PERMUTATIONS AND COMBINATIONS -- 2

EXERCISE – 4(e)

I.

1. **If** ${}^{n}C_{4} = 210,$ **find n**

Hint: ${}^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$

$= \dfrac{n(n-1)(n-2).....\left[n-(r-1)\right]}{1.2.3.....r}$

Sol: ${}^{n}C_{4} = 210$

$\Rightarrow \dfrac{n(n-1)(n-2)(n-3)}{1.2.3.4} = 10 \times 21$

$\Rightarrow n(n-1)(n-2)(n-3)$

$= 10 \times 21 \times 1 \times 2 \times 3 \times 4$

$= 10 \times 7 \times 3 \times 2 \times 3 \times 4$

$= 10 \times 9 \times 8 \times 7$

$\therefore n = 10$

2. **If** ${}^{12}C_{r} = 495$ **, find the possible values of r**

Hint ${}^{n}C_{r} = {}^{n}C_{n-r}$

Sol: ${}^{12}C_{r} = 495 = 5 \times 99$

$= 11 \times 9 \times 5 = \dfrac{12 \times 11 \times 9 \times 5 \times 2}{12 \times 2}$

$= \dfrac{12 \times 11 \times 10 \times 9}{1.2.3.4} = {}^{12}C_{4}$ *or* ${}^{12}C_{8}$

$\therefore r = 4$ *or* 8

3. **If** $10.\ {}^{n}C_{2} = 3.\ {}^{n+1}C_{3}$ **find n**

Sol: $10.\ {}^{n}C_{2} = 3.\ {}^{n+1}C_{3}$

$\Rightarrow 10 \times \dfrac{n(n+1)}{1.2} = \dfrac{3(n+1)(n-1)}{1.2.3}$

$\Rightarrow 10 = n + 1$

$\therefore n = 9$

4. **If** ${}^{n}P_{r} = 5040 \; and \; {}^{n}C_{r} = 210$ **find n and r.**

Hint: ${}^{n}P_{r} = r!\,{}^{n}C_{r}$ **and**

${}^{n}P_{r} = n(n-1)(n-2).....\left[n-(r-1)\right]$

Sol: ${}^{n}P_{r} = 5040, \; {}^{n}C_{r} = 210$

$r! = \dfrac{{}^{n}P_{r}}{{}^{n}C_{r}} = \dfrac{5040}{210} = \dfrac{504}{21} = 24 = 4!$

$\therefore r = 4$

${}^{n}P_{r} = 5040 \Rightarrow {}^{n}P_{4} = 5040 = 10 \times 504$

$= 10 \times 9 \times 56$

$= 10 \times 9 \times 8 \times 7$

$= {}^{10}P_{4}$

$\therefore n = 10$

$\therefore n = 10, \; r = 4$

5. **If** ${}^{n}C_{4} = {}^{n}C_{6},$ **find n.**

Hint: ${}^{n}C_{r} = {}^{n}C_{s} \Rightarrow r = s \; or \; r + s = n$

Sol: ${}^{n}C_{4} = {}^{n}C_{6}$

$\therefore n = 4 + 6 = 10, \; (\because 4 \neq 6)$

6. **If** ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$ **find r.**

Sol: ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$

$\Rightarrow 2r - 1 = 2r + 4 \; or \; (2r-1) + (2r+4) = 15$

i.e, $4r + 3 = 15 \Rightarrow 4r = 12 \Rightarrow r = 3$

$\therefore r = 3$

$\therefore 2r - 1 = 2r + 4 \Rightarrow -1 = 4$ **which is impossible**

$\therefore r = 3$

7. **If** ${}^{17}C_{2t+1} = {}^{17}C_{3t-5}$ **find t**

Sol: ${}^{17}C_{2t+1} = {}^{17}C_{3t-5}$

$\Rightarrow 2t = 1 = 3t - 5 \; or \; (2t+1) + (3t-5) = 17$

$\Rightarrow 1+5-t \text{ or } 5t-21$

$\Rightarrow 1+5-t \text{ or } 5t-21$

$\Rightarrow t=6 \text{ or } t=\frac{21}{5}$ **which is not an integer**

$\therefore$ **t = 6**

8. If $^{12}C_{r+1}=^{12}C_{3r-5}$**, find r**

Sol: $^{12}C_{r+1}=^{12}C_{3r-5}$

$\Rightarrow r+1=3t-5 \text{ or } (r+1)+(3r-5)=12$

$\Rightarrow 1+5=2r \text{ or } 4r-4=12$

$\Rightarrow 2r=6 \text{ or } 4r=16$

$\Rightarrow r=3 \text{ or } r=4$

$\therefore r=3 \text{ or } 4$

9. If $^{9}C_{3}+^{9}C_{5}=^{10}C_{r}$ **then find r**

Hint : $^{n}C_{r}=^{n}C_{n-r}$

Sol: $^{10}C_{r}=^{9}C_{3}+^{9}C_{5}$

$\therefore {}^{n}C_{r}+{}^{n}C_{r-1}={}^{(n+1)}C_{r}$

$\Rightarrow {}^{9}C_{3}+{}^{9}C_{5}={}^{9}C_{3}+{}^{9}C_{5}={}^{10}C_{6} \text{ or } {}^{10}C_{4}={}^{10}C_{r}$ **(given)**

$\Rightarrow r=4 \text{ or } 6$

10. Find the number of ways of forming a committee of 5 members from 6 men and 3 ladies.

Sol: Total number of persons $=6+3=9$

$\therefore$ **Number of ways of forming a committee of 5 members from 6 men and 3 ladies**

$={}^{9}C_{5}=\frac{9\times 8\times 7\times 6\times 5}{5\times 4\times 3\times 2\times 1}=126$

11. In questions no.10 how many committees contain atleast 2 ladies

Sol: Since a committee contains atleast 2 ladies, the members of the committee may be of the following two types

(i) 3men, 2 ladies (ii) 2 men, 3 ladies

The number of selections in the first type

$=6C_{3}\times 3C_{2}=20\times 3=60$

The number of selections in the second type

$= {}^{6}C_2 \times {}^{3}C_3 = 15 \times 1 = 15$

$\therefore$ **The required number of ways of selecting the committee containing atleast 2 ladies** $= 60 + 15 = 75$

12. **If** ${}^{n}C_5 = {}^{n}C_6$ **then find** ${}^{13}C_n$

Sol: $\because {}^{n}C_5 = {}^{n}C_6 \Rightarrow n = 6 + 5 = 11$

$${}^{13}C_n = {}^{13}C_{11} = {}^{13}C_2 = \frac{13 \times 12}{1 \times 2} = 78$$

II.

