 2\sqrt{\frac{gh(H-h)}{g}} = 2\sqrt{h(H-h)}$$



3. A tank is filled with water to a height H. At what depth h the hole may be made in the tank that the emerging horizontal stream of water ground at maximum distance from the base of the tank.

A. $x = 2\sqrt{h(H-h)}$

When x is maximum i.e. $\frac{dx}{dh} = 0 \Rightarrow \frac{d}{dh}(Hh - h^2) = 0 \Rightarrow h = \frac{H}{2}$

4. A tank is filled with water to a height H. At what depth h the hole may be made in the tank, that the emerging horizontal stream of water ground at what maximum distance from the base of the tank?

Ans. $x = 2\sqrt{h(H-h)}$

From the above x is maximum when $h = \frac{H}{2}$

$$\therefore \mathrm{X}_{\max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} \Rightarrow \mathrm{X}_{\max} = 2\times\frac{H}{2} = H$$

5. In the flow of a viscous liquid over a flat horizontal surface, the velocity is 0.5 cm/s at 1mm distance above the flat surface at rest. What is the velocity gradient?

A. $v_1 = 0$, $v_2 = 0.5$ cm/s = $0.5\times 10^{-2} m/s$; x = 1mm = 10^{-3} m

Velocity gradient $\frac{dv}{dx} = \frac{0.5\times 10^{-2} - 0}{10^{-3}} = 5 s^{-1}$

6. What is the radius of a small oil drop which falling through air acquires a terminal velocity of 1 mm/s. The density of oil and air are 0.91 g/cm^3 and 1.3g/litre respectively and $\eta = 1.8\times10^{-4}$ *poise for air*.

A. $$\text{Terminal velocity } V_T = \frac{2r^2 g(\rho-\sigma)}{9\eta}$$

$$r^2 = \frac{9v_T\,\eta}{2g(\rho-\xi)} = \frac{9\times10^{-1}\times1.8\times10^{-4}}{2\times980(0.91-1.3\times10^{-3})} \quad \Rightarrow \quad r = 3\times10^{-4}\ cm$$

7. An air bubble of radius 1cm rises through a liquid of density 1.5 g/cc, at a constant speed of 0.25 cm/s. What is coefficient of viscosity of liquid, neglecting the density of air?

A. r=1cm, ρ = 1.5 g/cc

v = 0.25 cm/s, η =?

$$\eta = \frac{2r^2 g\rho}{9v_T} = \frac{2\times1\times980\times1.5}{9\times0.25} = 1.30\times10^3 \quad poise$$

<u>ASSESS YOURSELF</u>

1. Do we have viscous force acting between the layers even if there is no relative motion?

A. No.

2. Water and castor oil are taken in two different bottles to the same level are shaken violently and kept on a table Which will come to rest first.

A. Castor oil, because it is a viscous fluid.

3. A sphere falls in air with terminal velocity of 50 m/s. If the same sphere is allowed to fall in vacuum. What is its terminal velocity?

A. There will be no terminal velocity.

4. A spherical ball is dropped in a long column of viscous liquid. Describe its motion.

A. First its speed increases then it decreases and after some time a constant velocity called terminal velocity is attained.

5. When does a solid dropped in a liquid attain terminal velocity?

A. When the net force on the solid is zero, it attains a terminal velocity.