PERMUTATIONS AND COMBINATIONS -1

IMPORTANT POINTS

1. **Fundamental principle:** If a work can be performed in m different ways and another work can be performed in n different ways, then the two works simultaneously can be performed in mn different ways. If n is a non - negative integer then i) 0 ! =- 1

2. If n is a non-negative integer then (i) 0! = 1

   (ii) n! = n (n-1) ! if n > 0

3. **The number of permutations of n dissimilar things taken 'r' at a time is denoted by** ${}^{n}P_{r}$ **and** ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ *for* $0 \le r \le n$

4. **If n, r positive integers and** $r \le n$**, then**

   (i) ${}^{n}P_{r} = n\,{}^{n-1}P_{r-1}$ *if* $r \ge 1$

   (ii) ${}^{n}P_{r} = n(n-1)^{(n-2)}P_{(r-1)}$ *if* $r \ge 2$

   (iii) ${}^{n}P_{r} = {}^{(n-1)}P_{r} + r.{}^{(n-1)}P_{(r-1)}$

5. The number of injections that can be defined from set A into set B is ${}^{n(B)}P_{n(A)}$ $n(A) \le n(B)$

6. The number of bijections that can be defined from set A into set B having same number of elements with A is $n(A)!$

7. The number of functions that can be defined from set A into set b in $\left[n(B)\right]^{n(A)}$

8. In the above, if 'O' is one among the given n digits, then the sum is

   ${}^{(n-1)}P_{(r-1)} \times$ sum of the digits ${}^{(n-1)}P_{(r-1)} \times$ (r times)

   ${}^{(n-2)}P_{(r-2)} \times$ sum of the digits $\times 111.....1[(r-1)$ times]

9. The number of permutations of n dissimilar things taken r at a time when repetitions are allowed any number of times is n^r

10. The number of circular permutations of n dissimilar things is $(n-1)!$

11. In the case of hanging type circular permutations like garlands of flowers, chain of bead etc., the number of circular permutations of n things is $\frac{1}{2}\left[(n-1)!\right]$

12. If in the given n things, p alike things are of one kind, q alike things are of the second kind, r alike things are of the third kind and the rest are dissimilar, then the number of permutations (of these n things) is $\frac{n!}{p!q!r!}$

13. The number of combinations of n things taken r at a time is denoted by ${}^{n}C_{r}$ and

    ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$ *for* $0 \le r \le n$

14. If n, r are integers and $0 \le n$ then $n_{C_r} = n_{C_{n-r}}$

15. Let n, r, s are integers and $0 \le r \le n, 0 \le s \le n$

    If ${}^{n}C_{r} = {}^{n}C_{s}$ then $r = s \; or \; r + s = n$

16. The number of ways of dividing 'm + n' things $(m \ne n)$ into two groups containing m, n things $(m+c)_{C_m} = (m+n)C_n = \frac{(m+n)!}{m!n!}$

17. The number of ways of dividing $(m+n+\rho)$ things (m, n, ρ are distinct) into 3 groups of m, n, ρ things is $\frac{(m+n+\rho)!}{m!\,n!\,\rho!}$

18. The number of ways of dividing mn things into m equal groups is $\frac{(mn)!}{(n!)^m m!}$

19. The number of ways if distributing mn things equally to m persons is $\frac{(mn)!}{(n!)^m}$

20. If p alike things are one kind, q alike things are of the second kind, and r alike things are of the third kind, then the number of ways of selecting one or more things out of them $(p+1)(q+1)(r+1)-1$

21. If m is a positive integer and $m = p_1^{\alpha}$, $p_1^{\alpha_2}$ ..... $p_k^{\alpha_k}$ where $p_1, p_2 ..... p_k$ are distinct primes and $\alpha_1, \alpha_2, ... \alpha_k$ are non-negative integers, then the number of divisors of m is $(\alpha_1 + 1)\ (\alpha_2 + 1) .....(\alpha_k + 1)$. [This includes 1 and m]

22. The total number of combinations of n different things taken any number of times is 2^n

23. The total number of combinations of n different things taken one or more at a time is



24. The number of diagonals in a regular polygon of n sides is $\frac{n(n-3)}{2}$

EXERCISE - 4(a)

1. If $^nP_3 = 1320$, find n,

Sol: Hint : $^nP_r = \frac{n!}{(n-r)!}$

$= n(n-1)(n-2) .....(n-r+1)$

$\because {}^nP_3 = 1320$

$= 10 \times 132$

$= 10 \times 12 \times 11$

$= 12 \times 11 \times 10 = {}^{12}P_3$

$\therefore n = 12$

2. If $^nP_7 = 42.\ ^nP_5$, find n

Sol:

$^nP_7 = 42\ .\ ^np_5$

$n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)$

$= 42\ .n(n-1)(n-2)(n-3)(n-4)$

$\Rightarrow (n-5)(n-6)=42$

$\Rightarrow (n-5)(n-6)=7\times 6$

$\Rightarrow n-5=7 \; or \; n-6=6$

$\therefore n=12$

3. **If** ${}^{(n+1)}P_5 : {}^{n}P_6 = 2:7$ **find n**

Sol:

$\Rightarrow \frac{(n+1)_{p_5}}{n_{p_6}}=\frac{2}{7}$

$\Rightarrow \frac{(n+1)n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)(n-5)}=\frac{2}{7}$

$\Rightarrow 7(n+1)=2(n-4)(n-5)$

$\Rightarrow 7n+7=2n^2-18n+40$

$\Rightarrow 2n^2-25n+33=0$

$\Rightarrow 2n(n-1)-3(n-11)=0$

$\Rightarrow (n-11)(2n-3)=0$

$\Rightarrow n=11 \; or \; \frac{3}{2}$

Since n is a positive integer, n = 11

4. **If** ${}^{12}P_5+5.{}^{12}P_4={}^{13}P_{r^1}$ **and r**

Sol: We have

${}^{(n-1)}P_r + r.{}^{(n-1)}P_{(r-1)} = {}^{n}P_r \; and \; r \le n$

${}^{12}P_5+5.{}^{12}P_4={}^{13}P_5={}^{13}P_r$ (given)