1. **Prove that for** $3 \le r \le n$

$${}^{(n-4)}C_r + 3^{(n-3)}C_{r-1} + 3^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} = {}^{n}C_r$$

Sol. **LHS**

$$= \left[{}^{(n-3)}C_r + {}^{(n-3)}C_{r-1}\right] + 2\left[{}^{(n-3)}C_{r-1} + {}^{(n-3)}C_{r-2}\right] + \left[{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3}\right]$$

$$= {}^{(n-3+1)}C_r + 2^{(n-3+1)}C_{r-1} + {}^{(n-3+1)}C_{r-2}$$

$$= {}^{(n-2)}C_r + 2^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}$$

$$= \left[{}^{n-2}C_r + {}^{(n-2)}C_{r-1}\right] + \left[{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}\right]$$

$$= {}^{(n-2+1)}C_r + {}^{(n-2+1)}C_{r-1}$$

$$= {}^{(n-1)}C_r + {}^{(n-1)}C_{r-1}$$

$$= {}^{(n-1+1)}C_r = {}^{n}C_r$$

$\therefore LHS = RHS$

2. **Find the value of** ${}^{10}C_5 + 2.\,{}^{10}C_4 + {}^{10}C_3$

Sol: ${}^{10}C_5 + 2.\,{}^{10}C_4 + {}^{10}C_3$

Hint : ${}^{n}C_r + {}^{n}C_{r-1} = {}^{(n+1)}C_r$

$$= \left({}^{10}C_5 + {}^{10}C_4\right) + \left({}^{10}C_4 + {}^{10}C_3\right)$$

$$= {}^{11}C_5 + {}^{11}C_4$$

$$= {}^{12}C_5 = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

3. Simplify $^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_r$

Sol: Hint: $^{n}C_r + {}^{n}C_{r-1} = {}^{(n+1)}C_r$

$$^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4 = {}^{34}C_5 + {}^{28}C_4 + {}^{27}C_4 + {}^{36}C_4 + {}^{35}C_4 + {}^{34}C_4$$

$$= \left({}^{34}C_5 + {}^{34}C_4\right) + {}^{35}C_4 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$$

$$= \left({}^{35}C_5 + {}^{35}C_4\right) + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$$

$$= \left({}^{37}C_5 + {}^{36}C_4\right) + {}^{27}C_4 + {}^{38}C_4$$

$$= \left({}^{37}C_5 + {}^{37}C_4\right) + {}^{38}C_4$$

$$= {}^{38}C_5 + {}^{38}C_4$$

$$= {}^{39}C_5$$

4. In a class there are 30 students. If each students plays a chess game with each of other students, then find the total number of chess games played by them.

Sol: Number of students in a class = 30

Since each student plays a chess game with each of the other students, the total number of chess games played by them $= {}^{30}C_2 = 435$

5. Find the number of ways of selecting 3 girls and 3 boys out of 7 girls and 6 boys

Sol: The number of ways of selecting 3 girls 3 boys out of 7 girls and 6 boys $= {}^{7}C_3 \times {}^{6}C_3 = 35 \times 20 = 700$

6. Find the number of way of selecting a committee of 6 members out of 10 members always including a specified member

Sol: Since a specified member always includes in a committee, remaining 5 members can be selected from remaining 9 members in $^{9}C_5$ **ways.**

$\therefore$ **Required number of ways selecting a committee** $= {}^{9}C_5 = 126$

7. Find the number of ways of selecting 5 books from 9 different mathematics books that a particular book is not included.

Sol: Since a particular book is not include in the selection, the 5 books can be selected in the selection, the 5 books can be selected from remaining 8 books in $^{8}C_5$ **ways.**

$\therefore$ **The required number of ways of selecting 5 books** $= {}^{8}C_5 = 56$

8. Find the number of ways of selecting 3 vowels and 2 consonants from the letters of the word equation.

Sol: The word EQUATION contains 5 vowels and 3 consonants.

The 3 vowels can be selected from 5 vowels in $^5C_3 = 10$ **ways**

The 2 consonants can be selected from 3 consonants in $^3C_2 = 3$ **ways**

$\therefore$ **The required number of ways of selecting 3 vowels and 2 consonants** $= 10 \times 3 = 30$

9. 10 persons are sitting around a circle. In how many ways can two persons out of them be selected so that they are not adjacent to each other?

Sol: Number of ways of selecting two persons from 10 persons $= {}^{10}C_2 = 45$

Number of ways of selecting two persons from 10 persons seated around a circle which are adjacent to each other = 10

$\therefore$ Required number of ways of selecting 2 persons out of 10 persons sitting around a circle so that they are not adjacent to each other = 45-10 = 35.

10. If n persons are sitting in a row, find the number of ways of selecting two persons out of them who are sitting adjacent to each other

Sol: The number of ways of selecting 2 persons out of n persons sitting in a row, who are sitting adjacent to each other = n - 1.

11. Find the number of ways of giving away 4 similar coins to 5 boys if each body can be given any number (less than or equal to 4) of coins.

Sol: The 4 similar coins can be divided into different number of groups as follows,

(i) One group containing 4 coins

(ii) Two groups containing 1, 3 coins respectively in

(iii) Two groups containing 2, 2 coins respectively

(iv) Two groups containing 3, 1 coins respectively

(v) Three groups containing 1, 1, 2 coins respectively

(vi) Three groups containing 1, 2, 1 coins respectively

(vii) Three groups containing 2, 1, 1 coins respectively

(viii) Four groups containing 1, 1, 1, 1 coins respectively

these groups can given away to 5 boys in

$$= {}^5C_1 + 2 \times {}^5C_2 + {}^5C_2 + {}^5C_3 \times \frac{3!}{2!} + {}^5C_4$$

$$= 5 + 20 + 10 + 30 + 5$$

$= 70$ **ways**

III.

