Analytical Geometry

Main Points:

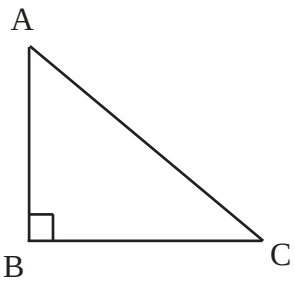
- The slope of X-axis is zero and the slope of Y-axis is undefined.
- The slope of a line passing through (x_1, y_1) and (x_2, y_2) i.e $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Distance between two points (or) length of the line segment = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Mid point of a line segment = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Internally dividing point = $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$
- Externally dividing point = $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$
- Centroid G(x, y) = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
- Arc of triangle $\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$ sq.units
- Slope form of a line is y = mx
- Slope intercept form of a line is y = mx + c
- Slope point form of a line is $y - y_1 = m(x - x_1)$
- Two-points form of a line is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Two intercepts form of a line is $\frac{x}{a} + \frac{y}{b} = 1$

4 Marks Questions

1. If the triangle formed with A (8, –10) B (7, –3) and C (O, P) is right angled with right angle at B, find 'P'.

Sol:

A (8, –10), B (7, –3), C (O, P)



A (8, –10) B (7, –3)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-8)^2 + (-3+10)^2}$

$= \sqrt{(-1)^2 + 7^2}$

$= \sqrt{50}$ units

B (7, –3) C (O, P)

$BC = \sqrt{(0-7)^2 + (P+3)^2}$

$= \sqrt{49 + (P+3)^2}$

A (8, –10) C (O, P)

$AC = \sqrt{(0-8)^2 + (P+10)^2}$

$= \sqrt{64 + (P+10)^2}$

Since ΔABC is right angled at B,

$AC^2 = AB^2 + BC^2$

$$\left[\sqrt{64 + (P+10)^2}\right]^2 = \left(\sqrt{50}\right)^2 + \left(\sqrt{49 + (P+3)^2}\right)^2$$

$$64 + P^2 + 20P + 100 = 50 + 49 + P^2 + 6P + 9$$

$20P - 6P = 108 - 164$

$14P = -56$

$\boxed{P = -4}$

2. Find the co-ordinates of the points of trisection of a segment joining A(–3, 2), B(9, 5).

Sol: A (–3, 2) B (9, 5)

Trisecting points divide the line segment in the ratio of 1:2 and 2:1

Case (i): m:n = 1:2, A(–3, 2), B(9, 5)

Internally dividing point $= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$$= \left(\frac{1(9) + 2(-3)}{1+2}, \frac{1(5) + 2(2)}{1+2}\right)$$

$$= \left(\frac{9-6}{3}, \frac{5+4}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{9}{3}\right)$$

$= (1, 3)$

Case (ii): m:n = 2:1, A(–3, 2) B(9, 5)

Internally dividing point $= \left(\frac{2(9) + 1(-3)}{2+1}, \frac{2(5) + 1(2)}{2+1}\right)$

$$= \left(\frac{18-3}{3}, \frac{10+2}{3}\right)$$

$$= \left(\frac{15}{3}, \frac{12}{3}\right)$$

$$= (5, 4)$$

$\therefore$ Required trisecting points are (1, 3) and (5, 4)

3. If the area of the triangle formed by the vertices (t, 2t), (–2, 6), (3, 1) is 5 sq. units then find 't'.

Sol: (t, 2t), (–2, 6), (3, 1), Δ = 5 sq. units

Area of triangle $\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1+y_2)\right|$ sq.units

$$5 = \frac{1}{2}\left|t[6-1]+(-2)[1-2t]+3[2t-6]\right|$$

$$5\times 12 = \left|t(5)+[-2+4t]+6t-18\right|$$

$$10 = \left|5t-2+4t+6t-18\right|$$

$$10 = \left|15t-20\right|$$

$$\Rightarrow \left|15t-20\right| = 10$$

$\Rightarrow$ 15t – 20 = 10 (or) 15t – 20 = –10
$\Rightarrow$ 15t = 10 + 20 (or) 15t = –10 + 20
$\Rightarrow$ 15t = 30 (or) 15t = 10