$\Rightarrow r = 5$

5. **If** ${}^{18}P_{r-1} : {}^{17}P_{r-1} = 9:7$ **, find r**

Sol: ${}^{18}P_{r-1} : {}^{17}P_{r-1} = 9:7$

$\Rightarrow \frac{{}^{18}P_{r-1}}{{}^{17}P_{r-1}} = \frac{9}{7}$

$\Rightarrow \frac{18!}{\left[18-(r-1)\right]!} \times \frac{\left[17-(r-1)\right]!}{17!} = \frac{9}{7}$

$\Rightarrow \frac{18!}{(19-r)!} \frac{(18-r)!}{17!} = \frac{9}{7}$

$\Rightarrow \frac{18 \times 17!(18-r)!}{(19-r)(18-r)!17!} = \frac{9}{7}$

$\Rightarrow 18 \times 7 = 9(19-r)$

$\Rightarrow 14 = 19 - r$

$\therefore r = 19 - 14 = 5$

6. **A man has 4 sons and there are 5 schools within his reach. In how many ways can he admit his sons in the schools so that no two of them will be in the same school.**

Sol:

The number of ways of admiting 4 sons into 5 schools if no two of them will be in the same $= {}^{5}P_{4} = 5 \times 4 \times 3 \times = 120$

II.

1. **If there are 25 railway stations on a railway line, how many single second class tickets must be printed *so* as to enable a passenger to travel from one station to another. Number of stations on a railway line**

Sol: Number of single second class tickets must be printed so as to enable a passenger to travel from one station to another $= {}^{25}P_{2} = 25 \times 24 = 600$

2. In a class there are 30 students. On the new year day, every student posts a greeting card to all his/her classmates. Find the total number of greeting cards posted by them.

Sol: Number of students in a class are 30,

$\therefore$ Total number of greeting cards posted by every student to all his/her classmates = ${}^{30}P_2$

$= 30 \times 29 = 870$

3. Find the number of ways of arranging the letters of the word TRIANGLE. So that the relative positions of the vowels and consonants are not distributed

Sol:

Vowels – A, E, I, O, U

In a given, word

Number of vowels is 3

Number of consonants is 5

C C V V C C C V

Since the relative positions of the vowels and consonants are not disturbed.

The 3 vowels can be arranged in their relative positions in 3! Ways and the 5 consonants can be arranged in their relative positions in 5! Ways.

$\therefore$ The number of required arrangements = $(3!)(5!) = (6)(120) = 720$

4. Find the sum of all 4 digited numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.

Sol: First method: The number of 4 digited numbers formed by using the digits 0, 2, 4, 7, 8 without repetition

$= {}^5P_4 - {}^4P_3 = 120 - 24 = 96$

Out of these 96 numbers,

${}^4P_3 - {}^3P_2$ numbers contain 2 in units place

${}^4P_3 - {}^3P_2$ numbers contain 2 in tens place

${}^4P_3 - {}^3P_2$ numbers contain 2 in hundreds place

${}^4P_3$ numbers contain 2 in thousands place

$\therefore$ The value obtained by adding 2 in all the numbers

$= ({}^4P_3 - {}^3P_2)\ 2 + ({}^4P_3 - {}^3P_2)\ 20 + ({}^4P_3 - {}^3P_2)\ 200 + {}^4P_3 \times 2000$

$= {}^4P_3\ (2 + 20 + 200 + 2000) - {}^3P_2\ (2 + 20 + 200)$

$-\quad 24 \times (2222) - 6\ (222)$

$= 24x2x1111-6x2x111$

Similarly, the value obtained by adding 4 is $24x4x1111-6x4x111$

The value obtained by adding 7 is

$24x7x1111-6x7x111$

The value obtained by adding 8 is

$24x8x1111-6x8x111$

$\therefore$ The sum of all the numbers

$= (24x2x1111-6x2x111) + (24 \times 4 \times 1111-6x4x111) + (24x7x1111-6x7 \times 111) + (24x8x1111-6x8x111) = 24 \times 111.1 \times (2+4+7+8) - 6 \times 111 \times (2+4+7+8)$

$= 26664\ (21) - 666\ (21$

$= 21\ (26664-666)$

$= 21\ (25998)$

$= 5,45,958.$

5. Find the number of numbers greater than 4000 which can be formed using the digits 0, 2, 4, 6, 8 without repetition.

Sol.

While forming any digit number with the given digits, zero cannot be filled in the first place. We can fill the first place with the remaining 4 digits. The remaining places can be filled with the remaining 4 digits.

All the numbers of 5 digits are greater than 4000. In the 4 digit numbers, the number starting with 4 or 6 or 8 are greater than 4000. The number of 4 digit numbers which begin with 4 or 6 or 8 $= 3 \times {}^4P_3$

$= 3x\ 24 = 72$

The number of 5 digit numbers $= 4 \times 4!$

= 4 x 24 - 96

∴ The number of numbers greater than 4000 is 72 + 96 = 168

6. Find the number of ways of arranging the letters of the word MONDAY so that no vowel occupies even place.

Sol: -

In the word MONDAY there are two vowels, 4 consonants and three even places, three odd places.

Since no vowel occupies even place, the two vowels can be filled in the three odd places in $3P_2$ ways. The 4 consonants can be filled in the remaining 4 places in 4! ways.

∴ The number of required arrangement.

$= {}^3P_2 \times 4! = 6 \times 24 = 144$

(i) Treat the 5 girls as one unit. Then we have 6 boys and 1 unit of girls. They can be arranged in 7! ways. The 5 girls can be arranged among themselves in 5! ways,

∴ The number of ways in which all girls are sit together - (7!) (5!)

(ii) First arrange the 6 boys in a row in 6! ways. Then there are 7 gaps between them as shown below by the letter X.

$\times B \times B \times B \times B \times B \times B \times$

Thus we have 7 gaps and 5 girls. They can be arranged in ${}^7P_5$

∴ The number of arrangements in which no two girls sit together = 6! x ${}^7P_5$

(iii) To arrange the 6 boys and 5 girls sit alternatively, the odd places will be occupied by 6 boys and even places by girls.

B G B G B G B G B G

The 6 boys can be arranged in 6 odd place in 6! ways and 5 girls in the 5 even places in 5!