1. **Prove that** $\frac{{}^{4n}C_{2n}}{{}^{2n}C_{n}} = \frac{1.3.5.....(4n-1)}{\left\{1.3.5.....(2n-1)^{2}\right\}}$

Sol: $\frac{{}^{4n}C_{2n}}{{}^{2n}C_{n}} = \frac{\frac{(4n)!}{(2n)!(2n)!}}{\frac{(2n)!}{n!n!}}$

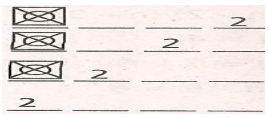
Hint : ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

$= \frac{(4n)!}{(2n)!^{2}} \times \frac{n!n!}{(2n)!}$

$= \frac{(4n)(4n-1)(4n-2)(4n-3)(4n-4).....5.4.3.2.1}{\left[(2n)(2n-1)(2n-2)(2n-3)(2n-4).....5.4.3.2.1\right]^{2}} \times \frac{n!n!}{(2n)!}$

$= \frac{\left[(4n-1)(4n-3).....5.3.1\right]\left[(2n)(2n-1)(2n-2).....2.1.2^{2n}\right]}{\left[\left\{(2n-1)(2n-3).....5.3.1\right\}\left\{n(n-1)(n-2)....2.1.2^{n}\right\}\right]^{2}} \times \frac{n!n!}{(2n)!}$

$= \frac{\left[(4n-1)(4n-3).....5.3.1\right](2n)!\,2^{2n}}{\left[(2n-1)(2n-3).....5.3.1\right]^{2}[n!]^{2}(2n)^{2}} \frac{(n!)^{2}}{(2n)!}$



$\therefore \frac{{}^{4n}C_{2n}}{{}^{2n}C_{n}} = \frac{1.3.5.....(4n-1)}{\left[1.3.5....(2n-1)^{2}\right]}$

2. If a set A has 12 elements. Find the number of subsets of A having (i) 4 elements (ii) atleast 3 elements (iii) atmost 3 elements

Sol. Number of elements in set A = 12

(i) Number of subsets of A with exactly 4 elements = $^{12}C_4 = 495$

(ii) The required subset contains atleast 3 elements.

Number of subsets of A with exactly 0 elements is $^{12}C_0$

Number of subsets of A with exactly 1 element is $^{12}C_1$

Number of subsets of A with exactly 2 element $^{12}C_2$

Total number of subsets of A formed = 2^{12}.

= 4096 - 79 = 401 7

(iii) The required subset contains almost 3 elements i.e., it may contain 0 or 1 or 2 or 3 elements.

Number of subsets of A with exactly 0 elements is $^{12}C_0$

Number of subsets of A with exactly 1 element is $^{12}C_1$

Number of subsets of A with exactly 2 elements is $^{12}C_2$

Number of subsets of A with exactly 3 elements is $^{12}C_3$

$\therefore$ Number of subsets of A with atmost 3 elements

$= {}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3$

$= 1 + 12 + 66 + 220 = 299$

3. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

Sol. Since the team consists of at least 5 bowlers, the selection may be of the following types.

		Batsmen (7)	Bowlers (6)
Type	1	6	5
Type	2	5	6

The number of selections in first type $= {}^{7}C_6 \times {}^{6}C_5 = 7 \times 6 = 42$

The number of selections in second type $= {}^{7}C_5 \times {}^{6}C_6 = 21 \times 1 = 21$

$\therefore$ The required number of ways selecting the cricket team = 42 + 21 = 63.

4. A person wants to take 9 children to an exhibition by taking 3 at a time in all possible ways. How many times does a particular child go to the exhibition and how many times does the person go to the exhibition.

Sol: Particular child will go to the exhibition only as many times as the other two can be taken out of the remaining 8 children.

$\therefore$ The number of times that particular child will go = ${}^{8}C_{2} = 28$

Since the person is to accompany each new group of children, he will have to go as many times as there are different groups of three children formed out of 9.

$\therefore$ The number of times that the person will have to go $= {}^{9}C_{3} = 84$

5. There are 8 railway stations along a railway line. In how many ways can a train be stopped at 3 of these stations such that no two of them are consecutive.

Sol: Number "of ways of selecting 3 stations out $8 = {}^{8}C_{3} = 56$

Number of ways of selecting 3 out of 8 stations such that 3 are consecutive = 6

Number of ways of selecting 3 out of 8stations such that 2 of them are consecutive

$= 2 \text{ x } 5 + 5 \text{ x } 4 = 10 + 20 = 30.$

$\therefore$ Number of ways of a train to be stopped at 3 of 8 stations such that no two of them are consecutive

$= 56 - (6 + 30) = 20$

6. Find the number of ways of forming a committee of 5 members out of 6 Indian and 5 Americans so that always the Indians will be in majority in the committee

Sol: Since committee contains majority of Indians, the members of the committee may be of the following types.

Indians (6).	Americans (5)
Type I	5 0
Type II	4 1
Type III	3 2

The number of selections in type I $= {}^{6}C_{5} \times {}^{5}C_{0} = 6 \times 1 = 6$

The number of selections in type $= {}^{6}C_{4} \times {}^{5}C_{1} = 15 \times 5 = 75$

The number of selections in type III $= {}^{6}C_{3} \times {}^{5}C_{2} = 20 \times 10 = 200$

$\therefore$ The required number ways of selecting a committee = 6 + 75 + 200 = 281.

7. A double decker minibus has 13 seats in a lower deck and 12 seats in the upper deck. In how many ways can a marriage party of 25 persons be arranged f 4 old people refuse to go the upper deck and 3 children wish to travel only in the upper deck.

Sol: The 4 old people who refuse to go to the upper deck must be accommodated in the lower deck & the 3 children who wish to travel in the upper deck must be accommodated in the upper deck.

The 9 vacant seats in the upper deck can be filled by the remaining 18 passengers in ${}^{18}C_9$ ways. After this is done, the remaining 7 vacant places in the lower deck can be accommodated by the remaining 7 passengers in ${}^{7}C_7 = 1$ways. After accommodating the seats in the upper and lower decks, the 12 passengers can be arranged in the upper deck in $(12)!$ ways and the 13 passengers in the lower deck can be arranged in $(13)!$ ways

$\therefore$ Required number of ways

$= {}^{18}C_9 \times 1 \times (13)! \times (12)!$

$= {}^{18}C_9 \times (13)! \times (12)!$

8. How many 5 Letters words can be formed using 3 consonants and 2 vowels from the letters of the word MIXTURE.

Sol: The word MIXTURE contains 3 vowels and 4 consonants.

