SURFACE TENSION

CAPILLARITY- EXCESS PRESSURE

POINTS TO REMEMBER

1. **Capillarity:** The rise (or) fall of the liquid in a capillary tube is called capillarity.

2. The surface tension of a liquid by capillary rise method $T = \dfrac{r\left(h + \dfrac{r}{3}\right)\rho g}{2\cos\theta}$

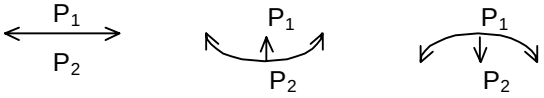
   If $\dfrac{r}{3}$ is negligible when compared to 'h'

   $T = \dfrac{rh\rho g}{2\cos\theta}$ ρ - density, h - height of the liquid in capillary tube,

   r - radius of the capillary tube, θ - angle of contact.

3. For any liquid $h \propto \dfrac{1}{r} \Rightarrow hr = \text{constant}$

4. If the liquid surface is plane, the net force on the liquid molecule on the surface is zero.



5. If the liquid meniscus is concave upwards, the net force is directed out wards.
6. If the liquid meniscus is convex upwards, the net force is directed inwards.
7. Excess pressure in side the liquid drop of radius R and surface tension T is

   $P = \dfrac{2T}{R}$

8. Excess pressure inside the soap bubble $P = \dfrac{4T}{R}$
9. If an air bubble of radius r is a lake

   1) At a depth h, $P = P_0 + hdg + \dfrac{2T}{R}$

   2) At the surface inside the liquid

   $$P = P_0 + \frac{2T}{R}$$

   3) Outside the surface $P = P_0 + \dfrac{4T}{R}$

10. Excess of pressure in liquid drop $P = \dfrac{2T}{r}$

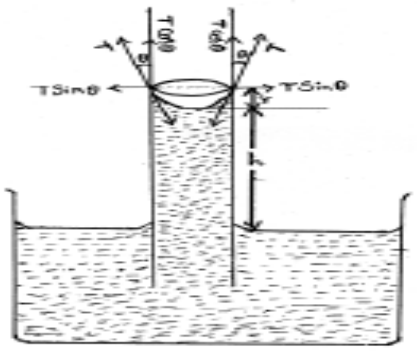
11. Excess of pressure in soap bubble $P = \frac{4T}{r}$

LONG ANSWER QUESTION

1) Describe how you determine the surface tension of a liquid using capillary rise method with relevant theory. (July 2001)

Ans: Surface tension - Capillary rise method:

Theory:-



Let a capillary tube of inner radius 'r' is dipped into a liquid water whose angle of contact (θ) is less than 90^0. Let the liquid rises in to the capillary tube to a height 'h' and let T be the surface tension of the liquid. Surface tension of the liquid is resolved into two components 1) $T\cos\theta$, parallel to glass wall and 2) $T\sin\theta$, perpendicular to glass wall of capillary tube.

The horizontal components of 'T' at the diametrically opposite points cancel each other. But the vertical components around the circumference of the circle of contact remain and will be added.

Net upward force $F = T\cos\theta \; x \; 2\pi r$

This net upward force due to surface tension is balanced by the weight of the liquid raised in the capillary tube.

Weight of the liquid in the tube = volume of the liquid of height h $\times$ density of liquid $\times$ g

$$= \left(\pi r^2 h + v\right) \rho g$$

Where v is the volume of the liquid in meniscus

Volume of liquid in the meniscus = volume of the cylinder of height r - volume of the hemisphere

$$= \pi r^2 \, r - \frac{2}{3}\pi r^3 = \pi r^3 - \frac{2}{3}\pi r^3 = \frac{\pi r^3}{3}$$

$\therefore$ Volume of the raised liquid $= \pi r^2 h + \frac{1}{3}\pi r^3 = \pi r^2\left(h + \frac{r}{3}\right)$

Weight of the liquid rose $= \pi\, r^2\left(h + \frac{r}{3}\right) \times \rho \times g$