$\Rightarrow \boxed{t = 2}$ or $\boxed{t = \frac{2}{3}}$

$\therefore$ t = 2 (or) t = 2/3

4. Find the area of a triangle enclosed between the co-ordinate axes and the line joining (8,–3) and (–4, 12).

Sol: (8, –3), (–4, 12)

Two points form of a line is $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$y+3 = \frac{12+3}{-4-8}(x-8)$$

$$y+3 = \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{-12}}}(x-8)$$

$$-4(y+3) = 5(x-8)$$
$$-4y-12 = 5x-40$$

$\Rightarrow 5x + 4y - 40 + 12 = 0$

$\Rightarrow 5x + 4y - 28 = 0$

Put x = 0	Put y = 0
4y –28 = 0	5x–28 = 0
4y = 28	5x = 28
y = 7	x = 28/5
$\therefore$ y-intercept (b) = 7	$\therefore$ x-intercept $(a) = \frac{28}{5}$

Area of triangle, enclosed between the co-ordinate axes = $\frac{1}{2}|ab|$ sq.units

$$\frac{1}{2}\left|\frac{28}{5}\times 7\right|$$

$$= \frac{1}{\cancel{2}}\times\frac{\overset{14}{\cancel{28}}}{5}\times 7$$

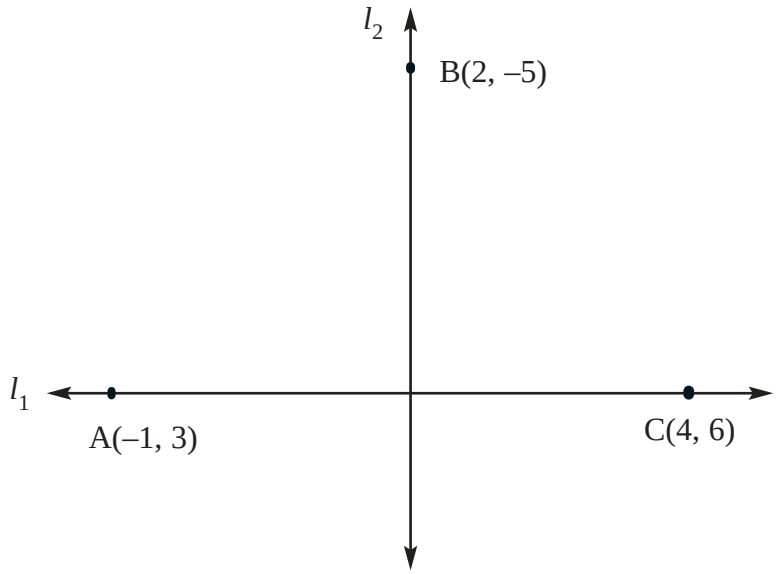
$$= \frac{98}{5}\text{ sq.units}$$

5. Find the equation of a line passing through B and is perpendicular to AC if A(–1, 3), B(2,–5) and C(4, 6)

Sol: Given line l_1

Passes through (–1, 3) and (4, 6)

l_1 slope ie $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{4-(-1)} = \frac{3}{5}$



Since l_2 is perpendicular to l_1

Then $m_1 \times m_2 = -1$

$$\frac{3}{5}\times m_2 = -1$$

$$m_2 = -1\times\frac{5}{3}$$

$$m_2 = \frac{-5}{3}$$

This l_2 line to passing through (2, –5)