The 2 vowels can be selected from 3 vowels $= 3C_2 = 3$

The 3 consonants can be selected from 4 consonants $= 4C_3 = 4$

The number of ways of selecting 2 vowels and 3 consonants from the letters of the word MIXTURE $= 3 \times 4 = 12$ ways and they can be arranged in 5! Ways

$\therefore$ The number of 5 letters words formed $= 12 \times 5!$

$= 12 \times 120 = 1440$

SOLVED PROBLEMS

1. If ${}^{n}P_4 = 1680$, find n

Sol: Given ${}^{n}P_4 = 1680$

But ${}^{n}P_4 = n(n-1)(n-2)(n-3)$

Thus, we are given $n(n-1)(n-2)(n-3)$

$= 1680 = 8 \times 7 \times 6 \times 5$

On comparing the largest integers on bothsides. We get n = 8

2. If $^{12}P_r = 1320$ **, find r**

Sol: $1320 = 12 \times 11 \times 10 = {}^{12}P_3$**. Hence** $r = 3$

3. If $^{(n+1)}P_5 : {}^{n}P_5 = 3:2$**, find n**

Sol: $^{(n+1)}P_5 : {}^{n}P_5 = 3:2$

$$\Rightarrow \frac{n+1}{n-4} = \frac{3}{2}$$

$$\Rightarrow 2n + 2 = 3n - 12$$

$$\Rightarrow n = 14$$

It can be verified that $n = 14$

Satisfies the given equation

4. If $^{56}P_{(r+6)} : {}^{54}P_{(r+3)} = 30800:1$**, find r**

Sol: $^{56}P_{(r+6)} : {}^{54}P_{(r+3)} = 30800:1$

$$\Rightarrow \frac{(56)!}{(56-(r+6))!} \times \frac{((54)-(r+3))!}{(54)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{(56)!}{(50-r)!} \times \frac{(51-r)!}{(54)!} = 30800$$

$$\Rightarrow (56)(55)(51-r) = 30800$$

$$\Rightarrow (51-r) = \frac{30800}{56 \times 55} = 10$$

$$\Rightarrow r = 41$$

It can be verified that $r = 41$ **satisfies the given equation**

5. Find the number of ways of arranging the letters of the word KRISHNA. In how many of them, all the vowels come together.

Sol: Since the world KRISHNA has 7 letters, they can be arranged in 7! = 5040 ways.

The world had two vowels I and A. Treat them as one unit. Then we have 5 consonants + 1 unit of vowels = 6 things

They can be arranged in 6! Ways. Now the vowels can be permuted among themselves in 2! ways. Therefore, the number arrangements in which the 2 vowels come together is $6! \times 2! = 720 \times 2 = 1440$

6. Find the numbers of ways of arranging 6 boys and 6 girls in a row. In how many of these arrangements.

(i) All the girls are together

(ii) No two girls are together

(iii) Boys and girls come alternately?

Sol: 6 boys and 6 girls are altogether 12 persons. They can be arranged in a row in (12)! ways.

(i) Treat the 6 girls as one unit. Then we have 6 boys and 1 unit of girls. They can be arranged in 7! ways. Now, the 6 girls can be arranged among themselves in 6.! ways. Thus the number of ways in which all 6 girls are together is (7! x 6!).

(ii) First arrange the 6 boys in a row in 6! ways. Then we can find 7 gaps between them (including the beginning gap and the ending gap) as shown below by the letter x :

$x\boxed{B}x\boxed{B}\ x\boxed{B}\ x\boxed{B}\ x\boxed{B}\ \ x\boxed{B}\ x$

Thus we have 7 gaps and 6 girls. They can be arranged in ${}^{7}P_{6}$ ways.

Hence, the number of arrangements which no two girls sit together is $6! \times {}^{7}P_{6} = 7.6!.6!$

(iii) The row may begin with either a boy or a girl, that is, 2 ways. If it begins with a boy, then odd places will be occupied by boys and even places by girls. The 6 boys can be arranged in 6 odd places in 6! ways and 6 girls in the 6 even places in 6! ways. Thus the number of arrangements in which boys and girls come alternately is 2 x 6! x 6!,

7. Find the number of 4 letter words **that can be formed using the letters of the word, MIRACLE. How many of them**

(i) begin with an vowel

(ii) begin and end with vowels

(iii) end with a consonant ?

Sol: The word MIRACLE has 7 letters. The number of 4 letter words that can be formed using these letters

$= {}^{7}P_{4} = 7 \times 6 \times 5 \times 4 = 840$

Now that 4 blanks

$\square\square\square$

(i) We can fill the first place with one of the 3 vowels {A, E, I} in ${}^3P_1 = 3$ ways.

Now the remaining 3 places can be filled using the remaining 6 letters in

${}^6P_3$ - 6 x 5 x 4 = 120 ways

∴ The number of 4 letter words that begin with an vowel = 3 x 120 = 360 ways.

Fill the first and last places with 2 vowels in ${}^3P_2 = 3 \times 2 = 6$ ways.

The remaining 2 places can be filled with the remaining 5 letters in ${}^5P_2 = 5 \times 4 = 20$ ways.

∴ The number of 4 letter words that begin and end with vowels = 6 x 20 = 120 ways.

(iii) We can fill the last place-with one of the 4 consonants {C, L, R, M} in ${}^4P_1 = 4$ ways.

The remaining 3 places can be filled with the remaining 6 letters in ${}^6P_3 = 6 \times 5 \times 4 = 120$ ways.

∴ The number of 4 letter words that end with a consonant is = 4 x 120 = 480 ways.

8. Find the number of ways of permuting all the letters of the word PICTURE, so **that**

(i) all vowels come together

(ii) no two vowels come together

Sol: The word PICTURE has 3 vowels {E, I, U} and 4 consonants {C, P, R, T}

(i) Treat the 3 vowels as one unit. Then we can arrange 4 consonants + 1 unit of vowels in 5! ways. Now 3 vowels among themselves can be permuted in 3! ways. Hence the number of permutations in which 3 vowels come together.

5! x3! = 120 x 6 = 720 ways.

(ii) No two vowels come together First arrange the4 consonants in 4! ways. Then in between the vowels, in the beginning and in the ending, there are 5 gaps as shown below by the x letter

$x \square x \square x \square x \square x$

In these 5 places we can arrange 3 vowels in ${}^5P_3$ ways.