∴ The number of arrangements in which boys and girls sit alternatively = 6! x 5!

(iv) First arrange the 5 girls in a row in 5! ways. Then there are 6 gaps between them as shown below by the letter X. Thus we have 6 gaps and 6 boys.

$\times G \times G \times G \times G \times G \times$

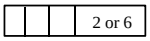
They can be arranged in 6! ways.

∴ The number of arrangements in which no two boys sit together = 5! x 6!

5. Find the number of 4 digited numbers that can be formed using the digits 1, 2, 5 6, 7, How many of them are divisible by (i) 2 (ii) 3 (iii) 4 (iv) 5 (v) 25.

Sol. The number of 4 digited numbers that can be formed using the digits 1, 2, 5, 6, 7 is ${}^5P_4 = 120$.

(i) A number is divisible by 2 when its unit place must be filled with an even digit from among the given integers. This can be done in 2 ways.



Now, the remaining 3 places can be filled with remaining 4 digits in ${}^4P_3 = 4 \times 3 \times 2 = 24$ ways.

$\therefore$ The number of 4 digited by numbers divisible by 2. $2 \times 24 = 48$

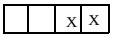
(ii) A number is divisible by 3 of the sum of the digits is divisible by 3. Sum of the given 5 digits $= = 1 + 2 + 5 + 6 + 7 = 21$.

The 4 digits such that their sum is a multiple of 3 from the given digits are 1, 2, 5, 7 (sum is 15)

They can be arranged in 4! Ways and all these 4 digited numbers are divisible by 3.

$\therefore$ The number of 4 digited numbers divisible by 3 = 4! =24

(iii) A number is divisible by 4 only when the last two places (tens and units places) of it is a multiple of 4.

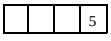


Hence the last two places with one of the following 12,16, 52,56,72 76. Thus the last two places can be filled in 6 ways.

The remaining two places can be filled by remaining 3 digits in ${}^3P_2 = 3 \times 2 = 6$ ways.

$\therefore$ The number of 4 digited numbers divisible by 4= 6 x 6 = 36.

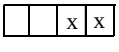
(iv) A number is divisible by 5 when its units place must be filled by 5 from the given integers 2, 5, 6, 7. This can be done in one way



The remaining 3 places can be filled with remaining 4 digits in ${}^4P_3 = 4 \times 3 \times 2$ ways.

$\therefore$ The number of 4 digited numbers divisible by 5 = 1 x 24 = 24

(v) A number is divisible by 25 when its last two places are filled with either 25 or 75.



Thus the last two places can be filled in 2 ways.

The remaining 2 places from the remaining 3 digits can be filled in ${}^3P_2 = 6$ ways.

$\therefore$ The number of 4 digited numbers divisible by $25 = 2 \times 6 = 12$

6. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the ranks of the words (i) REMAST (ii) MASTER.

Sol. (i) The alphabetical order of the letters of the given word is A, E, M, R, S, T

The number of words begin with A is 5! = 120

The number of words begin with E is 5! = 120

The number of words begin with M is 5! =120

The number of words begin with RA is 4! = 24

The number of words begin with REA is 3! = 6

The next word is REMAST .

$\therefore$ Rank of the word REMAST - 3 (120) + 24 + 6 + 1 = 360 + 31 = 391

(ii) The alphabetical order of the letters of the given word is A, E, M, R, S, T The number of words begin with A is 5! = 120

The number of words begin with E is 5! = 120

The number of words begin with MAE is 3! = 6

The number of words begin with MAR is 3! = 6

The number of words begin with MASE is 2! = 2

The number of words begin with MASR is 2! = 2

The next word is MASTER.

$\therefore$ Rank of the word MASTER = 2(120) + 2

(6) + 2(2) + 1

= 240 + 12 + 4 + 1

= 257.

7. If the letters of the permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word LUBER.

Sol:- The alphabetical order of the letters of the given word is B, E, L, R, U

The number of words begin with B is 4! = 24

The number of words begin with E is 4! = 24

The number of words begin with LB is 3! = 6

The number of words begin with LE is 3! = 6

The number of words begin with LR is 3! = 6

The next word is LUBER.

$\therefore$ The rank of the word LUBER

$= 2\ (24) + 3(6) + 1$

$= \ 48 + 18\ 1 = 67$

8. Find the sum of all 5 digited numbers that can be formed using the digits 1, 2, 4, 5, 6 without repetition.

Sol. The number of 4 digited number formed by using the digits 1, 2, 4, 5, 6 without repetition = ${}^5P_4 = 120$

Out of these 120 numbers,

${}^4P_3$ numbers contain 2 in units place

${}^4P_3$ numbers contain 2 in tens place

${}^4P_3$ numbers contain 2 in hundreds place

${}^4P_3$ numbers contain 2 in thousands place

$\therefore$ The value obtained by adding 2 in all the numbers.

$= {}^4P_3 \times 2 + {}^4P_3 \times 20 + {}^4P_3 \times 200 + {}^4P_3 \times 2000$

$= {}^4P_3\ (2 + 20 + 200 + 2000$

$= {}^4P_3 (2222)$

Similarly, the value obtained by adding 1 is ${}^4P_3$ x 1 x 1111

the value obtained by adding 4 is

${}^4P_{3\ x}$ 4 x 1111

the value obtained by adding 5 is

${}^4P_3$ x 5 x 1111

the value obtained by adding 6 is

${}^4P_3$ x 6 x 1111

The sum of all the numbers

$= {}^4P_3$ x 1 x 1111 + ${}^4P_3$ x 2 x 1111 + ${}^4P_3$ x 4 x 1111 + ${}^4P_3$ x 5 x 1111 + ${}^4P_3$ x 6 x 1111

$= {}^4P_3(1111)\ (1 + 2\ + 4\ + 5 + 6)$

= 24 (1111) (18)

= 4,79,952

9. Find the number of ways of arranging the letters of the word ORGANIC so that (i) all vowels come together (ii) no two vowels come together iii) the relative positions of vowels and consonants are not disturbed.

Sol: The word ORGANIC has 3 vowels and 4 consonants. ,

(i) Treat the 3 vowels as one unit. Then we have 4 consonants and 1 unit of vowels. These can be arranged in 5! ways. The 3 vowels can be arranged among themselves in 3! ways.