$\therefore$ Required l_2 equation is

$y - y_1 = m(x - x_1)$

$$y-(-5) = \frac{-5}{3}(x-2)$$

$3(y + 5) = -5x + 10$

$3y + 15 = -5x + 10$

$5x + 3y + 15 - 10 = 0$

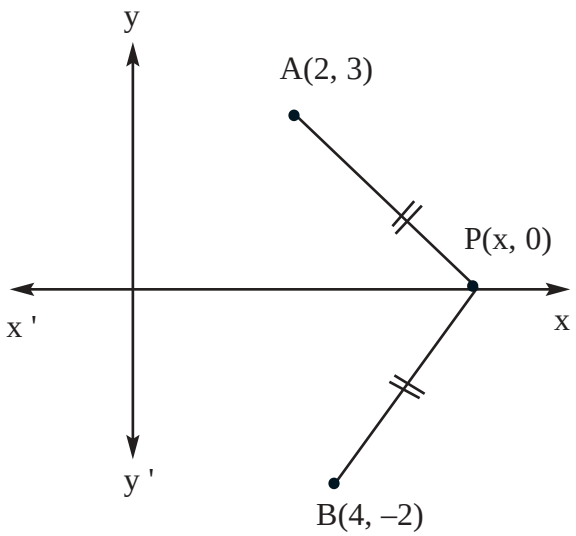
$5x + 3y + 5 = 0$ is required line equation.

2 Marks Questions

1. Find the point on X-axis that is equidistant from (2, 3) and (4, –2)

Sol: Let A(2, 3), B(4, –2) and P(x, 0) be the point on X-axis such that

PA = PB



P(x, 0) A(2, 3)

$$PA = \sqrt{(2-x)^2 + (3-0)^2}$$

$$PA = \sqrt{(2-x)^2 + 9} \quad \text{------ (1)}$$

P(x, 0) B(4, –2)

$$PB = \sqrt{(4-x)^2 + (-2-0)^2}$$

$$PB = \sqrt{(4-x)^2} \quad \text{------- (2)}$$

Given that PA = PB

$$\Rightarrow \sqrt{(2-x)^2 + 9} = \sqrt{(9-x)^2 + 4}$$

$$\Rightarrow 4 + \cancel{x^2} - 4x + 9 = 16 - 8x + \cancel{x^2} + 4$$

$\Rightarrow 8x - 4x = 20 - 13$

$\Rightarrow 4x = 7$

$\Rightarrow x = 7/4$

$\therefore$ Required point is $\left(\frac{7}{4}, 0\right)$

2. One end of a diameter of circle is (2, 3) and it's centre is (–2, 5) then find the co-ordinate of other end of the diameter.

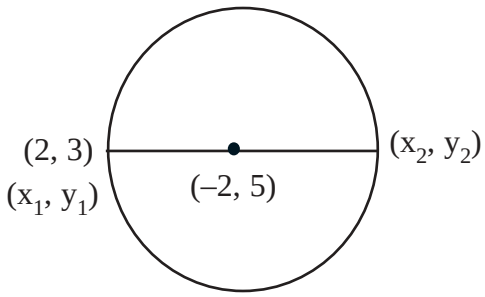
Sol: One end of diameter = (2, 3)

Centre of circle is (–2, 5)

Let the other end of diameter be (x_2, y_2)

Centre of the circle is mid point of diameter.

Mid point of diameter = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



$$(-2,5)=\left(\frac{2+x_2}{2},\frac{3+y_2}{2}\right)$$

$-2=\frac{2+x_2}{2}$	$5=\frac{3+y_2}{2}$
$-4 = 2 + x_2$	$10 = 3 + y_2$
$-4-2 = x_2$	$10-3 = y_2$
$-6 = x_2$	$7 = y_2$

$\therefore$ Other end of the diameter is $(-6,7)$

3. If the three points A (P, 2), B (–3, 4) C (7, –1) are collinear then find 'P'.

Sol: A (P, 2) B (–3, 4) C (7, –1), $\Delta = 0$

$$\Delta=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$0=\frac{1}{2}\left|P[4+1]+(-3)[-1-2]+7[2-4]\right|$$