∴ The number of words in which no two vowels come together

= 4! x ${}^5P_3$

=24 x 5 x 4 x 3 = 1440 ways.

9. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order. Find the ranks of the words.

(i) PRISON (ii) SIPRON

Sol. The letters of the given word in dictionary order are

I N O P R S

(i) In the dictionary order, first all the words that begin with I come. If I occupies the first place then the remaining 5 places can be filled with the remaining 5 letters in 5! ways.

Thus, there are 5! number of words that begin with I. On proceeding like this we get

I----------> 5! ways

N----------> 5! ways

O----------> 5! ways

P I----------> 4! ways

P N----------> 4!ways

P O ----------> 4! ways

P R I N----------> 2 ! ways

P R I O----------> 2! ways

P R I S N 0 ----------> 1 way

P R I S O N----------> 1 way

Hence the rank of PRISON is

3 x 5! + 3 x 4! + 2 x 2! + 1 x1

= 360 + 72 + 4 + 1 + 1

= 438

(ii) Now we find the rank of the word SIPRON as above.

I ----------> 5 ! ways

N----------> 5! ways

O ----------> 5! ways

P----------> 5! ways

R ----------> 5! ways

S I N----------> 3! ways

S I O ----------> 3! ways

S I P O----------> 2! ways

S I P O----------> 2! ways

S I P R N ----------> 1 way

S I P R O N ----------> 1 way

Therefore, the rank of SIPRON is

5x5! + 2x3! +2 x 2! + 1 x 1

= 600 + 12 + 4 + 2

= 618.

10. Find the number of 4 digit numbers that can be formed using the digits 2, 4, 5, 7, 8. How many of them are divisible by

(i) 2 (ii) 3 (iii) 4 (iv) 5 and (v) 25

Sol. Clearly, the number of 4 digit numbers that can be formed using the given 5 digits is $^5P_4$ = 120

(i) Divisible by 2

Units place must be filled with an even digit from among the given integers. This can be done in 3 ways. {2 or 4 or 8)

Now, the remaining 3 places can be filled with the remaining 4 digits in $^4P_3$ ways. Therefore, the number of 4 digited numbers divisible by 2 is

$3 \times {}^4P_3 = 3 \times 24 = 72$

(ii) Divisible by 3

We know that a number is divisible by 3 only when the sum of the digits in that number is a multiple of 3. Here, the sum of the given 5 digits is 26. We can select 4 digits such that their sum is a multiple of 3 in 3 ways. They are

4, 5, 7, 8 (sum is 24) (leaving 2)

2, 4, 7, 8 (sum is 21) (leaving 5)

2,4, 5,7 (sum is 18) (leaving 8)

In each case, we can permute them in 4! ways and all these 4 digited numbers are divisible by 3. Thus, the number of the 4 digited numbers divisible by 3 is 3 x (4!)

= 3 x 24 = 72.

(iii) Divisible by 4

A number is divisible by 4 only when the number formed by the digits in the last two places (tens and units places) is a multiple of 4.

$\square\square x x$

Therefore, the last two places should be filled with one of the following.

24, 28, 48, 52, 72, 84

Thus the last two places can be filled in 6 ways. Then we are left with 2 places and 3 digits. They can be filled ${}^3P_2$ ways. Thus the number of 4 digits numbers divisible by 4 is 6 x ${}^3P_2 = 36$.

(iv) Divisible by 5

A number is divisible by 5, if the units place is '0' or 5. But '0' is not in the given digits. Hence we must fill the units place by 5 only.

$\square\square\square 5$

Now, the remaining 3 places can be filled with the remaining 4 digits in ${}^4P_3$ ways. Therefore, the number of 4 digited numbers divisible by ${}^4P_3 = 24$

(v) Divisible by 25

A 4 digited number formed by using the given digits is divisible by 25 if the last two places (tens and units places) are filled as either 25 or 75.

$\square\square x x$

Thus, the last two places can be filled in '2' ways. Now, the remaining 2 places with the remaining '3' digits can he filled in ${}^3P_2$ ways.

Therefore, the number of 4 digited numbers that are divisible by 25 is $2x{}^3P_2 = 12$.

11. Find the sum of all 4 digited numbers that can be formed using the digits

1, 3, 5, 7, 9.

Sol. We know that the number of 4 digited numbers that can be formed using the digits

1, 3, 5, 7, 9 is ${}^5P_4 = 120$.

We have to find their sum. We first find the sum of the digits in the units place of all the 120 numbers. Put 1 in the units place.

$\square\square\square 1$

The remaining 3 places can be filled with the remaining 4 digits in ${}^4P_3$ ways. Which means that there are ${}^4P_3$ number of 4 digited numbers with 1 in the units place. Similarly, each of the other digits 3, 5, 7, 9 appears in the units place ${}^4P_3$ times. Hence, by adding all these digits of the units place, we get the sum of the digits in the units place.

${}^4P_3 \times 1 + {}^4P_3 \times 3 + {}^4P_3 \times 5 + {}^4P_3 \times 7 + {}^4P_3 \times 9$

$= {}^4P_3\left(1+3+5+7+9\right)$

$= {}^4P_3\left(25\right)$

Similarly, we get the sum of all digits in 10's place also as ${}^4P_3$ x 25. Since it is in 10's place, its value is ${}^4P_3 \times 25 \times 10$

Like this the values of the sum of the digit in 100's place and 1000's place are respectively

${}^4P_3 \times 25 \times 100$ *and* ${}^4P_3 \times 25 \times 1000$

On adding all these sums, we get the sum of all the 4 digited numbers formed by using the digits 1, 3, 5, 7, 9. Hence the required sum is

${}^4P_3 \times 25 \times 1 + {}^4P_3 \times 25 \times 10 + {}^4P_3 \times 25 \times 100 + {}^4P_3 \times 25 \times 1000$

$= {}^4P_3 \times 25 \times 1111$

$= 24 \times 25 \times 1111 = 6,66,600$

12. Find the number of ways of arranging 4 boys and 3 girls in a row so that the row always begins with a boy and ends with a girl

Sol: The first place can be filled with any body in 4 ways and similarly, the last place can be filled with a girl in 3 ways



Now, the remaining 5 places can be filled with the remaining 5 persons (3 boys and 2 girls) in 5! ways. Hence, the number of the required arrangements is

$4 \times 3 \times 5! = 1440$

13. Find the number of injections of a set with 'm' elements into a set with 'n' elements.

Sol: Let A be a set with m elements and B a set with n elements. If m > n, then we cannot define an injection from A into B. Hence the number of injections is 'o'

Now, suppose $m \le n$. Then the required number of the injections is ${}^n p_m$

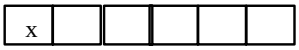
14. Find the number of injections of set A with 5 elements to a set B with 7 elements.