∴ The number of ways in which all vowels come together = 5! x 3!

$= 120 \times 6$

$= 720$

(ii) First arrange the 4 consonants in 4! ways Then there are 5 gaps between them a: shown below by the letter X

x [C] x [C] x [C] x [C] x

In these 5 gaps, the 3 vowels can be arranged in ${}^{5}P_{3}$ ways.

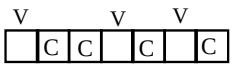
∴ The number of ways in which no two vowels come together

$= 4! \times {}^{5}P_{3}$

$= 24 \times 5 \times 4 \times 3$

= 1440

(iii) The 3 vowels can be arranged in their relative positions in 3! ways and the 4 consonant can be arranged in their relative positions in 4! ways.



∴ The number of required arrangements

= 3! x 4! = 6 x 24 = 144

EXERCISE – 4(b)

1. Find the number of 4 digited numbers that can be formed using the digits 1, 2, 4, 5, 7, 8 when repetition is allowed.

Sol: The number of 4 digited numbers that can be formed using the digits 1, 2, 4, 5, 7, 8 when repetition is allowed $= 6^4 = 1296$

$\boxed{x}\boxed{x}\quad\boxed{x}\boxed{x}$

6 6 6 6

2. Find the number of 5 letter words that can be formed using the letters of the word RHYME if each letter can be used any number of times.

$\boxed{x}\boxed{x}\boxed{x}\quad\boxed{x}\boxed{x}$

5 5 5 5 5

Sol: The number of 5 letter words that can be formed using the letters of the word RHYME if each letter can be used any number of times $= 5^5 = 3125$

3. Find the number of functions from set A containing 5 elements into a set B containing 4 elements

Sol: Set A contains 5 elements

Set B contains 4 elements

Hint : The total number of functions from set A containing m elements to set B containing n elements in n^m.

For the image of each of the 5 elements of the set A has 4 choices.

$\therefore$ The number of functions from set a containing 5 elements into a set B containing 4 elements

$= 4 \times 4 \times ..... 4(5 times)$

$= 4^5 = 1024$

4. Find the number of injections from set A containing 4 elements into a set B containing 6 elements.

Sol: Set A contains 4 elements.

Set B contains 6 elements.

Hint : The number of injections from set A containing m elements to set B containing n element is ${}^nP_m$

Let A = $\{a,, a_2, a_3, a_4\}$

B= $\{b_1\ b_2, b_3\ b_4, b_5, b_6|$

Here the element a_1 has 6 choices for its image, a_2 has 5 choices for its image, a_3 has 4 choices for its image and a_4 has 3 choices for its image.

$\therefore$ **The number of injections from set A containing 4 elements into a set B containing 6 elements**

= 6 x 5 x 4 x 3 or $^6P_4$

= 360

II

1. Find the number of surjections from set A containing 6 elements into a set B containing 2 elements.

Sol: Let $A=\{a_2, a_2, a_3, a_4, a_5, a_6\}\ B=\{x, y\}$

Hint : The number of surjections from set A containing n elements to a set B with 2 elements is $2^n - 2$

The total number of functions from

A to B = 2^5

For a surjection, both the elements of x, y of

B must be in the range,

$\therefore$ A function is not a surjection if the range contains only x (or y). There are only two such functions.

$\therefore$ The number of surjection from A to B

$= 2^6\text{-}2 = 64 - 2 = 62$

2. Find the number of 4 digited telephone numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 with atleast one digit repeated.

Sol. The number of 4 digited numbers formed using the digits 1, 2, 3, 4, 5, 6 when repetition is allowed = 6^4.

The number of 4 digited numbers formed using the digits 1, 2, 3, 4, 5, 6 when repetition is not allowed = $^6P_4$

$\therefore$ The number of 4 digited telephone numbers in which atleast one digit repeated

$= 6^4 - {}^6P_4 \qquad \left[{}^6P_4 = 6\times5\times4\times3\right]$

$= 1296 - 360 = 936$

3. Find the number of 5 digited numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 if each digit can be used any number of times.

Sol. The digit '0' cannot be occupied the first place

x	x	x	x	x
5	6	6	6	6

∴ The number of 5 digited numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 if each digit can be used any number of times = $5 \times 6^4 = 5 \times 1296 = 6480$

4. Find the number of bijections from a set A containing 7 elements onto itself.

Sol: -

Hint : The number of bijections from set A with n elements to net B with same number of elements in A is n !

Let A = [a_1 a_2, a_3, a_4, a_5, a_6, a_7]

For a bijection, the element a_1 has 7 choices

for its image, the element a_2 has 6 choices

for its image, the element a_3 has 5 choices

for its image, the element a_4 has 4 choices

for its image, the element a_5 has 3 choices

for its image, the element a_6 has 2 choices

for its image and the element a_7 has 1 choice

for its image.

∴ The number of bijections from set A with 7 elements onto itself

= 7 x 6 x 5 x 4 x 3 x 2 x 1 or 7!

= 5040

5. Find the number of ways of arranging r things in a line using the given 'n[1] different things in which at least one thing is repeated.

Sol: The number of ways of arranging, r things in a line using the given n different things

(i) when repetition is allowed is n^r

(ii) when repetition is not allowed is ${}^nP_r$

∴ The number of ways of arranging 'r' things in a line sing the 'n' different things in which atleast one thing is repeated = $n^r - {}^nP_r$

6. Find the number of 5 letter words that can be formed using the letters of the word NATURE that begin with N, when

Sol: First we can fill up the first place with N in one way.

N	x	x	x	x
1	6	6	6	6

The remaining 4 places can be filled with any one of the 6 letters in 6 x 6 x 6 x 6 = 6^4 ways.

∴ The number of 5 letter words that can be formed using the letters of the word NATURE that begin with N when repetition is allowed = 1 x 6^4 = 1296

7. Find the number of 5 - digit numbers divisible by 5 that can be formed using the digits 0, 1, *2*, 3, 4, 5 when repetition is allowed.

Sol. The unit place of 5 digited number which can be divisible by 5 using the given digits can be filled by either 0 or 5 in two ways.