$$0=\left|P(5)+(-3)(-3)+7(-2)\right|$$

$$0=\left|5P+9-14\right|$$

$$0=\left|5P-5\right|$$

$\Rightarrow |5P-5|=0$ $\qquad [|x|=0 \Rightarrow x=0]$

$\Rightarrow 5P - 5 = 0$

$\Rightarrow 5P = 5$

$\Rightarrow \boxed{P=1}$

4. If P (2, 5) Q(x, –7) find the possible values of *x* so that PQ = 13

Sol: P(2, 5) Q(x, –7) and PQ = 13

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=13$$

$$\sqrt{(x-2)^2+(-7-5)^2}=13$$

S.O.B.S

$(x-2)^2 + (-12)^2 = 13^2$

$(x-2)^2 = 169-144$

$(x-2)^2 = 25$

$(x-2)^2 = (5)^2$

$x-2 = \pm 5$

i) If $x - 2 = 5$

$\Rightarrow x = 5 + 2$

$\Rightarrow x = 7$

ii) If $x - 2 = -5$

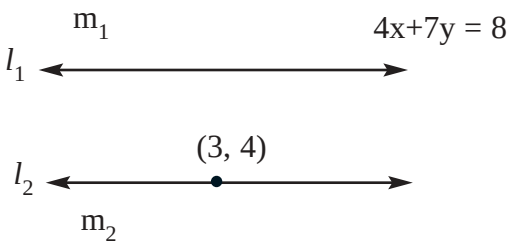
$\Rightarrow x = -5 + 2$

$\Rightarrow x = -3$

$\therefore x = 7$ (or) -3

5. Find the equation of a line passing through (3, 4) and parallel to 4x + 7y = 8

Sol: Given line $l_1 \rightarrow 4x + 7y = 8$



$7y = -4x + 8$

$y = \frac{-4}{7}x + \frac{8}{7}$

$m_1 = \frac{-4}{7}$

Since $l_2 \,//\, l_1 \Rightarrow = m_2 = m_1$

$\Rightarrow m_2 = -4/7$, point (3, 4)

Slope point form of line is

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-4}{7}(x - 3)$

$7y - 28 = -4x + 12$

$4x + 7y - 28 - 12 = 0$

$4x + 7y - 40 = 0$ is required line equation.

1 Mark Questions

1. Find the point of intersection of median of triangle whose vertices are (–1, 0),(5, –2) and (8, –2).

Sol: (–1, 0) (5, –2) (8, –2)

Centroid $G(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{-1+5+8}{3}, \frac{0-2-2}{3}\right)$

$$= \left(\frac{12}{3}, \frac{-4}{3}\right)$$

$$= \left(4, \ -\frac{4}{3}\right)$$

2. Find the externally dividing point of the line joining (2, –4) and (5, 6) in the ratio of 5:3.

Sol: (2, –4), (5, 6), m:n = 5:3

x_1 y_1 x_2 y_2

Externally dividing point = $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - my_1}{m-n}\right)$

$$= \left(\frac{5(5)-3(2)}{5-3}, \frac{5(6)-3(-4)}{5-3}\right)$$

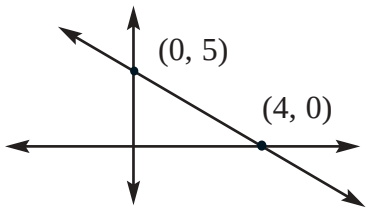
$$= \left(\frac{25-6}{2}, \frac{30+12}{2}\right)$$

$$= \left(\frac{19}{2}, 21\right)$$

3. Find the area of triangle enclosed between the co-ordinate axes and the line joining (4,0) and (0, 5).

Sol: Here x-intercept (a) = 4

y-intercept (b) = 5



$\therefore$ area of triangle $\Delta = \frac{1}{2}|ab|$

$$= \frac{1}{2}|4 \times 5|$$

$$= \frac{1}{2} \times 20$$

= 10 sq.units.