Sol: If a set A has m elements and the set B has n elements $(m \le n)$, then the number of injections from A into $B = {}^{n}P_{m}$

$\therefore$ The number of injections from set A with 5 elements into set B with 7 elements.

$= {}^{7}P_{5} = 2,520$

15. Find the number of 5 letter words that can be formed using the letters of the word EXPLAIN which begin with an vowel when repetitions are allowed.

Sol: We have to fill up 5 blanks with the given letters. 

First place can be filled with an vowel in 3 ways (E or A or I). Then we are left with 7 letters and 4 blanks. They can be filled in 7^4 ways. Hence the number of the required 5 letter words is 3×7^4

16. a) Find the number of functions from a set A containing m elements to a set B containing n elements.

Sol: Let $A = \{a_1, a_2, ... a_m\}$ and $B = \{b_1, b_2 ..... b_n\}$

First to define the image of a_1 we have n choices (any element of B). Then to define the image of a_2 we again have n choices (since a_1, a_2 can have the same image). Thus we have n choices for the image of each of the m elements of the set A. Therefore, the number of different ways of defining the images of elements of A(with images in B) is $n \times n \times ..... \times n\,(m\ times) = n^m$

b) Find the number of surjections from a set A with n elements to a set B with 2 elements.

Sol: Let $A = \{a_1, a_2, ...a_n\}$ and $b = \{x, y\}$. The total number of functions from A to B is 2^n (by problem 2). For a surjection, both the elements x, y of B must be in the range. Therefore, a function is not a surjection if the range contains only x (or y). There are only two such functions. Hence, the number of surjections from A to B is $2^n - 2$

17. Find the number of permutations of 4 digit numbers that can be formed using the digit 1, 2, 3, 4, 5, 6 when repetition is allowed.

Sol: $\square\square\square\square$
6 6 6 6

The number of permutations of 4 digit number that can be formed using the given 6digit
$= 6\times 6\times 6\times 6 = 6^4 = 1,296$

18. Find the number of 4 digited numbers using the digits 1, 2, 3, 4, 5, 6 that are divisible by (i) 2 (ii) 3 when repetitions are allowed.

Sol: (i) Number divisible by 2

Take 4 blank places, First the units place

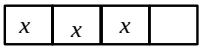


Can be filled by an even digit in 3 ways (2 or 4 or 6). The remaining three places can be filled with the 6 digits in 6 ways each. Thus they can be filled in $6\times 6\times 6 = 6^3$ ways . Therefore, the number of 4 digited numbers divisible by 2 is

$3\times 6^3 = 3\times 216 = 648$

(ii) Numbers divisible by 3

First we fill up the first 3 places 6 digits in 6^3 ways



After filling up the first 3 places, if we fill the units place with the given 6 digits, we get 6 consecutive positive integers. Out of these six consecutive integers exactly 2 will be divisible by '3'. Hence the units place can be filled in '2' ways. Therefore, the number of 4 digited numbers divisible by 3

$= 6^3\times 2 = 216\times 2 = 432$

19. Find the number of4 letter words that can be formed using the letters of the word PISTON in which atleast one letter is repeated.

Sol: The word PISTON has 6 letters

The number of 4 letter words that can be formed using these 6 letters

(i) When repetition is allowed $= 6^4$

(ii) When repetition is not allowed $= {}^6P_4$

$\therefore$ The number of 4 letter words in which at least one letter repeated is

$= 6^4 - {}^6P_4 = 1296 - 360 = 936$

20. Find the number of 5 letter words that can be formed using the letters of the word MIXTURE which begin with an vowel when repetition are allowed.

Sol: The word MIXTURE has 7 letters 3 vowels {E, I, U} and 4 consonants {E, M, R, X} we have to fill up 5 blanks

□ □ □ □ □

Fill the first place with one of the 3 vowels in 3 ways

Each of the remain 4 places can be filled in 7 ways (since repetition is allowed)

$\therefore$ The number of 5 letter words

$= 3 \times 7 \times 7 \times 7 \times 7 = 3 \times 7^4$

21. Find the number of 5 letter words that can be formed using the letters of the word EXPLAIN that begin and end with a vowel, when repetitions are allowed.

Sol: The word EXPLAIN has 7 letters, among them 3 vowels {A, E, I} and 4 cosonants

{L, P, N, X}

□ □ □ □ □

3 3

We can fill the first and last places with vowels each in 3 ways.

Now each of the remaining 3 places can be filled in 7 ways (using any letter of given 7 letters).

Hence the number of 5 letter words which begin and end with vowels

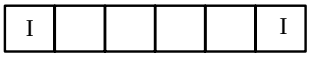
= 3x7x7x7x3 = 3,087

22. Find the number of ways of arranging the **letters of the word SINGING so that**

(i) they begin and end with I

(ii) the two G's come together

Sol: The word SINGING has 2 I's, 2 G's and 2 N's and one S. Total 7 letters.



(i) First, we fill the first and last places with I's in $\frac{2!}{2!}=1$ way as shown below.

Now we fill the remaining 5 places with the remaining 5 letters in $\frac{5!}{2!2!}=30$ ways.

Hence the number of required permutations = 30

(ii) Treat two G's as one unit. Then we have 5 letters 2 I's, 2 N's and one S + one unit of 2 G's = 6 can be arranged in $\frac{6!}{2!2!}=\frac{720}{2\times 2}=180$ ways.

Now the two G's among themselves can be arranged in one way. Hence the number of received permutations = 180 x 1 = 180.

23. Find the number of ways of arranging a^4, b^3, C^5 in its expanded form

Sol. The expanded form of $a^4\, b^3\, C^5$ is

aaaa bbb ccccc

There are 4 + 3 + 5 = 12 letters

There can be arranged in $\frac{(12)!}{4!3!5!}$ ways

24. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order. Find the rank of the word EAMCET.

Sol: The dictionary order of the letters of the word EAMCET IS

A C E E M T

In the dictionary order first gives the words which begins with the letters A.

