SIMPLE HARMONIC MOTION

ENERGY IN S.H.M.

POINTS TO REMEMBER

1. **Potential Energy :**

$P.E. = \frac{1}{2}m\omega^2x^2 + U_0 = \frac{1}{2}m\omega^2A^2\sin^2\theta + U_0$

Where m = mass of S.H.M.

x = displacement of S.H.M. from its mean position

A = amplitude of oscillation

θ = phase angle from its mean position

U_o= P.E. of S.H.M. at its mean position.

During one complete vibration average potential Energy is given by = $1/4\ m\omega^2A^2$

2. **Kinetic Energy:**

i) The K. E. of a particle in S.H.M is given by K. E = $1/2\ m\omega^2 (A^2 - y^2) = 1/2\ m\omega^2A^2 \cos^2(\omega t \pm \phi)$

ii) At mean position (y = o) K. E is maximum

iii) At extreme position (y = A) K. E. is zero.

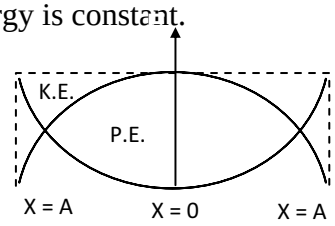
iv) During one complete vibration average kinetic Energy = $1/4\ m\omega^2A^2$

3. **Total Energy:**

i) T. E. = P. E. + K. E. = $1/2\ m\omega^2A^2 + U_0$

ii) When a particle is in S. H. M. At any position T. total energy is constant.

Energy and displacement curve.



4. In SHM, average kinetic energy = average potential energy = half of the total energy, when friction is zero.

5.

Displacement(y)	KE	PE	TE
0	$\frac{1}{2}m\omega^2A^2$	0	$\frac{1}{2}m\omega^2A^2$
0 to $\pm A$	$\frac{1}{2}m\omega^2\left(A^2-y^2\right)$	$\frac{1}{2}m\omega^2y^2$	$\frac{1}{2}m\omega^2A^2$
$\pm A$	0	$\frac{1}{2}m\omega^2A^2$	$\frac{1}{2}m\omega^2A^2$

SHORT ANSWER QUESTIONS :

4. Derive the equation for the kinetic energy of simple harmonic oscillator.

Ans. The velocity of the particle in simple harmonic motion at any position is given by

$V = A\omega\cos\omega t$

The kinetic energy of the particle $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2\cos^2\omega t$

Also, $V = \omega\sqrt{A^2 - y^2}$

$\therefore$ The kinetic energy of the particle $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$

Or $K.E = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$

5. Derive an expression for the potential energy of a simple harmonic oscillator.

Ans. When the displacement of a particle executing S.H.M increases, the restoring force also increases. The restoring force is in the opposite direction to the displacement. Hence work is done in displacing the body against the restoring force. If 'F' is the force at the displacement 'y'

Restoring force F = -m $F = ma = m\omega^2 y$

Work is done in displacing the body through a small displacement dy is given by

W = -F dy = $-m\omega^2 y\,dy$

Now the total work done in displacing the body from 0 to y is given by

$W = \int dW = \int_0^y -m\omega^2 y\,dy = \frac{1}{2}m\omega^2 y^2$

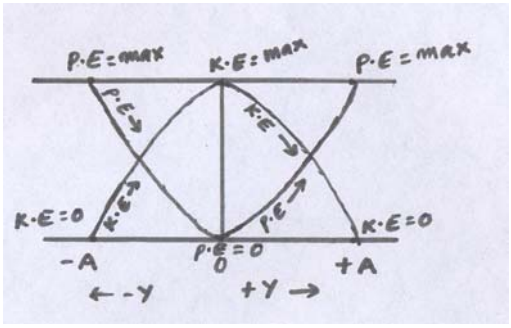
This work done is stored as potential energy

$\therefore P.E = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 Sin^2\omega t$

6. Show that the total energy of a particle in simple harmonic motion is constant at any point on its path.

Ans: Consider a particle of mass 'm' making to and fro motion between the extreme positions A and -A about the mean position 'O' as shown in diagram.