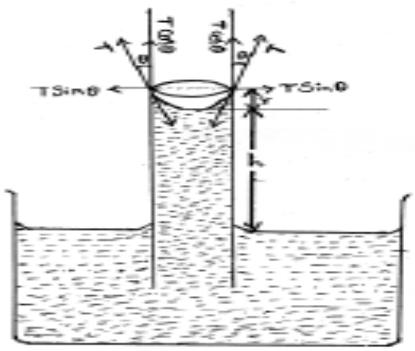
$\therefore\ T = \dfrac{r\left(h + \frac{r}{3}\right)\rho\, g}{2\cos\theta}$

For a narrow tube, $T = \dfrac{rh\rho g}{2\cos\theta}$

SHORT ANSWER QUESTIONS:-

1. Deduce an expression for the rise of liquid in a capillary tube. (June 2003)

Ans:



Let a capillary tube of inner radius 'r' is dipped into liquid water whose angle of contact (θ) is less than 90^0. Let the liquid rises in to the capillary tube to a height 'h' and let T be the surface tension of the liquid. Surface tension of the liquid is resolved into two components 1) $T\cos\theta$, parallel to glass wall and 2) $T\sin\theta$, perpendicular to glass wall of capillary tube.

The horizontal components of 'T' at the diametrically opposite points cancel each other. But the vertical components around the circumference of the circle of contact remain and will be added.

Net upward force $F = T\cos\theta\, x\ 2\pi r$

This net upward force due to surface tension is balanced by the weight of the liquid raised in the capillary tube.

Weight of the liquid = volume of the liquid of height h $\times$ density of liquid $\times$ g

$$= \left(\pi r^2 h + v\right)\rho g$$

Where v is the volume of the liquid in meniscus

Volume of liquid in the meniscus = volume of the cylinder of height r - volume of the hemisphere

$$= \pi r^2 \, r - \frac{2}{3}\pi r^3 = \pi r^3 - \frac{2}{3}\pi r^3 = \frac{\pi r^3}{3}$$

$\therefore$ Volume of the raised liquid $= \pi r^2 h + \frac{1}{3}\pi r^3 = \pi r^2 \left(h + \frac{r}{3} \right)$

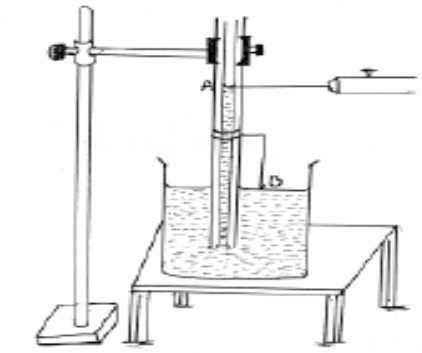
Weight of the liquid rose $= \pi \, r^2 \left(h + \frac{r}{3} \right) \times \rho \times g$

$$\therefore \quad T = \frac{r\left(h + \frac{r}{3} \right) \rho \, g}{2\cos\theta}$$

For a narrow tube, $T = \frac{rh\rho g}{2\cos\theta}$ or $h = \frac{2T\cos\theta}{r\rho g}$

2. Describe an experiment for the determination of surface tension of a liquid? (March 03, Mar 02)

Ans: A long wire bent twice at right angles is taken and is attached to a clean dry glass capillary tube. This tube is vertically dipped in a glass beaker containing the liquid and the liquid rises into the tube upto a certain extent. The height of the tube is adjusted so that the tip of wire B, just touches the surface of the liquid in the beaker. Microscope is focused to the lower meniscus 'A' of liquid in the tube and reading R_1 is noted .Then the liquid beaker is slowly removed without disturbing the tube and wire. Then the microscope is focused against tip of 'B' and reading R_2 is noted.



The difference between the two readings of microscope gives the height of raised liquid (h). The diameter of the capillary tube is measured using microscope by keeping the tube horizontal and radius (r) is determined. The surface tension 'T' of the given liquid can be determined on substituting the values of h, r, d and g in the equation, Where is density of the liquid, g - acceleration due to gravity

$$T = \frac{\left(h + \frac{r}{3}\right) r\rho g}{2}$$

3. Explain the significance of capillarity in daily life? (March 2006)

Ans: a . Oil ascends in a wick due to capillarity.

b. Flow of ink through a nib is due to capillarity.
c. A painter's brush under water has its hair spread but on withdrawal from water they adhere to each other due to surface tension.
d. Ploughing of land brings moisture to the top by capillary action.
e. Sap rises up to the various parts of a plant through the capillaries in the plant.