If we fill the first place with A, remaining 5 letters can be arranged in $\frac{5!}{2!}$ ways (since there are 2 E's) on proceeding like this, we get

A ----------> $\frac{5!}{2!}$ words

C ----------> $\frac{5!}{2!}$ words

E A C ----------> 3! words

E A E ----------> 3! words

E A M C E T ----------> 1 word

Hence the rank of the word EAMCET is

$$= 2 \times \frac{5!}{2!} + 2 \times 3! + 1$$

$$= 120 + 12 + 1 = 133$$

25. Find the number of ways of arranging 8 men and 4 women around a circular table. In how many of them

(i) all the women come together

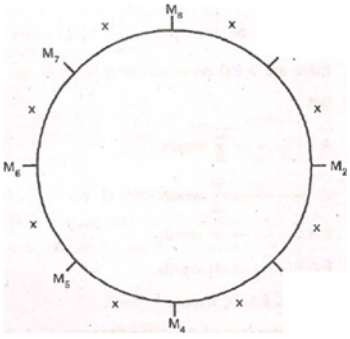
(ii) no two women come together

Sol. Total number of persons = 12 (8 men + 4 women)

Therefore, the number of circular permutations is (11) !

(i) Treat the 4 women as one unit. Then we have 8 men + 1 unit of women = 9 entities. Which can be arranged around a circle in 8! ways. Now, the 4 women among themselves can be arranged in 4! ways. Thus, the number of required arrangements is 8! x 4!.

(ii) First arrange 8 men around a circle in 7! ways. Then there are 8 places in between them as shown in fig by the symbol x (one place in between any two consecutive men). Now, the 4 women can be arranged in these 8 places in ${}^{8}P_{4}$ ways.



Therefore, the number of circular arrangements in which no two women come together is 7! X $^{8}P_{4}$.

26. Find the number of ways of seating 5 Indians, 4 Americans and 3 Russians at a round table so that

(i) all Indians sit together

(ii) no two Russians sit together

(iii) persons of the same nationality sit together

Sol. (i) Treat the 5 Indians as a single unit. Then we have 4 Americans, 3 Russians and 1 unit of Indians. That is, 8 entities in total. Which can be arranged at a round table in (8 - 1)! = 7! ways.

Now, the 5 Indians among themselves can be arranged in 5! ways. Hence, the number of required arrangements is 7! x 5!

(ii) First we arrange the 5 Indians + 4 Americans around the table in (9 - 1) ! = 8! ways.

Now, we can find 9 gaps in between these 9 persons (one gap between any two consecutive persons).

The 3 Russians can be arranged in these 9 gaps in $^{9}P_{3}$ ways. Hence, the number of required arrangements is 8! x $^{9}P_{3}$.

(iii) Treat the 5 Indians as one unit, the 4 Americans as the second unit and the 3 Russians as the third unit. These 3 units can be arranged at round table in (3 - 1)! = 2! ways.

Now, the 5 Indians among themselves can be permuted in 5! ways. Similarly, the 4 Americans in 4! ways and the 3 Russians in 3! ways. Hence the number of required arrangements is 2! x 5! x 4! x 3!

27. Find the number of chains that can be prepared using 7 different coloured beads.

Sol: We know that the number of circular permutations of hanging type that can be formed using n things is $\frac{1}{2}\{(n-1)!\}$. Hence the number of chains is $\frac{1}{2}\{(7-1)!\}$

$= \frac{1}{2}(6!) = 360$

28. Find the number of different ways of preparing a garland using 7 distinct red roses and 4 distinct yellow roses such that no two yellow roses come together.

Sol: First arrange 7 red roses in a circular form (garland form) in (7 - 1)! = 6! ways. Now, there are 7 gaps and 4 yellow roses can be arranged in these 7 gaps in 4 $^{7}P_{4}$ ways.

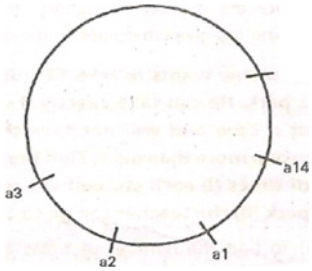
Thus, the total number of circular permutations is 6! x $^{7}P_{4}$.

But, in the case of garlands, clock-wise and anti-clock-wise arrangements look alike. Hence, the number of required ways is.

$\frac{1}{2}\left(6!\times {}^{7}P_4\right)$)

29. 14 persons are seated at a round table. Find the number of ways of selecting two persons out of them who are not seated adjacent to each other.

Sol. The seating arrangement of given 14 persons at the round table as shown below.



Number of ways of selecting 2 persons out of 14 persons $= {}^{14}C_2 = \frac{14\times13}{1\times2} = 91$

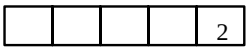
In the above arrangement two persons sitting adjacent to each other can be selected in 14 ways, (they are $a_1a_2, a_2a_3, ..., a_{13}a_{14}, a_{14}, a_1$)

$\therefore$ The required no. of ways = 91 - 14 = 77

30. Find the number of 5 digited numbers that can be formed using the digits. 1, 1, 2, 2, 3. How many of them are even?

Sol. In the given 5 digits, there are two 1's and two 2's. Hence the number of 5 digited numbers that can be formed is $\frac{5!}{2!2!} = 30$

Now, for the number to be even, it should end with 2.



After filling the units place with 2, the remaining 4 places can be filled with the remaining 4 digits 1,1, 2, 3 in $\frac{4!}{2!} = 12$ ways. Thus, the number of 5 digited even numbers that can be formed using the digits. 1, 1,2, 2, 3, is 12.

31. There are 4 copies (alike) each of 3 different books. Find the number of ways of arranging these 12 books in a shelf in a single row.

Sol. We are given 12 books in which 4 alike books are of one kind, 4 alike books are of the second kind and 4 alike books are of the third kind. Hence, the number of ways of arranging the given 12 books is $\frac{12!}{4!4!4!}$

32. Find the number of ways of forming a committee of 4 members out of 5 boys and 5 girls such that there is atleast one girl in the committee.

Sol. The number of ways of forming a committee of 4 members out of the given 10 members is ${}^{10}C_4$. Out of these, the number of committees having only boys is ${}^{5}C_4$ (that is, selecting all the 4 members from the 5 boys).

Thus, the number of committees with atleast one girl is

$${}^{10}C_4 - {}^{5}C_4 = 210 - 5 = 205$$

33. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 5 bowlers and 3 wicket keepers with atleast 3 bowlers and 2 wicket keepers.