If the body is in the S.H.M and its displacement is 'y' then



Potential energy $P.E = \frac{1}{2} m\omega^2 y^2$ And

Kinetic energy $KE = \frac{1}{2} m\omega^2 \left(A^2 - y^2\right)$

Case - 1: When the body is at displacement 'y' from the mean position,

Total energy is given by E = PE + KE

$$E = \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} m\omega^2 \left(A^2 - y^2\right)$$

$$E = \frac{1}{2} m\omega^2 A^2 \text{ .....(1)}$$

Case-2 : When the body is at the mean position, then y = 0

$$PE = \frac{1}{2} m\omega^2 y^2 = 0$$

$$KE = \frac{1}{2} m\omega^2 \left(A^2 - 0\right) = \frac{1}{2} m\omega^2 A^2$$

Total energy = KE + PE $= \frac{1}{2} m\omega^2 A^2 + 0$

$\therefore$ E = $\frac{1}{2} m\omega^2 A^2 \text{.....}(2)$

Case-III: When the body is at the extreme position y = A

$$PE = \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 A^2$$

$$KE = \frac{1}{2} m\omega^2 \left(A^2 - A^2\right) = 0$$

TE = KE + PE $= 0 + \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m\omega^2 A^2$ .....(3)

From equations (1) (2) & (3) it was proved that the total energy is constant. Thus the total energy of an object under S.H.M is constant at any point on its path.

VERY SHORT ANSWER QUESTIONS

15) How does the energy of a simple pendulum vary as it moves from one extreme to the other during its oscillations?

A): Total energy remains the same from one extreme position to other. Velocity is maximum at mean position.

Exercise 1

11. A particle of mass 10 g executes a linear SHM of amplitude 5 cm with a period of 2 s. Find the P.E. and K.E. $\frac{1}{6}$ s after it has passed through the mean position.

Sol. $m = 10\,g = 10\times10^{-3} = 1\times10^{-2}\,kg$

T = 2 s ; $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$ rad/s.

A = 5 cm $= 5\times10^{-2}\,m$

When $t = \frac{1}{6}s$,

K.E. $= \frac{1}{2}mA^2\omega^2\cos^2\omega t = \frac{1}{2}\times1\times10^{-2}\times\left(5\times10^{-2}\right)^2\left(\pi^2\right)\cos^2\frac{\pi}{6} = 9.255\times10^{-5}\,J$

P.E. $= \frac{1}{2}mA^2\omega^2\sin^2\omega t = \frac{1}{2}\times1\times10^{-2}\times\left(5\times10^{-2}\right)^2\pi^2\sin^2\frac{\pi}{6} = 3.085J\times10^{-5}\,J$

$= 0.11832\,J$

12. A particle is executing SHM with amplitude of 0.2 m. At what distance from the mean position the potential energy of the particle will be equal to its kinetic energy?

Sol. $P.E_y = \frac{1}{2}m^2\omega^2y^2$ and $K.E_y = \frac{1}{2}m^2\omega^2\left(A^2 - y^2\right)$

But $PE_y = KE_y$

$\therefore \frac{1}{2}m^2\omega^2y^2 = \frac{1}{2}m\omega^2\left(A^2 - y^2\right) \Rightarrow y = \frac{A}{\sqrt{2}}$

But A = 0.2 m

$\therefore y = \frac{0.2}{\sqrt{2}} = 0.1414\,m$

13. When a particle is performing linear SHM its K.E. is two times its P.E. at a position A and its P.E. is two times its K.E. at another position B. Find the ratio of K.E. at A to K.E. at B.

Sol. TE at A = K.E. + P.E.

But, K.E. at A = 2 (P.E. at A) $\Rightarrow$ P.E. at A = $\frac{1}{2}$ (K.E. at A)

$\therefore$ T.E. at A = K.E. + $\frac{1}{2}$ (K.E.) = $\frac{3}{2}$ K.E. at A

Similarly, T.E. at B = K.E. + P.E.