The first place can be filled any one of the given digits except '0' in 5 ways. The remaining 3 places can be filled by any one of the given digits in 6 x 6 x 6 ways (∵ repetition is allowed)

x	x	x	x	x
5	6	6	6	2

∴ The number of 5 digited numbers divisible by 5 that can be formed using the given digits when repetition is allowed = 2 x 5 x 6 x 6 x 6 = 2160 ways.

8. Find the number of numbers less than 2000 that can be formed using the digits 1, 2, 3, 4, if repetition is allowed.

Sol. All the single digited numbers, two digited numbers, three digited numbers and the four digited numbers started with 1 are the numbers less than 2000 using the digits 1 , 2, 3,4.

The number of single digited numbers formed using the given digits - 4

x
4

The number of two digited numbers formed using the given digits when repetition is allowed =

4 x 4 = 1 6

x	x
4	4

The number of three digited numbers formed using the given digits = 4x4x4 = 64

x	x	x
4	4	4

The number of 4 digited numbers started with 1 formed using the given digits

= 4x4x4= 64

1 [x] [x] [x]
  4 4 4

∴ The total number of numbers less than 2000 that can be formed using the digits 1, 2, 3, 4 if repetition is allowed

= 4 + 1 6 + 64 + 64 = 148

III

1. 9 different letters of an alphabet are given. Find the number of 4 letter words that can be formed using these 9 letters which have (i) no letter repeated (ii) atleast one letter repeated.

Sol. The number of 4 letter words can be formed using the 9 different letters of an alphabet when repetition is allowed = 9^4

(i) The number of 4 letter words can be formed using the 9 different letters of an alphabet in which no letter is repeated = $^9P_4$

= 9 x 8 x 7 x 6 = 3024

(ii) The number of 4 letter words can be formed using the 9 different letters of an alphabet in which atleast one letter repeated - 9^4 - $^9P_4$

= 6561 - 3024 = 3537.

2. **Find the number of 4 digited even numbers that can be formed using the digits 0, 2, 5, 7, 8 when repetition is allowed.**

Sol: First place can be filled by either 2 or 5 or 7 or 8 in 4 ways.

[x] [x] [x] [x]
4 5 5 3

Second place can be filled by any one of the given digits in 5 ways.

Third place can be filled by any one of the given digits in 5 ways.

Last place (or units place) can be filled by either 0 or 2 or 8 in 3 ways.

∴ The number of 4 digited even numbers that can be formed using the digits 0, 2, 5, 7, 8 when repetition is allowed = 4 x 5 x 5 x 3 = 300

3. Find the number of 5 digited numbers that can be formed using the digits 0, 1, 2, 3, 4 that are divisible by 4 when repetition is allowed.

Sol: Since a number is divisible by 4, the last two places should be filled [x][x] [x][x][x]
4 5 5 5 8

with one of the 00, 04, 12, 20, 24, 32, 40, 44 = 8 ways

The first place can be filled any one of the given digits except '0' in 4 ways. The remaining 2 places can be filled by any one the given digits in 5 x 5 ways.

$\therefore$ The number of 5 digited numbers that can be formed using the digits 0, 1, 2, 3, 4 that are divisible by 4 when repetition is allowed = $4 \times 5^2 \times 8 = 800$

4. **Find the number of 4 digited numbers that can be formed using the digits 0, 1, 2, *3*, 4, 5 which are divisible by 6 when repetition of the digits is allowed.**

Sol. The first place of the number can be filled by any one of the given digits except '0' in 5 ways. The 2nd and 3rd places can be filled by any one of the given 6 digits in 6^2 ways.

x	x	x	x
5	6	6	1

After filling up the first 3 places, if we fill the units place with the given 6 digits, we get 6 consecutive positive integers. Out of these 6 consecutive integers exactly one will be divisible by '6'. Hence the units place can be filled in oneway.

$\therefore$ The number of 4 digited numbers formed using the given digits which are divisible by 6 when repetition is allowed. = $5 \times 6^2 \times 1 = 180$

EXERCISE – 4(C)

1. Find the number of ways of arranging 7 persons around a circle.

Sol: Number of persons, n = 7

$\therefore$ The number of ways of arranging 7 persons around a circle = (n - 1)! = 6! = 720

2. Find the number of ways of arranging the chief minister and 10 cabinet ministers at a circular table so that the chief minister always sit in a particular seat.

Sol: Total number of persons = 11

The chief minister can be occupied a particular seat in one way and the remaining 10 seats can be occupied by the 10 cabinet ministers in (10)! ways.

$\therefore$ The number of required arrangements = (10)! X 1 = (10)! = 36,28,800

The number of ways of preparing a chain with 6 different coloured beads.

$$= \frac{1}{2}(6-1)!$$

$$= \frac{1}{2} \times 5! = \frac{1}{2} \times 120 = 60$$

II.

1. Find the number of ways of arranging 4 boys and 3 girls around a circle so that all the girls sit together.

Sol: Treat ail the 3 girls as one unit. Then we have 4 boys and 1 unit of girls. They can be arranged around a circle in 4! ways. Now, the 3 girls can be arranged among themselves in 3! ways.

$\therefore$ The number of required arrangements

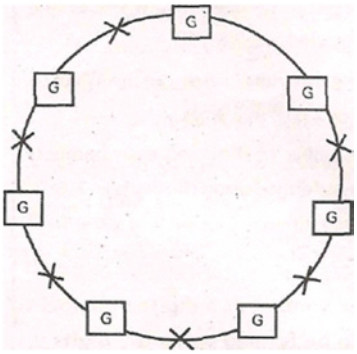
$= 4! \times 3!$

$= 24 \times 6 = 144$

2. Find the number of ways of arranging 7 gents and 4 ladies around a circular table if no two ladies wish to sit together.

Sol: - First arrange the 7 gents around a circular table in 6! ways.

Then we can find 7 gaps between them. The 4 ladies can be arranged in these 7 gaps in ${}^{7}P_{4}$ ways.



$\therefore$ The number of required arrangements

$= 6! \times {}^{7}P_{4}$

$= 720 \times 7 \times 6 \times 5 \times 4$

$= 6,04,800$

3. Find the number of ways of arranging 7 guests and a host around a circle, if 2 particular guests wish to sit on either side of the host.