4. Explain the phenomenon of capillarity.

Ans: **Capillarity:** The rise or fall of a liquid in a capillary tube due to surface tension is called capillarity.

a)When a glass capillary tube is dipped into a liquid of angle of contact less than 90º; (ex: water, acetone etc.) the liquid rises up in the tube, till the equilibrium is attained. Adhesive forces are greater than cohesive forces.

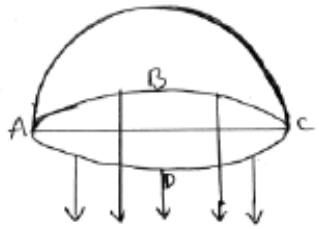
b) When a glass capillary tube is dipped into a liquid of angle of contact greater than 90º (ex: mercury), the liquid gets depressed in the tube till the equilibrium is attained. Cohesive forces are greater than adhesive forces.

c) When a glass capillary tube is dipped into a liquid of angle of contact greater than 0º (ex: Pure water in a silver vessel) the liquid neither rises nor depresses. Cohesive and adhesive forces are equal.

5. Obtain an expression for the excess pressure inside a liquid drop. (March 2000)

Ans: Considering the surface tension alone the potential energy will be minimum for minimum surface area and hence the small liquid drops acquire a spherical shape. The net force on the molecules present near the surface due to surface tension of a spherical liquid drop is inwards . Hence pressure inside the drop will be greater than the pressure outside.

Now consider the equilibrium along any diametrical plane of spherical drop due to excess pressure P acting on the area πr^2 .



Under equilibrium, the upward force due to excess pressure 'P' is equal to the downward force due to surface tension, $\therefore\ P(\pi r^2) = T(2\pi r)$

$\therefore$ Excess pressure in liquid drop $P = \frac{2T}{r}$

VERY SHORT ANSWER QUESTIONS

1) Water rises to a length of 6cm in a capillary tube of certain radius on earth, Find the height up to which the water rises in the same capillary tube on the moon, where the acceleration due to gravity is $\frac{1}{6}$ th that of the earth ?

A) $h_1 = 6$ cm ; $h_2 = ?$; $g_1 = g$; $g_2 = g/6$

$h\alpha\frac{1}{g} \Rightarrow h_1 g_1 = h_2 g_2 \Rightarrow 6 \times g = h_2 \times \frac{g}{6}$

$h_2 = 36$ cm

2) State two examples of capillarity.

A) a . Oil ascends in a wick due to capillarity.

b. Flow of ink through a nib is due to capillarity.

3) What is the excess pressure on the air bubble inside the liquid?

A) $P = \frac{2T}{r}$ Where T = Surface Tension and r = radius of air bubble

Exercise

1. What should be the radius of a capillary tube if the water has to rise to a height of 6cm in it? Surface tension of water = 7.2×10^{-2} Nm^{-1}. (Density of water $10^3 kg/m^3$, $g = 9.8\ m/s^2$)

Sol: $h = 6$ cm $= 6 \times 10^{-2}$ m ; $T = 7.2 \times 10^{-2}$ Nm^{-1} ; $\theta = 0$

$$T = \frac{hrdg}{2\cos\theta} \Rightarrow 7.2\times10^{-2} = \frac{(6\times10^{-2})(r)(1000)(9.8)}{2\times1} \Rightarrow r = 0.24 \times 10^{-3}\ \text{m}$$

2. In an experiment with capillary tube a liquid of density 0.75×10^3 kgm^{-3} rises to a height of 19.8×10^{-3} m in a capillary tube of diameter 2mm. Find the surface tension of the liquid (angle of contact is 8º).

Sol: $r = = 1mm = 1\times10^{-3}m$; $h = 19.8\times10^{-3}m$; $d = 0.75\times10^{3}kg/m^{3}$; $\theta = 8^{o}$

$$T = \frac{hrdg}{2\cos\theta} = \frac{(19.8\times10^{-3})\times(1\times10^{-3})\times(0.75\times10^{3})\times9.8}{2\times0.9903}$$

$$\therefore T = 73.5\times10^{-3}\,Nm^{-1}$$

3. The rise of liquid in a capillary tube of certain radius when immersed in a liquid is 4.2cm. What is the rise in the liquid when a capillary tube of radius 3/2 times the previous radius is used?