4. Find the intercepts made by the line 2x – 6y + 7 = 0 on the co-ordinate axes.

Sol: 2x – 6y + 7 = 0

Put x = 0

–6y + 7 = 0

–6y = –7

y = 7/6

$\therefore$ y-intercept (b) = 7/6

Put y = 0

2x + 7 = 0

$2x = -7$

$x = -7/2$

$\therefore$ x-intercept (a) = $-7/2$

5. Find the slope of a line joining (5, –2) and (3, 11).

Sol: (5, –2), (3, 11)

Slope (m) = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{11-(-2)}{3-5}$

$= \frac{13}{-2} = -13/2$

Slope (m) = $-13/2$

Multiple Choice Questions

1. The distance between the points (acosθ, 0) and (0, asinθ) is ———

a) a b) $\sqrt{a}$ c) a^2 d) 0

2. The line x = my + C cuts the y-axis at ———

a) $\left(0, \frac{C}{m}\right)$ b) $\left(0, -\frac{C}{m}\right)$ c) $\left(\frac{m}{C}, 0\right)$ d) $\left(-\frac{m}{C}, 0\right)$

3. The centroid of the triangle formed by the sides x = 0, y = 0 and x + y = 6 is ———

a) (0, 0) b) (2, 2) c) (3, 3) d) (6, 6)

4. The slope of line joining (–a, a) and (0, a + a$\sqrt{3}$)

a) $\sqrt{3}$ b) $1/\sqrt{3}$ c) 2a d) 0

5. ——— introduced "Analytical Geometry"

a) J.J. Sylvestor b) Cramer

c) Rene Descartes d) Newton

6. A point on X-axis is ———

a) (2, 3) b) (0, 2) c) (2, 0) d) (0, –3)

7. The slope of a line perpendicular to 5x – 2y + 4 = 0 is

a) 2/5 b) –2/5 c) 2 d) 5/2

8. The y-intercept of 2x + 3y – 6 = 0 is ———

a) 1/3 b) 1/2 c) 3 d) 2

9. ax + by + c = 0 represents a straight line if and only if ———

a) $|a| + |b| = 0$ b) $|a| \neq |b|$ c) $|a| + |b| > 0$ d) $|a| + |b| \neq 0$

10. Gradient form of a line is ———

a) $y - y_1 = m(x - x_1)$ b) $y = mx + c$

c) $\frac{x}{a} + \frac{x}{b} = 1$ d) $y = mx$

11. The slope of a line making an angle 45° with the positive direction of X-axis is ———

a) 1 b) 2 c) 3 d) $\sqrt{3}$

12. The midpoint of $(\sin^2\alpha, \sec^2\alpha)$ and $(\cos^2\alpha, -\tan^2\alpha)$ is ———

a) (1, –1) b) (–1, 1) c) (1, 1) d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

13. Two lines are parallel, if their slopes are ———

a) Zero b) Unequal c) Equal d) Undefined

14. Which of the following is parallel to 2x – 3y = – 8

a) 3x – 2y + 8 = 0 b) 3x – 2y + 6 = 0

c) 2x – 3y + 4 = 0 d) 2x + 3y + 8 = 0

15. If two lines intersect at right angle, then the product of their slopes is ——

a) 0 b) 1 c) –1 d) ∞

KEY

1) a	2) b	3) b	4) a	5) c
6) c	7) b	8) d	9) d	10) d
11) a	12) d	13) b	14) c	15) c

Fill in the Blanks

1. The slope of a line parallel to 3x – 2y + 1 = 0 is ———
2. The line y = 5 is parallel to ——— axis
3. The slope of Y-axis is ———
4. Centroid divides the median in the ratio of ——— from the midpoint of opposite side.
5. $\Delta = 0 \Leftrightarrow$ the gives points are ———
6. Parallel lines differ only by ———
7. The slope of $\frac{x}{a} + \frac{y}{b} = 1$ is

8. Two intercepts form of line is ———
9. $ax + by + c = 0$ represents a ——— if $|a| + |b| \neq 0$
10. Slope of a line which makes 150° with the positive directions of X-axis is ———

KEY

1) 3/2	2) X	3) Undefined	4) 1:2
5) Collinear	6) Constant	7) –b/a	8) $\frac{x}{a}+\frac{y}{b}=1$
9) Straight line	10) $-\frac{1}{\sqrt{3}}$		