But, P.E. at B = 2 (K.E. at B)

T.E. at B = K.E. + 2(K.E.) = 3 K.E. at B

From the law of conservation of energy

T.E. at A = T.E. at B

$\frac{3}{2}$ K.E. at A = 3 K.E. at B

$$\frac{K.E.\,at\,A}{K.E.\,at\,B}=\frac{2}{1}$$

Exercise 2

17. **A body of mass 15 g oscillates about a fixed point with a SHM of amplitude 8 cm. If the body is attracted towards the fixed point, when at a distance of 4 cm from it with a force equal to the weight of 10 g, find (a) the period of oscillation of the mass about the fixed point and (b) the maximum KE of the mass.**

Sol: a) $g = 10 m/s^2$

$$\therefore F = mg = \left(\frac{10}{1000}\right)10 = \frac{1}{10} N$$

But, $F = m\omega^2 x$

$$\Rightarrow \frac{1}{10} = \frac{15}{1000} \times \omega^2 \times \frac{4}{100} \Rightarrow \omega^2 = \frac{10^4}{60}$$

$$\Rightarrow T = \sqrt{\frac{60 \times 4\pi^2}{10^4}} = \frac{\sqrt{240} \times \pi}{100} = 0.4863 \text{ sec}$$

b) Maximum $K.E = \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} . \frac{15}{1000} . \frac{10^4}{60} . \left(\frac{8}{100}\right) . \left(\frac{8}{100}\right) = \frac{8}{1000} = 8 \times 10^{-3} J$

23. A small body of mass 10 gram is making simple harmonic oscillations along a straight line with a time period of $\pi/4$ s and the maximum displacement is 10cm. Find the energy of oscillations. What is the average kinetic energy during one oscillation?

Sol : m = 10 g = $= 0.01kg$; $T = \frac{\pi}{4}\sec$; A = 10 cm = 0.1 m

Total energy $= \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(0.01)\left(\frac{4\pi^2}{\pi^2/16}\right)(0.1)^2 = 32\times10^{-4}J$

Average velocity $= \frac{Total\ dis\tan ce}{total\ time} = \frac{4A}{T} = \frac{4\times0.1}{\pi/4} = \frac{1.6}{\pi}m/\sec$

Average K.E = $\frac{1}{2}\times m\times v_{avg}^2$

$K.E_{avg} = \frac{1}{2}\times0.01\times\frac{1.6}{\pi}\times\frac{1.6}{\pi} = \frac{2.56\times10^{-2}}{19.7} = 12.99\times10^{-4}J$

24. At what displacement is the KE of a particle performing SHM of amplitude 10 cm, three times its PE?

Sol: K.E = 3 P.E and A = 10cm

$\frac{1}{2}m\omega^2\left(A^2 - x^2\right) = 3\left(\frac{1}{2}m\omega^2x^2\right)$

$\therefore x = \frac{10}{2} = 5cm$

25. A particle of mass 2 g is executing SHM of frequency 10 Hz. Find its KE, PE and TE when its displacement is 1 cm. Its amplitude is 4cm.

Sol: m = 0.002 kg ; $n = 10Hz$; $x = 1cm = 0.01m$; A = 0.04 m

$K.E = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

$= \frac{1}{2}\times(0.002)\left(4\pi^2n^2\right)\left[(0.04)^2 - (0.01)^2\right] = 5.91\times10^{-3}J$

P.E $= \frac{1}{2}m\omega^2x^2 = \frac{1}{2}(0.002)\left(4\pi^2\times100\right)(0.01)^2 = 0.394\times10^{-3}J$

$\therefore T.E = P.E + K.E = (5.91 + 0.394)\times10^{-3}J = 6.318\times10^{-3}J$

26) **A particle of mass 0.5 kg is executing SHM along a straight line. Its path length is 10 cm and time period is 8 s. Calculate its KE, PE and TE when its phase angle is $\frac{\pi}{6}$ radian.**

Sol : m = 0.5 kg ; 2A = 10 cm $\Rightarrow A = 5cm = 0.05m$; T = 8 sec; $\phi = \frac{\pi}{6}$

$$x = A\sin\phi = 0.05 \times \sin\left(\frac{\pi}{6}\right) = \frac{0.05}{2} = 0.025m$$

$$K.E = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2} \times 0.5 \times \left(\frac{2\pi}{8}\right)^2\left[(0.05)^2 - (0.025)^2\right]$$

Or $K.E = 2.88 \times 10^{-4} J$

$$P.E = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2} \times 0.5 \times \frac{4\pi^2}{64} \times 0.000625 = 0.962 \times 10^{-4} J$$

ASSESS YOURSELF

1. What is the frequency of KE or PE if the frequency of displacement is f?

A. 2 f is the frequency of both PE and KE.