Sol. Number of guests = 7

Treat the two particular guests along the host as one unit. Then we have 5 guests and one unit of 2 particular guests along the host. They can be arranged around a circle in 5! ways. The two particular guests can be arranged on either side of the host in 2 ! ways. .-. Number of required arrangements

$= 5! \times 2! = 120 \times 2 = 240$

4. Find the number of ways of preparing a garland with 3 yellow, 4 white and 2 red roses of different sizes such that the two red roses come together.

Sol. Treat that 2nd roses of different sizes as one unit. Then we have 3 yellow, 4 white and one unit of red roses. Then they can be arranged in garland form in $\frac{1}{2}(8-1)! = \frac{1}{2}(7!)$ ways.

Now 2 red roses in one unit can be arranged among themselves in 2! ways.

$\therefore$ The number of ways of preparing a garland

$$= \frac{1}{2}(7!) \times 2! = \frac{1}{2} \times 5040 \times 2 = 5040$$

III.

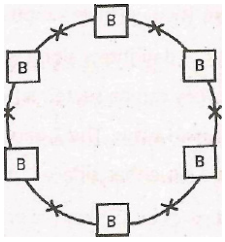
1. Find the number of ways of arranging 6 boys and 6 girls around a circular table so that (i) all the girls sit together (ii) no two girls sit together iii) boys and girls sit alternatively.

Sol. (i) Treat all the 6 girls as one unit. Then we have 6 boys and 1 unit of girls. They can be arranged around a circular table in 6! ways. Now, the 6 girls can be arranged among themselves in 6! ways.

$\therefore$ The number of required arrangements

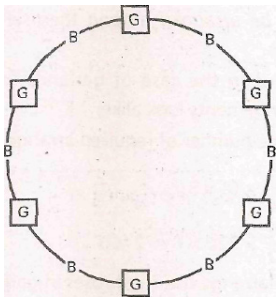
= 6! x 6! = 720 x 720 = 5,18,400

(ii) First arrange the 6 boys around a circular table in 5! ways. Then we can find 6 gaps between them. The 6 girls can be arranged in these 6 gaps in 6! ways.



$\therefore$ The number of required arrangements = 5! x 6!= 120x720 = 86,400

(iii) Here the number of girls and number of boys are same.



Hence the arrangements of boys and girls sit alternatively in same as the arrangements of no two girls sit together or arrangements of no two boys sit together.

First arrange the 6 girls around a circle table in 5! ways. Then we can find 6 gaps between them. The 6 boys can be arranged in these 6 gaps in 6! ways.

$\therefore$ The number of required arrangements

= 5! X 6! = 12 0x 720 = 86,400

2. Find the number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland. In how many of them (i) all the yellow roses are together (ii) no two yellow roses are together

Hint : The number of circular permutations like the garlands of flowers, chains of beads etc., of n things

$= \frac{1}{2}(n-1)!$

Sol: Total number of roses = 6 + 3 = 9

$\therefore$ The number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland

$= \frac{1}{2} \times (9-1)! = \frac{1}{2} \times 8!$

$= \frac{1}{2} \times 40.320 = 20,160$

(i) Treat all the 3 yellow roses as one unit. Then we have 6 red roses and one unit of yellow roses. They can be arranged in garland form in (7 - 1)! = 6! ways. Now, the 3 yellow roses can be arranged among themselves in 3! ways.

But in the case of garlands, clockwise arrangements look alike.

$\therefore$ The number of required arrangements.

$= \frac{1}{2} \times 6! \times 3!$

$= \frac{1}{2} \times 720 \times 6 = 2160$

(ii) First arrange the 6 red roses in garland form in 5! ways. Then we can find 6 gaps between them. The 3 yellow roses can be arranged in these 6 gaps in ${}^6P_3$ ways.

But in the case of garlands, clock-wise and anti-clockwise arrangements look alike.

$\therefore$ The number of required arrangements

$= \frac{1}{2} \times 5! \times {}^6P_3$

$= \frac{1}{2} \times 120 \times 6 \times 5 \times 4 = 7200$

3. A round table conference is attended by 3 Indians, 3 Chinese, 3 Canadians and 2 Americans. Find the number of ways of arranging them at the round table so that the delegates belonging to same country sit together.

Sol: Since the delegates belonging to the same country sit together, first arrange the 4 countries in a round table in 3! ways. Now, 3 Indians can be arranged among themselves in 3! ways,

3 Chinese can be arranged among themselves in 3! ways,

3 Canadians can be arranged among themselves in 3! ways, and

2 Americans can be arranged among themselves in 2! ways.

$\therefore$ The number of required arrangements

– 3! x 3! x 3! x 3! x 2i

= 6 x 6 x 6 x 6 x 2 = 2592

4. A chain of beads is to be prepared using 6 different red coloured beads and 3 different blue coloured beads. In how many ways can this be done so that no two blue coloured beads come together ?

Sol: First arrange the 6 red coloured beads in the form of chain of beads in (6- 1)! = 5! ways. Then there are 6 gaps between them, The 3 blue coloured beads can be arranged in these 6 gaps in $6P_3$ ways.

Then the total number of circular permutations = 5! x $6P_3$

But in case of chain of beads, clock-wise and anti-clockwise arrangements look alike.

$\therefore$ The number of required arrangements

$= \frac{1}{2} \times 5! \times {}^{6}P_{3}$

$= \frac{1}{2} \times 120 \times 6 \times 5 \times 4 = 7200$

5. A family consists of a father, a mother, 2 daughters and 2 sons. In how many different ways can they sit at a round table, if the 2 daughters wish to sit on either side of the father?

Sol: Total number of persons in a family = 6

Treat the 2 daughters along with a father as one unit. Then we have a mother, 2 sons and one unit of daughters along with father in a family. They can be seated around a table in (4 - 1)! =3! ways. The 2 daughters can be arranged an either side of the father in 2! ways.