Sol. The required teams can contain the following compositions

Bowlers (5)-	Wickt Keepers (3)	Bats men (7)	Number of ways of selecting them
3	2	6	${}^{5}C_3 \times {}^{3}C_2 \times {}^{7}C_6 = 210$
4	2	5	${}^{5}C_4 \times {}^{3}C_2 \times {}^{7}C_5 = 315$
5	2	4	${}^{5}C_5 \times {}^{3}C_2 \times {}^{7}C_4 = 105$
3	3	5	${}^{5}C_3 \times {}^{3}C_3 \times {}^{7}C_5 = 210$
4	3	4	${}^{5}C_4 \times {}^{3}C_3 \times {}^{7}C_4 = 175$
5	3	3	${}^{5}C_5 \times {}^{3}C_3 \times {}^{7}C_3 = 35$

Therefore, the number of required ways

= 210 + 315 + 105 + 210 + 175 + 35

= 1050.

34. If a set of 'm' parallel lines intersect another set of 'n' parallel lines (not parallel to the lines in the first set), then find the number of parallelograms formed in this lattice structure.

Sol. To form a parallelogram, we have to select 2 lines from the first set which can be done in ${}^{m}C_2$ ways and *2* lines from the second set which can be done in ${}^{n}C_2$ ways. Thus, The number of parallelograms formed is ${}^{m}C_2 \times {}^{n}C_2$

35. There are m points in a plane out of which p points are collinear and no three of the points are collinear unless all the three are from these p points. Find the number of different

(i) straightlines passing through pairs of distinct points

(ii) triangles formed by joining these points (by line segments).

Sol. (i) From the given m points, by drawing straight lines passing through 2 distinct points at a time, we are supposed to get ${}^{m}C_2$ number of lines. But, since p out of these m points are collinear, by forming lines passing through these p points 2 at a time we get only one line instead of getting ${}^{P}C_2$. Therefore, the number of different lines passing through pairs of distinct points is ${}^{m}C_2 - {}^{P}C_2 + 1$.

(ii) From the given m points, by joining 3 points at a time, we are supposed to get ${}^{m}C_3$ number of triangles. Since p out of these m points are collinear by joining these p points 3 at a time we do not get any triangle when as we are supposed to get ${}^{P}C_3$ number of triangles. Hence the number of triangles formed by joining the given m points $= {}^{m}C_3 - {}^{p}C_3$

36. A teacher wants to take 10 students to a park. He can take exactly students at a time and will not take the same group more than once. Find the number of times (i) each student can go to the park (ii) the teacher can go to the park.

Sol. (i) To find the number of times a specific student can go to the park, we have to select 2 more students from the remaining 9 students. This can be done in ${}^{9}C_2$ ways.

Hence each student can go to the park ${}^{9}C_2$ times $= \frac{9 \times 8}{1 \times 2} = 36$ times

(ii) The no. of times the teacher can go to park = The no. of different ways of selecting 3 students out of $10 = {}^{10}C_3 = 120$

37. A double decker minibus has 8 seats in the lower and 10 seats in the upper deck. Find the number of ways of arranging 18 persons in the bus, if 3 children want to go to the upper deck and 4 old people cannot go to the upper deck.

Sol: Allowing 3 children to the upper deck and 4 old people to the lower deck, we are left with 11 people and 11 seats (7 seats in the upper deck and 4 in the lower deck). We can Select 7 people for the upper deck out of 11 people in $^{11}C_?$ ways. The remaining 4 persons go to the lowerdeck.

Now we can arrange 10 persons (3 children and 7 others) in the upper deck and 8 persons (4 old people and 4 others) in the lower deck in (10)! and (8)! ways respectively.

Hence the required number of arrangements $= {}^{11}C_7 \times 10! \times 8!$

38. Prove that

(i) ${}^{10}C_3 + {}^{10}C_6 = {}^{11}C_4$

(ii) ${}^{25}C_4 + \sum_{r=0}^{4} {}^{(29-r)}C_3 = {}^{30}C_4$

Sol: (i) ${}^{10}C_3 + {}^{10}C_6 = {}^{10}C_3 + {}^{10}C_4$ (since ${}^{n}C_r = {}^{n}C_{n-r}$) $= {}^{11}C_4$

(ii) ${}^{25}C_4 + \sum_{r=0}^{4} {}^{(29-r)}C_3$

$= {}^{25}C_4 + \left\{ {}^{25}C_3 + {}^{27}C_3 + {}^{28}C_3 + {}^{29}C_3 \right\}$

$= {}^{26}C_4 + {}^{26}C_3 + {}^{27}C_3 + {}^{28}C_3 + {}^{29}C_3$

(since ${}^{25}C_3 + {}^{25}C_4 = {}^{26}C_4$)

$= {}^{27}C_4 + {}^{27}C_3 + {}^{28}C_3 + {}^{29}C_3$

$= {}^{28}C_3 + {}^{28}C_3 + {}^{29}C_3$

$= {}^{29}C_4 + {}^{29}C_3$

$= {}^{30}C_4$

39. (i) If ${}^{12}C_{5+1} = {}^{12}C_{2s-5}$**, then find s.**

(ii) If ${}^{n}C_{21} = {}^{n}C_{27}$**, then find** ${}^{49}C_n$

Sol: (i) ${}^{12}C_{s+1} = {}^{12}C_{2s-5} \Rightarrow$ either $s+1 = 2s-5$ or $s+1+2s-5 = 12$

$\Rightarrow s = 6$ *or* $3s = 16$

$\therefore s = 6$ (since 's' is non negative integer)

(ii) ${}^{n}C_{21} = {}^{n}C_{27} \Rightarrow n = 21 + 27 = 48$

Hence ${}^{49}C_n = {}^{49}C_{48} = {}^{49}C_1 = 49$

40. If there are 5 alike pens, 6 alike pencils and 7 alike erasers, find the number of ways of selecting any number of (one or more) things out of them.

Sol: The required number of ways

$= (5+1)(6+1)(7+1) - 1 = 335$

41. Find the number of positive division of 1080.

Sol: $1080 = 2^3 \times 3^3 \times 5^1$

$\therefore$ The number of positive divisions of 1080

$= (3+1)(3+1)(1+1)$

$= 4 \times 4 \times 2 = 32$