Sol: $r_2 = \frac{3}{2}r_1$; $h_1 = 4.2cm$; $h_2 = ?$; $T = \frac{hrdg}{2\cos\theta}$.

$\therefore$ hr = constant or $h_1 r_1 = h_2 r_2$ or $h_2 = \frac{h_1 r_1}{r_2} = \frac{4.2\times r_1}{3r_1}\times2 = 2.8\,\text{cm}$

4. The diameter of a soap bubble is 8 mm and the surface tension of soap solution is 4×10^{-2} Nm^{-1}. Find the excess pressure inside the bubble.

Sol: $r = 4\times10^{-3}$ m ; $T = 4\times10^{-2}$ Nm^{-1}

$$P = \frac{4T}{r} = \frac{4\times4\times10^{-2}}{4\times10^{-3}} = 40\,Nm^{-2}$$

5. The radius of mercury drop at 20ºC is 3 mm. If the surface tension of mercury at this temperature is 4.65×10^{-1} Nm^{-1}. Find the excess pressure inside the liquid (mercury) drop (March 2009, March 01)

Sol: $T = 4.65x10^{-1}$ Nm^{-1} ; $r = 3mm = 3\times10^{-3}$ m

$$\therefore P = \frac{2T}{r} = \frac{2\times4.65\times10^{-1}}{3\times10^{-3}} = 310\,Nm^{-2}$$

EXERCISE 2

1. How high methyl alcohol rise in a glass tube of 0.06 cm in diameter? The surface tension of methyl alcohol is 0.023 N/m and the density is 0.8 gm cm^{-3}. Assume that the angle of contact is zero.

Sol: D = 0.06 cm = 0.06×10^{-2} m ; r = 0.03×10^{-2} m ; T = 0.023 N/m ; d = 0.8 gm $cm^{-3} = 0.8\times10^{3}$ kg/m^3

$\theta = 0^0$

$$T = \frac{hrdg}{2\cos\theta} \Rightarrow h = \frac{2T\cos\theta}{rdg} = \frac{2\times\left(\cos 0^0\right)}{0.03\times10^{-2}\times 0.8\times10^{3}\times 9.8} = 2\times10^{-2}\ m\ =\ 2\ cm$$

2. What should be the radius of a capillary tube ,if the water has to rise to a height of 6 cm in it. Surface tension of water is $7.2\ X10^{-2}\ N/m.\ \left(g = 10m/\sec^2, d = 10^3 kg/m^3\right)$ **(March 2005)**

Sol: h = 6cm = $6\ X10^{-2}$ m ; T = $7.2\ X10^{-2}$ N/m ; $\theta = 0^0$, d = 1000 kg/m^3 ; r = ?

$$T = \frac{hrdg}{2} \Rightarrow r = \frac{2T}{hdg} = \frac{2\ X\ 7.2\ X\ 10^{-2}}{6\ X\ 10^{-2}\ X\ 1000\ X\ 10} = 0.24\ mm$$

3. Calculate the surface tension of water assuming that the density of water is $10^3\ kg/m^3$**, angle of contact is 0º and acceleration due to gravity is** $9.8\ m/sec^2$**, radius of capillary tube is 0.2 mm, and rise of water in the capillary tube is 6.6 cm.**

Sol: d = 10^3 kg/m^3 ; $\theta = 0^0$; g = 9.8 m/sec^2 ; r = 0.2 mm = 0.2×10^{-3} m ; h = 6.6 cm = 6.6×10^{-2} m

$$T = \frac{hrdg}{2\cos\theta} = \frac{6.6\times10^{-2}\times 0.2\times10^{-3}\times10^{3}\times 9.8}{2} = 6.468\ X\ 10^{-2}\ N/m$$

4. Determine the height to which water rises when capillary tube of radius 0.2 mm is dipped vertically in beaker containing water. Surface tension of water is 7.0×10^{-2} N/m. Angle of contact is 0º and density of water is 10^3 kg/m^3. [g = 10m/s^2]

Sol: r = 0.2 mm = 0.2×10^{-3} m ; T = 7.0×10^{-2} N/m ; $\theta = 0^0$; d = 10^3 kg/m^3 ; $g = 10m/s^2$ h = ?