$\therefore$ Number of required arrangements

$= 3! \times 2! = 6 \times 2 = 12$

EXERCISE – 4(d)

I

1. Find the number of ways of arranging the letters of the word.

(i) INDEPENDENCE

(ii) MATHEMATICS

(iii) SINGING **(iv) PERMUTATION**

(iv) COMBINATION **(vi) INTERMEDIATE**

Sol: (i) The word INDEPENDENCE contains 12 letters in which there are 3 N's are alike, 2 D's are alike, 4 E's are alike and rest are different,

$\therefore$ The number of required arrangements

$= \frac{(12)!}{4!3!2!}$

(ii) The word MATHEMATICS contains 11 letters in which there are 2 M's are alike, 2 A's are alike, 2 T's are alike and rest are different.

$\therefore$ The number of required arrangements

$= \frac{(11)!}{2!2!2!}$

(iii) The word SINGING contains 7 letters in which there are 2 I's are alike, 2 N's are alike, 2 G's are alike and rest is different.

$\therefore$ The number of required arrangements $= \frac{7!}{2! \times 2! \times 2!}$

(iv) The word PERMUTATION contains 11 letters in which there are 2 T's are alike and rest are different $\therefore$ The number of required arrangements $= \frac{(11)!}{2!}$

(v) The word COMBINATION contains 11 letters in which there are 2 O's are alike, 2 I's are alike, 2 N's are alike and rest are different.

$\therefore$ The number of required arrangements

$= \frac{11!}{2!2!2!}$

(vi) The word INTERMEDIATE contains 12 letters in which there are 2 I's are alike, 2 T's are alike , 3 E's are alike and rest are different,

$\therefore$ The number of required arrangements

$= \frac{(12)!}{2!2!3!}$

2. Find the number of different words that can be formed using $4\,As, 3\,B's, 2C's$ **and one D.**

Sol: Total number of given letters are 10 in which 4 A's are alike of one kind, 3 B's are alike of second kind, 2 C's are alike of third kind and rest of the letter D is different

$\therefore$ The number of required arrangements

$= \frac{(10)!}{4!3!2!}$

3. Find the number of 7 digited numbers that can be formed using *2,2,2,3,*3,4,4.

Sol. In the given 7 digits, there are three 2 's, two 3's and two 4's.

$\therefore$ The number of 7 digited numbers that can be formed using the given digits

$$= \frac{7!}{3!2!2!}$$

II

I. Find the number of 4 letter words that can be formed using the letters of the word RAMANA

Sol: The given word RAMANA has 6 letters in which there are 3 A's are alike and rest are different. Using these 6 letters, 3 cases arises to form 4 letter words.

Case I: All different letters R, A, M, N Number of 4 letter words formed - 4! = 24

Case II: Two like letters A, A and two out of R, M, N

The two different letters can be chosen from 3 letters in $3C_2 = 3$ ways.

$\therefore$ Number of 4 letters word formed $= 3 \times \frac{4!}{2!}$

$= 3 \times 12 = 36$

Case III : Three like letters A, A, A and one out of R, M, N.

One letter can be chosen from 3 different letters in $3C_1 = 3$ ways.

$\therefore$ Number of 4 letter words formed

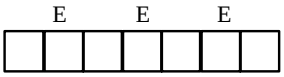
$= 3 \times \frac{4!}{3!} = 3 \times 4 = 12$

$\therefore$ Total number of 4 letter words formed from the word RAMANA = 24 + 36 + 12 = 72

2. How many numbers can be formed using all the digits 1, *2*, 3, 4, 3, *2*, 1 such that even digits always occupy even places?

Sol. In the given 7 digits, there are two 1's, two 2's, two 3's and one 4.

The 3 even places can be occupied by the even digits 2, 4, 2, in $\frac{3!}{2!}$. (Even place is shown by E)



The remained odd places can be occupied by the odd digits 1,3,3,1 in $\frac{4!}{2!2!}$ ways.

$\therefore$ The number of required arrangements

$= \frac{3!}{2!} \times \frac{4!}{2! \times 2!} = 3 \times 6 = 18$

3. In a library, there are 6 copies of one book, 4 copies each of two different books 5 copies each of three different books and 3 copies each of two different books. Find the number of ways of arranging all the books in a shelf in a single row.

Sol. Total number of books in a library are 6 + (4 x 2) + (5 x 3) + (3 x 2) = 35

$\therefore$ The number of required arrangements

$$= \frac{(35)!}{6!(4!)^2(5!)^3(3!)^2}$$

4. A book store has 'm' copies each, 'n' different books. Find the number of ways of arranging the books in a shelf in a single row.

Sol. Total number of books in a book store are

$= m \times n = mn$

$\therefore$ The number of required arrangements

$$= \frac{(mn)!}{(m!)^n}$$

5. Find the number of 5-digit numbers that can be formed using all the digits 0,1,1, 2,3.

Sol. 'O' can also be taken as one digit, the number of 5 digited number formed $= \frac{5!}{2!} = 60$

(Among them, the numer that starts with zero is only 4 digit number. The number of 4! numbers start with zero $= \frac{4!}{2!} = 12$

Hence the number of 5 digit numbers that can be formed by using all the given digits = 60 - 12 = 48

6. In how many ways can the letters of the word CHEESE be arranged so that no two E's come together?

Sol. The given word contains 6 letters in which one C, one H, 3 E's and one S.

Since no two E's come together, first arrange the remaining 3 letters in 3! ways. Then we can find 4 gaps between them. The 3 E's can be arranged in these 4 gaps in $\frac{{}^4P_3}{3!}$

$\therefore$ The number of required arrangements

$=$ **3! X 4= 24**

7. There are S copies of 4 different books. Find the number of ways of arranging these books in a shelf in a single row.

Sol. Total number of books - 4 x 5 = 20

$\therefore$ The required number of arrangements

$$= \frac{(20)!}{5!5!5!5!} = \frac{(20)!}{(5!)^4}$$

III.

1. Find the number of ways of arranging the letters of the word ASSOCIATIONS. In how many of them (i) all the three S's come together ii) The two A's do not come together.

Hint : The number of linear permutations of 'n' things in which 'p' alike things of one kind, 'q' alike things of 2nd kind, 'r' alike things of 3rd kind and the rest are different is $\frac{n!}{p!q!r!}$

Sol: - The given word ASSOCIATIONS has 12 letters in which there are 2 A's are alike, 3 S's are alike, 2 O's are alike 2 1's are alike and rest are different.

$\therefore$ They can be arranged $= \frac{(12)!}{2!3!2!2!}$

(i) Treat the 3 S's as one unit. Then we have 9 + 1 = 1 0 entities in which there are 2A's are alike, 20's are alike, 2 1's are alike and rest are different.

They can be arranged in $\frac{(10)!}{2!2!2!}$ ways

The 3 S's among themselves can be arranged in $\frac{3!}{3!} = 1$ way.

$\therefore$ The number of required arrangements

$$= \frac{(10)!}{2!2!2!}$$

(ii) Since 2 A's do not come together, first arrange the remaining 1 0 letters in which there are 3 S's are alike, 2 O's are alike 2 I's are alike and rest are different in $\frac{(10)!}{3!\,2!\,2!}$ ways. Then we can find 11 gaps between them. The 2 A's can be arranged in these 11 gaps in $\frac{^{11}P_2}{2!}$ ways.

$$= \frac{(10)!}{3!\,2!\,2!} \times \frac{^{11}P_2}{2!}$$

$\therefore$ The number of required arrangements

2. Find the number of ways of arranging the letters of the word ARRANGE so that (i) the 2 R's come together (ii) A occurs at the beginning and at the end (iii) relative positions of the vowels and consonants are not disturbed.

Sol: The given word ARRANGE has 7 letters in which 2 A's are alike, 2R's are alike and rest are different.

(i) Treat the 2 R's as one unit. Then we have $5 + 1 = 6$ entities in which there are 2 A's are alike and rest are different. They can be arranged in $\frac{6!}{2!} = 360$ ways. The 2R's among themselves can be arranged in $\frac{2!}{2!} = 1$ ways.

$\therefore$ The number of required arrangements = 360 x 1 = 360

(ii) Since A occurs at the beginning and at the end, the remaining 5 letters in which there are 2 R's, are alike and rest are different can be arranged in the remaining 5 places in $\frac{5!}{2!}$ ways. A--------- A

$\therefore$ Required number of arrangements

$$= \frac{5!}{2!} = 60$$

(iii) In the word ARRANGE, there are 3 vowels of which there are 2 A's are alike and one is different and there are 4 consonants of which 2 R's are alike and rest are different.

V C C V C C V

The positions originally occupied by vowels must be occupied by vowels among themselves in $\frac{3!}{2!}$ ways and those occupied by consonants must be occupied by consonants among themselves in $\frac{4!}{2!}$

$\therefore$ The number of required arrangements

$$= \frac{3!}{2!} \times \frac{4!}{2!} = 3! \times 12 = 36$$

3. Find the number of ways of arranging letters of the word MISSING so that two S's are together and the two i's are together

Sol. In the given word MISSING contains 7 letters in which there are 2 *I's* are alike, 2 S's are alike and rest are different.

Treat the 2 S's as one unit and 2 I's as one unit. Then we have 3 + 1 + 1 = 5 entities.

These can be arranged in 5! ways. The 2 S's can be arranged among themselves in $\frac{2!}{2!} = 1$ ways and the 2 I's can be arranged among themselves in $\frac{2!}{2!} = 1$ ways

$\therefore$ The number of required arrangements

$= 5!$ X 1 x 1 = 120

4. How many ways can the letters of the word ENGINEERING be arranged so that the 3 N's come together but the 3 E's do not come together.

Sol. Given word contains 11 letters of which 3 E's, 3 N's, 2 G's, 2 I's and one R.

Treat all the 3 N's as one unit. Then the number of arrangements in which all the 3 N's are come together.

$$= \frac{9!}{3!2!2!} \times \frac{3!}{3!} = \frac{9!}{3!2!2!}$$

Treat all the 3 N's as one unit and 3 E's as one unit. Then the number of arrangements in which 3 N's come together and 3 E's come together and 3 E's come together $= \frac{7!}{2!2!}$

$\therefore$ The number of arrangements in which 3 N's come together and 3 E's do not come together $= \frac{9!}{3!2!2!} = \frac{7!}{2!2!}$

$$= 15120 - 1260 = 13860$$

5. How many ways can word BANANA be arranged so that (i) all the A's come together (ii) no two A's come together.

Sol: Given word contains 6 letters in which 3 A's are alike, 2 N's are alike and one B.

(i) Treat the 3 A's as one unit. Then we have $3 + 1 = 4$ entities in which 2 N's are alike and rest are different. They can be arranged in $\frac{4!}{2!}$ ways

The 3 A's among themselves can be arranged in $\frac{3!}{3!} = 1$ way.

$\therefore$ The number of required arrangements

$$= \frac{4!}{2!} \times 1 = 12$$

(ii) Since no two A's come together, first arrange the remaining 3 letters in which 2 N's and one is different in $\frac{3!}{2!} = 3$ ways. Then we can 4gaps between them. The 3 A's can be arranged in these 4 gaps in $\frac{4P_3}{3!} = 4$ ways.

$\therefore$ The number of required arrangements

$= 3x4 = 12$

6. If the letters of the word AJANTA are permited in all possible ways and the words then formed are arranged in dictionary order. Find the rank of the words i) AJANTA ii) JANATA

Sol: - The dictionary order of the letters of the word AJANTA is

A A A J N T

(i) In the dictionary order first comes that words which begin with the letter A. If we fill the first place with A, we may set the word AJANTA. Second place can be filled with A, the remaining 4 places can be filled in 4! = 24 ways.

On proceeding like this, we get

AA --- = 4 ! = 24

AJAA = 2 ! = 02

AJ ANA----- = 1 = 0 1

AJANTA = 1 = 01

$\therefore$ Rank of the word AJANTA

$= 24 + 02 + 01 + 01 = 28$

(ii) In the dictionary order first comes that words which begin with the letter A. If we fill the first place with A, remaining 5 letters can be arranged in $\frac{5!}{2!}$ ways (since there 2 A's remain)

On proceeding like this, we get

A ------ = $\frac{5!}{2!}$ = 60

JAA- = 3 ! = 6

JANAA - - = 1 = 1

JANATA = 1 = 1

$\therefore$ Rank of the word JANATA is

= 60 + 6 + 1 + 1 = 68.