$$T = \frac{hrdg}{2\cos\theta} \Rightarrow h = \frac{2T\cos\theta}{rdg} = \frac{2\times 7\times10^{-2}\times\cos 0^0}{0.2\times10^{-3}\times10^{3}\times 10} = 7\times10^{-2}\ m = 7\ cm$$

5. Find the depression of the meniscus in the capillary tube when a capillary tube of diameter 0.4 mm is dipped in a beaker containing mercury. Density of mercury is

13.6×10^3 kg/m^3. Angle of contact is 130^o and surface tension of mercury is 0.49 N/m (g = 9.8 m/sec^2)

Sol: D = 0.4 mm = 0.4×10^{-3} m ; r = 0.2×10^{-3} m ; d = 13.6×10^3 kg/m^3 ; $\theta = 130^0$;cos (130) = -0.6428

T = 0.49 N/m ; h = ?

$$T = \frac{hrdg}{2\cos\theta},\ h = \frac{2T\cos\theta}{rdg} = \frac{2\times 0.49\ (-0.6428)}{0.2\times 10^{-3}\times 13.6\times 10^3\times 9.8} = -0.024 \text{ m}$$

6. The tube of a mercury barometer has an internal diameter of 4 mm. How much error does surface tension introduce in readings? Density of mercury = 13. 6 × 10^3 Kg m^{-3}, surface tension = 0.49 Nm^{-1} and angle of contact is 135^o.

Sol : d = $13.6\times10^3\, kg\, m^{-3}$; T = 0.49 Nm^{-1} ; $\theta = 135°$

$$T = \frac{hrdg}{2\cos\theta} \Rightarrow h = \frac{2T\cos\theta}{rdg},$$

$$h = \frac{2\times0.49\times\cos135°}{2\times10^{-3}\times13.6\times10^{-3}\times9.8} = 0.024 \text{ m.}$$

7. If the diameter of a soap bubble 10mm and its surface tension is 0.04 N/m. Find the excess pressure inside the bubble.

Sol: D = 10 mm ; r = 5 mm = 5×10^{-3} m ; T = 0.04 N/m

$$T = \frac{4T}{r} = \frac{4\ X\ 0.04}{5\ X\ 10^{-3}} = 32 \text{ N/m}^2$$

8. A liquid rises to a height of 50 cm in a capillary tube of diameter 0.04mm. If the density of the liquid is 0.8 × 10^3 kg m^{-3} and angle of contact is 20^0 find the surface tension of the liquid?

Sol: h = 50 cm = 50×10^{-2} m ; D = 0.04 mm; r =0.02 mm = 0.02×10^{-3} m; d = 0.8×10^3 Kg m^{-3}

$\theta = 20°$,

$$T = \frac{hrdg}{2\cos\theta} = \frac{50\times10^{-2}\times0.02\times10^{-3}\times0.8\times10^3\times10}{2\cos20^o} = 0.42 \text{N/m}$$

9. Two soap bubbles of radii R_1 and R_2 are in contact. What is radius of curvature of the contact surface?

Sol : $\frac{4T}{R}=\frac{4T}{R_2}-\frac{4T}{R_1}\Rightarrow\frac{1}{R}=\frac{1}{R_2}-\frac{1}{R_1}\Rightarrow R=\frac{R_1R_2}{R_1-R_2} if\ R_1>R_2$

10. Two soap bubbles have radii R_1 and R_2 in vacuum. If they coalesce under isothermal conditions, what is the new radius of the bubble if T is surface tension of the soap solution.

Sol : $P_1V_1+P_2V_2=PV\Rightarrow\frac{4T}{R_1}.\frac{4}{3}\pi R_1^3+\frac{4T}{R_2}.\frac{4}{3}\pi R_2^3=\frac{4T}{R}.\frac{4}{3}\pi R^3\Rightarrow R^2=R_1^2+R_2^2\Rightarrow R=\sqrt{R_1^2+R_2^2}$

ASSESS YOURSELF

1. A capillary tube immersed in water is taken to moon, where the value of g is g/6. What happens to the rise of the liquid (h) on the moon?

Ans: Height becomes 6h, provided the capillary tube is that long. If it is not that long, it rises to the height of the capillary tube.

2. A capillary tube immersed in water is kept in an ascending lift whose acceleration is equal to g. What happens to the rise in the capillary tube h?

Ans: h/2